Nonlinear Tax Incidence and Optimal Taxation in General Equilibrium*

Dominik Sachs LMU Munich

Aleh Tsyvinski Yale University

Nicolas Werquin Toulouse School of Economics

April 17, 2019

Abstract

We study the incidence of nonlinear labor income taxes in an economy with a continuum of endogenous wages. We derive the effects of reforming nonlinearly an arbitrary tax system in closed form, by showing that this problem can be formalized as an integral equation. Our tax incidence formulas are valid both when the underlying assignment of skills to tasks is fixed or endogenous. We show qualitatively and quantitatively that contrary to conventional wisdom, if the tax system is initially suboptimal and progressive, the general-equilibrium "trickle-down" forces raise the benefits of increasing the marginal tax rates on high incomes. We finally derive a parsimonious characterization of optimal taxes.

*We thank Laurence Ales, Costas Arkolakis, Andy Atkeson, Alan Auerbach, Felix Bierbrauer, Carlos Da Costa, Cécile Gaubert, François Geerolf, Austan Goolsbee, Piero Gottardi, Nathan Hendren, James Hines, Claus Kreiner, Tim Lee, Arash Nekoei, Michael Peters, Emmanuel Saez, Florian Scheuer, Hakan Selin, Chris Sleet, Stefanie Stantcheva, Philip Ushchev, Gianluca Violante, Ivan Werning, Sevin Yeltekin and Floris Zoutman for comments and discussions.

Introduction

We study the incidence and the optimal design of nonlinear income taxes in a general equilibrium Mirrlees (1971) economy. Our analysis connects two classical strands of the public finance literature that have so far been somewhat disconnected: the tax incidence literature (Harberger (1962); Kotlikoff and Summers (1987); Fullerton and Metcalf (2002)), and the literature on optimal nonlinear income taxation in partial and general equilibrium (Mirrlees (1971); Stiglitz (1982); Diamond (1998); Saez (2001); Rothschild and Scheuer (2013)). The objective of the tax incidence analysis is to characterize the first-order effects of locally reforming a given, potentially suboptimal, tax system on the distribution of individual wages, labor supplies, and utilities, as well as on government revenue and social welfare. We provide closed-form analytical formulas for the incidence of any tax reform in the environment with arbitrarily nonlinear taxes and a continuum of endogenous wages. A characterization of optimal taxes in general equilibrium is then obtained immediately, by imposing that no tax reform has a positive impact on social welfare.

In our baseline environment, there is a continuum of skills that are imperfectly substitutable in production. The aggregate production function uses as inputs the labor effort of all skills. Agents choose their labor supply optimally given their wage and the tax schedule. The wage, or marginal product of labor, of each skill type is endogenous. Specifically, it is decreasing in the aggregate labor effort of its own skill if the marginal productivity of labor is decreasing, and increasing (resp., decreasing) in the aggregate labor effort of those skills that are complements (resp., substitutes) in production. We then microfound this production structure in an environment with a technology over a continuum of tasks and endogenous assignment of skills to tasks as in Costinot and Vogel (2010) and Ales et al. (2015). Thus our approach is in the sufficient-statistic tradition (see Chetty (2009a)): our formulas hold whether the underlying structure of the assignment is exogenous or endogenous.

For simplicity of exposition, we start by focusing on the incidence of general tax reforms in a model where the utility function is quasilinear. When wages are exogenous, the effects of a tax change on the labor supply of a given agent can be easily derived as a function of the elasticity of labor supply of that agent (Saez, 2001). The key difficulty in general equilibrium is that a change in labor supply in turn impacts the wage, and thus the labor supply, of every other individual. This further affects the wage distribution, which influences labor supply decisions, and so on. Solving for the fixed point in the labor supply adjustment of each agent is the key step in the tax incidence analysis and the primary technical challenge of our paper.

We show that this a priori complex problem of deriving the effects of an arbitrary tax reform on individual labor supply can be mathematically formalized as solving an integral equation. The tools of the theory of integral equations allow us to derive an analytical solution to this problem for a general production function. Furthermore, this solution has a clear economic interpretation. Specifically, it can be represented as a series: its first term is the partial-equilibrium impact of the reform, and each of its subsequent terms captures a successive round of cross-wage feedback effects in general equilibrium. These are expressed in terms of meaningful elasticities for an arbitrary production function, i.e., for any pattern of complementarities between skills in production. Finally, these series reduce to particularly simple closed-form expressions in some cases – namely, when the cross-wage elasticities are multiplicatively separable, which is the case if the technology is CES or in the larger HSA class introduced by Matsuyama and Ushchev (2017). Once we have characterized the incidence of tax reforms on labor supply, it is straightforward to derive the incidence on individual wages and indirect utilities. Importantly, the elasticities we uncover in general equilibrium (in particular, the cross-effect of an increase in labor supply of a given skill on the wage of another skill) can be estimated in the data and are sufficient statistics: conditional on these parameters, our incidence formulas are valid whether the assignment of worker skills to production tasks is fixed as in Heathcote, Storesletten, and Violante (2016), or endogenous as in Costinot and Vogel (2010).

Next, we analyze the aggregate effect of tax reforms on government revenue and social welfare. We derive a general formula that shows that, in response to an increase in the marginal tax rate at a given income level, the standard deadweight loss obtained in the model with exogenous wages is modified to include a general-equilibrium term that depends on the covariation between the shape of the marginal tax rates in the initial economy, and the pattern of production complementarities with the skill where the tax rate has been perturbed. We derive further implications of this general formula by focusing on specific functional forms for the initial economy's tax schedule and the production function. When the elasticity of labor supply is constant, the initial tax schedule has a constant (positive) rate of progressivity, and the production function has a constant elasticity of substitution (CES), we obtain that the benefits of reforming the tax schedule in the direction of higher progressivity are larger (the excess burden is smaller) in general equilibrium than the conventional formula that assumes exogenous wages would predict. We show moreover that this insight continues to hold in the model with endogenous assignment of skills to tasks, and is robust to various extensions of our baseline environment. This result shows that the conventional "trickle-down" forces (Stiglitz (1982), Rothschild and Scheuer (2013)) may imply that higher tax rates on high income are more desirable than in partial equilibrium if the tax system to which the tax reform is applied is initially not optimal and resembles the U.S. tax code.

To understand the intuition for this result, suppose that the government raises the marginal tax rate at a given income level. This disincentivizes the labor supply of agents who initially earn that income, which in turn raises their own wage (since the marginal product of labor is decreasing), and lowers the wage of the skills that are complementary in production. Since the production function has constant returns to scale, Euler's homogeneous function theorem implies that the impact of these wage adjustments on aggregate income is equal to zero, even after labor supplies adjust if the corresponding elasticity is constant. If moreover the tax schedule is linear, so that the marginal tax rate is originally the same for all agents, we immediately obtain that the impact of the reform on government revenue is zero – that is, the general-equilibrium forces have no impact on aggregate government revenue beyond those already obtained assuming exogenous wages. If instead the tax schedule is initially progressive, then an increase in the marginal tax rate on high incomes raises their wage and hence government revenue by a larger amount (since the marginal tax rate is initially higher) than do the equivalent wage losses at all other income levels. In other words, starting from a progressive tax code, the general equilibrium forces raise the revenue gains from further increasing the progressivity of the tax schedule. We finally provide numerical simulations to quantify these results. We find that the gains from raising the marginal tax rates on high incomes are significantly affected by the endogeneity of wages. In the U.S., assuming exogenous wages would imply that 33 percent of the revenue from a given tax increase is lost through behavioral responses; instead, for our preferred calibrations, only 17 percent to 29 percent is lost once the general equilibrium effects are taken into account.

We then consider various generalizations of our baseline model, and show that the methodology we used to analyze our baseline environment extends to more sophisticated frameworks with no further technical difficulties. We first allow for general individual preferences with income effects. Second, we let agents choose their labor supply both on the intensive (hours) and the extensive (participation) margins.

Third, we analyze an economy with several sectors or education levels (Roy model as in Rothschild and Scheuer (2013)), with a continuum of skills within each group (and consequently overlapping wage distributions). For each of these extensions, we derive closed-form tax incidence formulas and show that the main qualitative insights we derived in our baseline model carry over.

Finally, we derive the implications of our analysis regarding the optimal tax schedule. Our tax incidence analysis immediately delivers a general characterization of optimal taxes, by equating the effects of tax reforms on social welfare to zero. In the main body of the paper, we focus on deriving a novel characterization that depends on a parsimonious number of parameters which can be estimated empirically. To do so, we specialize our production function to have a constant elasticity of substitution (CES) between pairs of types. This leads to particularly sharp and transparent theoretical insights. First, we obtain an optimal taxation formula that generalizes those of Diamond (1998) and Saez (2001). There are two key differences between our formula and those derived assuming exogenous wages. First, because of the decreasing marginal productivity of labor, the relevant labor supply elasticity is smaller, implying lower disincentive effects of raising the marginal taxes, and hence higher optimal rates. This is because a higher tax rate reduces labor supply, which in turn raises the wage, and hence the labor supply, of these agents. Second, marginal tax rates should be lower (resp., higher) for agents whose welfare is valued less (resp., more) than average. This is because an increase in the marginal tax rate of a given skill type increases her wage at the expense of all other types. We show that the general equilibrium forces reinforce the U-shaped pattern of optimal taxes obtained by Diamond (1998). We finally extend the closed-form optimal top tax rate formula of Saez (2001) in terms of the labor supply elasticity, the Pareto parameter of the income distribution, and the elasticity of substitution between skills in production.

Related literature. This paper is related to the literature on tax incidence: see, e.g., Harberger (1962) and Shoven and Whalley (1984) for the seminal papers, Hines (2009) for emphasizing the importance of general equilibrium in taxation, and Kotlikoff and Summers (1987) and Fullerton and Metcalf (2002) for comprehensive surveys. Our paper extends this framework to an economy with a continuum of labor inputs with arbitrary nonlinear tax schedules, i.e., we study tax incidence in the workhorse model of optimal nonlinear labor income taxation of Mirrlees (1971); Diamond (1998).

The optimal taxation problem in general equilibrium with arbitrary nonlinear tax instruments has originally been studied by Stiglitz (1982) in a model with two types. The key result of Stiglitz (1982) is that at the optimum tax system, general equilibrium forces lead to a lower (resp., higher) top (resp., bottom) marginal tax rate. In the recent optimal taxation literature, there are two strands that relate to our work. First, a series of important contributions by Scheuer (2014); Rothschild and Scheuer (2013, 2014); Scheuer and Werning (2017), Chen and Rothschild (2015), Ales, Kurnaz, and Sleet (2015), Ales and Sleet (2016), and Ales, Bellofatto, and Wang (2017) form the modern analysis of optimal nonlinear taxes in general equilibrium.[1] Specifically, Rothschild and Scheuer (2013, 2014) generalize Stiglitz (1982) to a setting with N sectors and a continuum of (infinitely substitutable) skills in each sector, leading to a multidimensional screening problem. Ales, Kurnaz, and Sleet (2015) and Ales and Sleet (2016) microfound the production function by incorporating an assignment model into the Mirrlees framework and study the implications of technological change and CEO-firm matching for optimal taxation.

Our baseline model is simpler than those of Rothschild and Scheuer (2013, 2014) and Ales, Kurnaz, and Sleet (2015). In particular, different types earn different wages (there is no overlap in the wage distributions of different types, as opposed to the framework of Rothschild and Scheuer (2013, 2014)), and the assignment of worker skills to tasks involved in production is fixed (in contrast to Ales, Kurnaz, and Sleet (2015)).[2] The first key distinction is that these papers focus on optimal taxation by applying the methods of mechanism design, whereas our study of tax incidence is based on a variational, or "tax reform" approach, introduced by Piketty (1997); Saez (2001, 2002) and extended to several other contexts by, e.g., Kleven, Kreiner, and Saez (2009) and Golosov, Tsyvinski, and Werquin (2014). In this paper we extend these techniques to the general equilibrium framework with endogenous wages. Our use of the variational approach and integral equations allows us to study more generally the incidence of reforming an arbitrary initial tax system in any direction. We show that the tax system to which the reform is applied (say, the U.S. tax code)

[1] Rothstein (2010) studies the desirability of EITC-type tax reforms in a model with heterogenous labor inputs and nonlinear taxation. He only considers own-wage effects, however, and no cross-wage effects. Further he treats intensive margin labor supply responses as occurring along linearized budget constraints.

[2] Finally, our setting is distinct from those of Scheuer and Werning (2016, 2017), whose modeling of the technology is such that the general equilibrium effects cancel out at the optimum tax schedule, so that the formula of Mirrlees (1971) extends to their general production functions. We discuss in detail the difference between our framework and theirs in Appendix A.4.4 .

is a crucial determinant of the direction and size of the general equilibrium effects. Second, our characterization of optimal income tax rates is novel: assuming a simpler production function leads to a parsimonious and transparent formula that generalizes the U-shape result of Diamond (1998), and a closed-form expression for the top tax rate that generalizes that of Saez (2001).[3] Third, we also analyze extensions of our baseline framework to production structures which induce endogenous assignment of skills to tasks as in Sattinger (1975); Teulings (1995); Costinot and Vogel (2010); Ales, Kurnaz, and Sleet (2015), and overlapping wage distribution as in Roy (1951); Rothschild and Scheuer (2013, 2014).[4] Crucially, we show that conditional on the wage elasticities that we introduce, our baseline tax incidence formulas remain identical in these alternative production structures. Therefore our paper differs from those mentioned above in that it is in the sufficient statistic tradition (Saez, 2001; Chetty, 2009b): our main results are valid for several underlying primitive environments.

Finally, our paper is related to the literature that characterizes optimal government policy, within restricted classes of nonlinear tax schedules, in general equilibrium extensions of the continuous-type Mirrleesian framework. Heathcote, Storesletten, and Violante (2016) study optimal tax progressivity in a model where agents face idiosyncratic risk and can invest in their skills. Itskhoki (2008) and Antras, de Gortari, and Itskhoki (2016) characterize the impact of distortionary redistribution of the gains from trade in an open economy. Their production functions are CES with a continuum of skills and restrict the tax schedule to be of the CRP functional form. On the one hand, our model is simpler than their framework as we study a static and closed economy with exogenous skills. On the other hand, for most of our theoretical analysis we do not restrict ourselves to a particular functional form for taxes nor the production function. Our papers share, however, one important goal: to derive simple closed form expressions for the effects of tax reforms in general-equilibrium Mirrleesian environments. Our baseline modeling of the production function, which is the same as that of Heathcote, Storesletten, and Violante (2016), is also motived by an empirical literature that estimates the impact of immigration on the native wage

[3] Our generalization of the optimal top tax rate to the case of endogenous wages is related to Piketty, Saez, and Stantcheva (2014), who extend the Saez (2001) top tax formula to a setting with a compensation bargaining channel using a variational approach. More generally, Rothschild and Scheuer (2016) study optimal taxation in the presence of rent-seeking. In this paper we abstract from such considerations and assume that individuals are paid their marginal productivity.

[4] We also use the calibration of Ales, Kurnaz, and Sleet (2015) in our quantitative analysis with endogenous assignment.

distribution and groups workers according to their position in the wage distribution (Card, 1990; Borjas, Freeman, Katz, DiNardo, and Abowd, 1997; Dustmann, Frattini, and Preston, 2013). The empirical literature on immigration is a useful benchmark because it studies the impact of labor supply shocks of certain skills on relative wages, which is exactly the channel we want to analyze in our tax setting (except that in our model the labor supply shocks are induced by tax reforms). An alternative in the immigration literature is to group workers by education levels (Borjas, 2003; Card, 2009). We fully extend our analysis and results to a production function with different education groups in Sections D.3 and D.4.

This paper is organized as follows. Section 1 describes our framework and defines the relevant elasticity variables. In Section 2 we analyze the incidence of nonlinear tax reforms on individual variables (labor supply, wages, utilities). In Section 3 we derive the effects of tax reforms on aggregate variables (government revenue, social welfare). In Section 4, we calibrate the model and evaluate our main results quantitatively. In Section 5 we analyze various generalizations of our baseline environment. Finally, in Section 6 we derive optimal taxes with a CES production function. The proofs of our formulas and additional results are gathered in the Appendix.

1 The baseline environment

In this section we set up a model to derive our main results most transparently. We extend our analysis to more general environments in Section 5 and Appendix C.2.

1.1 Initial equilibrium

Individuals. There is a continuum of mass 1 of workers indexed by their skill $\theta \in \Theta = [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}_+$, distributed according to the pdf $f(\cdot)$ and cdf $F(\cdot)$. Individual preferences over consumption c and labor supply l are represented by the quasilinear utility function $c - v(l)$, where the disutility of labor $v : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, strictly increasing and strictly convex. An individual with skill θ earns a wage $w(\theta)$ that she takes as given. She chooses her labor supply $l(\theta)$ and earns taxable income $y(\theta) = w(\theta)\, l(\theta)$. Her consumption is equal to $y(\theta) - T(y(\theta))$, where $T : \mathbb{R}_+ \to \mathbb{R}$ is a twice continuously differentiable income tax schedule. Her optimal labor supply choice $l(\theta)$ is the solution to the first-order condition:[5]

$$v'(l(\theta)) = [1 - T'(w(\theta) l(\theta))] w(\theta). \tag{1}$$

We denote by $U(\theta)$ the utility attained by the agent, and by $L(\theta) \equiv l(\theta) f(\theta)$ the total amount of labor supplied by individuals of type θ.

Firms. There is a continuum of mass 1 of identical firms that produce output using the labor of every skill type $\theta \in \Theta$. We posit a constant returns to scale aggregate production function $\mathcal{F}(\mathscr{L})$ over the continuum of labor inputs $\mathscr{L} \equiv \{L(\theta)\}_{s\in\Theta}$.[6] In equilibrium, firms earn no profits and the wage $w(\theta)$ is equal to the marginal productivity of type-θ labor, that is,

$$w(\theta) = \frac{\partial}{\partial L(\theta)} \mathcal{F}(\mathscr{L}). \tag{2}$$

Remark (*Monotonicity*). Without loss of generality we order the skills θ so that the wage function $\theta \mapsto w(\theta)$ is strictly increasing given the tax schedule T.[7] We show in Appendix A.2 that, by the Spence-Mirrlees condition, the pre-tax income function $\theta \mapsto y(\theta)$ is then also strictly increasing. There are therefore one-to-one maps between skills θ, wages $w(\theta)$, and pre-tax incomes $y(\theta)$.[8] We denote by $f_Y(y(\theta)) = (y'(\theta))^{-1} f(\theta)$ the density of incomes and by F_Y the corresponding c.d.f.

Government. The government chooses the twice-continuously differentiable tax function $T : \mathbb{R}_+ \to \mathbb{R}$. Tax revenue is given by

$$\mathcal{R} = \int_\Theta T(y(\theta)) f(\theta) d\theta.$$

[5] The dependence of labor supply on the tax schedule T is left implicit for simplicity. Whenever necessary, we denote the solution to (1) by $l(\theta; T)$.

[6] In Section 1.3 below we provide a microfoundation of this production function. An alternative interpretation of our framework is that different types of workers produce different types of goods that are imperfect substitutes in household consumption (see, e.g., Acemoglu and Autor (2011)).

[7] Moreover, we can w.l.o.g. assume that the skill type Θ is the interval $[0, 1]$ and that the distribution $f(\theta)$ is uniform. In this case, θ indexes the agent's percentile in the wage distribution

[8] When we perturb the tax system, the ordering of wages may generally change. Our analysis does not require that the initial ordering remains unaffected by the tax reforms we consider.

We define the local rate of progressivity[9] of the tax schedule T at income level y as (minus) the elasticity of the retention rate $1 - T'(y)$ with respect to income y,

$$p(y) \equiv -\frac{\partial \ln[1 - T'(y)]}{\partial \ln y} = \frac{yT''(y)}{1 - T'(y)}.$$

In particular, the tax schedule has a constant rate of progressivity (CRP) if

$$T(y) = y - \frac{1-\tau}{1-p} y^{1-p}, \tag{3}$$

for $p < 1$.[10] This tax schedule is linear (resp., progressive, regressive), i.e., the marginal tax rates $T'(y)$ and the average tax rates $T(y)/y$ are constant (resp., increasing, decreasing), if $p = 0$ (resp., $p > 0$, $p < 0$).

Equilibrium. An equilibrium given a tax function T is a schedule of labor supplies $\{l(\theta)\}_{\theta \in \Theta}$, labor demands $\{L(\theta)\}_{\theta \in \Theta}$, and wages $\{w(\theta)\}_{\theta \in \Theta}$ such that equations (1) and (2) hold, the labor markets clear: $L(\theta) = l(\theta) f(\theta)$ for all $\theta \in \Theta$, and the goods market clears: $\mathcal{F}(\mathscr{L}) = \int_\Theta w(\theta) L(\theta) d\theta$.

Examples: CES and HSA production functions. We conclude this section by presenting useful special cases of production functions that we use for some of our results below. The technology is CES if

$$\mathcal{F}(\mathscr{L}) = \left[\int_\Theta a(\theta) (L(\theta))^{\frac{\sigma-1}{\sigma}} d\theta\right]^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

for some constant elasticity of substitution $\sigma \in [0, \infty)$ and technological parameters $a(\cdot) \in \mathbb{R}_+$. The wage schedule is given by $w(\theta) = a(\theta) (L(\theta) / \mathcal{F}(\mathscr{L}))^{-1/\sigma}$. The cases $\sigma = 1$ and $\sigma = 0$ correspond respectively to the Cobb-Douglas and Leontieff production functions, and wages are exogenous if $\sigma = \infty$. More generally, the class of HSA (homothetic demand with a single aggregator) technologies has been introduced by Matsuyama and Ushchev (2017). They are defined non-parametrically as follows:

[9] See Musgrave and Thin (1948).

[10] See, e.g., Bénabou (2002); Heathcote, Storesletten, and Violante (2016).

for all $\theta \in [0,1]$, there exists a function $s(\cdot;\theta) : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$\frac{w(\theta)\,L(\theta)}{\mathscr{F}(\mathscr{L})} \;=\; s\left(\frac{w(\theta)}{\mathcal{A}(\boldsymbol{w})};\theta\right), \tag{5}$$

where we denote $\boldsymbol{w} \equiv \{w(\theta)\}_{\theta\in\Theta}$ and $\mathcal{A}(\boldsymbol{w})$ is the solution to $\int s(\frac{w(\theta)}{\mathcal{A}};\theta)d\theta = 1$, which ensures constant returns to scale.[11] That is, the labor share of output of skill θ is given by a (skill-specific) function of its own wage normalized by the common aggregator $\mathcal{A}$. The HSA class contains as a strict special case the CES production function with $s(x;\theta) = (a(\theta))^{\sigma} x^{1-\sigma}$, as well as technologies with variable elasticities of substitution. Several examples, including the separable Translog cost function, are discussed in Appendix A.4.3 and in Matsuyama and Ushchev (2017).

1.2 Elasticities

In this section we define the elasticity parameters that determine the economy's adjustment to tax reforms. First, it is useful to consider the labor market of a given skill θ in isolation, i.e., to reason in partial equilibrium. We denote by $\varepsilon_w^S(\theta)$ and $\varepsilon_w^D(\theta)$ the elasticities of the labor supply and labor demand curves in this market. Second, in general equilibrium, a perturbation of the labor market for skill θ affects all other markets $\theta' \neq \theta$ through cross-price effects, which we denote by $\gamma(\theta',\theta)$. We proceed to formally define each of these elasticities, starting with the latter.

Cross-wage elasticities. Consider first two distinct labor markets for skills θ and $\theta' \neq \theta$. We define the elasticity of the wage of type θ', $w(\theta')$, with respect to the aggregate labor of type θ, $L(\theta)$, as

$$\gamma(\theta',\theta) \;\equiv\; \frac{\partial \ln w(\theta')}{\partial \ln L(\theta)} \;=\; \frac{L(\theta)\,\mathscr{F}''_{\theta',\theta}(\mathscr{L})}{\mathscr{F}'_{\theta'}(\mathscr{L})}, \quad \forall \theta' \neq \theta \tag{6}$$

where $\mathscr{F}'_{\theta'}$ and $\mathscr{F}''_{\theta',\theta}$ denote the first and second partial derivatives of the production function $\mathscr{F}$ with respect to the labor inputs of types θ' and θ. The cross-wage elasticity between two skills θ, θ' is non-zero if they are imperfect substitutes in production. We say that the skills are Edgeworth complements if $\gamma(\theta',\theta) > 0$ and substitutes if

[11] Matsuyama and Ushchev (2017) provide necessary and sufficient restrictions on the labor share mappings $s(\cdot;\theta)$ such that there exists a well-defined production function $\mathscr{F}$ generating the demand system (5).

$\gamma(\theta', \theta) < 0$.

Labor demand elasticities. Next, consider the labor market for a given skill θ. Note that in the previous paragraph we have only defined $\gamma(\theta', \theta)$ for skills $\theta' \neq \theta$.[12] Now the impact of the aggregate labor effort of skill θ on its own wage, $\frac{\partial \ln w(\theta)}{\partial \ln L(\theta)}$, may be different than its impact on the wage of its close neighbors $\theta' \approx \theta$, $\lim_{\theta' \to \theta} \frac{\partial \ln w(\theta')}{\partial \ln L(\theta)} \equiv \gamma(\theta', \theta)$. That is, the function $\theta' \mapsto \frac{\partial \ln w(\theta')}{\partial \ln L(\theta)}$ may be discontinuous at $\theta' = \theta$. This is the case, e.g., if the production function is CES. We denote by $\gamma(\theta, \theta) \equiv \lim_{\theta' \to \theta} \frac{\partial \ln w(\theta')}{\partial \ln L(\theta)}$ the complementarity between θ and its neighboring skills, and define the inverse elasticity of labor demand for skill θ, $1/\varepsilon_w^D(\theta)$, as the jump between $\frac{\partial \ln w(\theta)}{\partial \ln L(\theta)}$ and $\gamma(\theta, \theta)$. Formally,

$$\frac{\partial \ln w(\theta')}{\partial \ln L(\theta)} \equiv \gamma(\theta', \theta) - \frac{1}{\varepsilon_w^D(\theta)} \delta(\theta' - \theta), \quad \forall (\theta, \theta') \in \Theta^2, \tag{7}$$

where $\delta(\cdot)$ denotes the Dirac delta function. Intuitively, this captures the fact that the marginal productivity of a given skill is a non-constant (e.g., decreasing) function of the aggregate labor of its own type. Note that the tax incidence formulas we will derive are valid whether this discontinuity indeed occurs (e.g., if the production function is CES, see below), or not (e.g., in the microfoundation of Section 1.3 below, where $1/\varepsilon_w^D(\theta) = 0$ for all θ).

Labor supply elasticities. Finally, we define the elasticities of labor supply $l(\theta)$ with respect to the retention rate $r(\theta) \equiv 1 - T'(y(\theta))$ and the wage $w(\theta)$ as[13]

$$\varepsilon_r^S(\theta) \equiv \frac{\partial \ln l(\theta)}{\partial \ln r(\theta)} = \frac{e(\theta)}{1 + p(y(\theta)) e(\theta)}, \quad \varepsilon_w^S(\theta) \equiv \frac{\partial \ln l(\theta)}{\partial \ln w(\theta)} = (1 - p(y(\theta))) \varepsilon_r^S(\theta), \tag{8}$$

where $e(\theta) \equiv \frac{v'(l(\theta))}{l(\theta) v''(l(\theta))}$. These variables differ from the standard elasticity $e(\theta)$ as they account for the fact that if the tax schedule is nonlinear, a change in individual labor supply $l(\theta)$ causes endogenously a change in the marginal tax rate $T'(y(\theta))$

[12] We assume throughout that the map $\theta' \mapsto \gamma(\theta', \theta)$ is continuous on $\Theta \setminus \{\theta\}$.

[13] Since there is a one-to-one map between types θ and incomes $y(\theta)$, one can equivalently index these elasticities by income as $\varepsilon_r^S(y(\theta)) = \varepsilon_r^S(\theta)$. We use these two notations interchangeably in the sequel, and analogously for the labor demand elasticities $\varepsilon_r^D(\theta), \varepsilon_w^D(\theta)$ defined above. On the other hand, the natural change of variables between types θ and incomes $y(\theta)$ for the cross-wage elasticities reads $\gamma(y(\theta_1), y(\theta_2)) = (y'(\theta_2))^{-1} \gamma(\theta_1, \theta_2)$. See Appendix A.1.2 for details.

captured by the rate of progressivity $p\left(y\left(\theta\right)\right)$ of the tax schedule, and hence a further labor supply adjustment $e\left(\theta\right)$. Solving for the fixed point leads to the correction term $p\left(y\left(\theta\right)\right)e\left(\theta\right)$ in the denominator of $\varepsilon_r^S\left(\theta\right)$ and $\varepsilon_w^S\left(\theta\right)$.[14]

Examples: CES and HSA production functions. Consider the CES production function defined by (4). The cross-wage elasticities are then given by $\gamma\left(\theta',\theta\right)=\frac{1}{\sigma}a\left(\theta\right)\left(L\left(\theta\right)/\mathscr{F}\left(\mathscr{L}\right)\right)^{\frac{\sigma-1}{\sigma}}$ and the own-wage elasticities are given by $\varepsilon_w^D\left(\theta\right)=\sigma$ for all θ',θ.[15] Note that $\varepsilon_w^D\left(\theta\right)>0$ is constant and that $\gamma\left(\theta',\theta\right)>0$ does not depend on θ', implying that a change in the labor supply of type θ has the same effect, in percentage terms, on the wage of every type $\theta'\neq\theta$. Consider next the HSA production function defined by (5). We then have $\gamma\left(\theta',\theta\right)=\sigma(\frac{w(\theta')}{\mathcal{A}(\boldsymbol{w})};\theta')^{-1}\frac{w(\theta)L(\theta)}{\mathscr{F}(\mathscr{L})}$, where $\sigma\left(x;\theta\right)\equiv 1-\frac{xs_1'(x;\theta)}{s(x;\theta)}$ denotes the elasticity of the labor share function. Note that these cross-wage elasticities are multiplicatively separable between θ and θ'.

1.3 Microfoundation and sufficient statistics

In this section we argue that the production function we introduced in Section 1.1 can be microfounded as the reduced form of an underlying model of assignment between worker skills and tasks involved in production. Thus our analysis is more general and encompasses both the cases of fixed and endogenous assignment. To show this, we set up a model that, analogous to Ales, Kurnaz, and Sleet (2015), extends Costinot and Vogel (2010) by allowing workers to choose their labor supply endogenously (as in (1)) and the government to tax labor income nonlinearly. All of the technical details are gathered in Appendix A.3.

Formally, the final consumption good is produced using a CES technology over a continuum of *tasks* $\psi\in\Psi$, indexed by their skill intensity (e.g., manual, routine, abstract, etc.). The output of each task is in turn produced linearly using the labor of the skills $\theta\in\Theta$ that are endogenously assigned to this task. Assuming that high-skill workers have a comparative advantage in tasks with high skill intensities, the market clearing conditions for intermediate goods determine a one-to-one matching function $M:\Theta\to\Psi$ between skills and tasks in equilibrium – there is positive assortative

[14] See Appendix A.1.1 for further details. See also Jacquet and Lehmann (2017) and Scheuer and Werning (2017).

[15] Denoting by $\sigma\left(\theta',\theta\right)=-[\frac{\partial\ln\left(w\left(\theta'\right)/w\left(\theta\right)\right)}{\partial\ln\left(L\left(\theta'\right)/L\left(\theta\right)\right)}]^{-1}$ the elasticity of substitution between any two labor inputs, we have $\sigma\left(\theta',\theta\right)=\sigma$ for all $\left(\theta',\theta\right)\in\Theta^2$.

matching. It is straightforward to show that this model admits a reduced-form representation where the production of the final good is performed by a technology over skills. This reduced-form technology inherits the CES structure (4) of the original production function over intermediate tasks, except that the technological coefficients $a(\cdot)$ now depend on the matching function M, and are thus endogenous to taxes.

Crucially, we show that tax reforms affect the equilibrium assignment M *only* through their effect on individual labor supply choices $\{L(\theta)\}_{\theta\in\Theta}$. Mathematically, this is a consequence of the fact that, fixing labor supplies, none of the equations that determine the equilibrium depend explicitly on the tax schedule T.[16] Intuitively, this is because individuals always choose the task that maximizes their net wage. But a tax reform does not alter directly the ranking of wages, as long as marginal tax rates are below 100 percent. Therefore taxes affect the equilibrium sorting of skills only indirectly, through the labor supply responses that they induce. It follows from this result that the technological coefficients $a(\cdot; M)$ of the reduced-form technology described above can be written without loss of generality as $a(\cdot; \{L(\theta)\}_{\theta\in\Theta})$. Substituting these parameters into (4) yields a production function with the general form postulated in Section 1.1, $\mathcal{F}(\{L(\theta)\}_{\theta\in\Theta})$.

This reasoning implies in turn that the cross-wage elasticities $\gamma(\theta', \theta) \propto \frac{\partial^2 \mathcal{F}}{\partial L(\theta) \partial L(\theta')}$ introduced in (6), where $\mathcal{F}$ denotes the reduced-form production function over worker skills, already account for the potential reassignment of workers into new tasks.[17] That is, they represent the impact of an increase in the labor supply of skill θ on the marginal productivity of skill θ', leaving everyone else's labor supply unchanged *and*, if assignment is endogenous, letting workers be reassigned into different tasks (i.e., taking into account the adjustment of the technological coefficients $a(\cdot, \{L(\theta)\}_{\theta\in\Theta})$). Therefore, these cross-wage elasticities are *sufficient statistics*: once expressed as a function of these parameters, the tax incidence formulas that we will derive in Sections 2 and 3 are valid both when the underlying structure of assignment is fixed and when it is endogenous to tax reforms. They can be either estimated in the data directly, or

[16] In particular, total spending on the final good (sum of individual disposable incomes plus government tax revenue) is equal to agents' total gross income, so that the market clearing condition for the final good does not depend directly on the tax schedule.

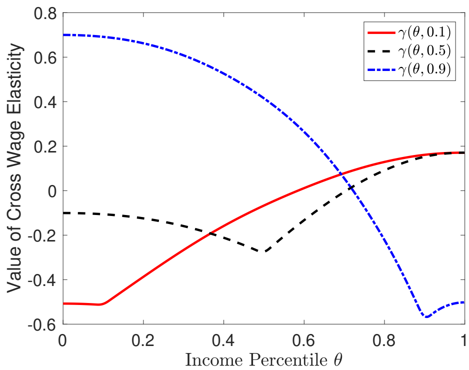
[17] Note moreover that, while in a setting with exogenous assignment the inverse labor demand elasticities $1/\varepsilon_w^D$ are generally non-zero (i.e., there is a discontinuity in the schedule of elasticities $\frac{\partial \ln w(\theta')}{\partial \ln L(\theta)}$ as $\theta' \approx \theta$), instead with costless reassignment such a discontinuity would cause workers to migrate to neighboring tasks, leading to perfectly elastic labor demand curves (i.e., $1/\varepsilon_w^D = 0$). Our tax incidence formulas are naturally valid in both cases.

derived by specifying the underlying structural model (see below).[18]

Graphical representation. We now represent graphically the cross-wage elasticities that arise in the model we just described, assuming a CES production function over tasks, and endogenous (costless) assignment of worker skills to these tasks. We use the calibration of Ales, Kurnaz, and Sleet (2015) who assume a Cobb-Douglas technology over tasks. We compare these elasticities with those obtained in our baseline model of Section 1.1, assuming a CES production function over skills. In this setting we propose two calibrations. The first consists of simply shutting down the endogenous reassignment channel in the previous model while keeping all of the other parameters unchanged, hence assuming a Cobb-Douglas production function over skills ($\sigma = 1$). The second, more relevant, calibration consists of directly estimating a CES production function over labor supplies: we then use the value $\sigma = 3.1$ in (4) estimated by Heathcote, Storesletten, and Violante (2016). The calibration is described in more detail in Section 4.

The left panel of Figure 1 plots the resulting cross-wage elasticities $\gamma(y, y^*)$ in the model of endogenous assignment, for changes in labor supply at the 10^{th}, 50^{th} and 90^{th} percentiles of the wage distribution. They are V-shaped and increasing in the distance $|y - y^*|$. A higher labor effort of agents y^* lowers wages on a non-degenerate interval of incomes around y^* and raises those of much higher or much lower incomes. Note that the shape of the cross-wage elasticities in Figure 1 is similar to those of Teulings (2005). The right panel compares these elasticities with those obtained with a CES production function (4) and fixed assignment, for both values $\sigma = 1$ (dashed line) and $\sigma = 3.1$ (solid line). In this case, the wages of agents y^* decrease, while those of everyone else increase, by the same amount in percentage terms (recall that $\gamma(y, y^*)$ depends only on y^* if the technology is CES). The discontinuity at y^* is represented by the Dirac arrows in the figure. Letting workers be reassigned to different tasks in response to an exogenous increase in the labor supply of agents y^* thus spreads out the cross-wage effects around $y \approx y^*$ and removes the discontinuity that arises when matching is kept fixed.

[18] We extend our baseline environment to a multi-dimensional Roy model as in Rothschild and Scheuer (2013) in Section 5 and show that the same sufficient-statistic insight continues to hold.



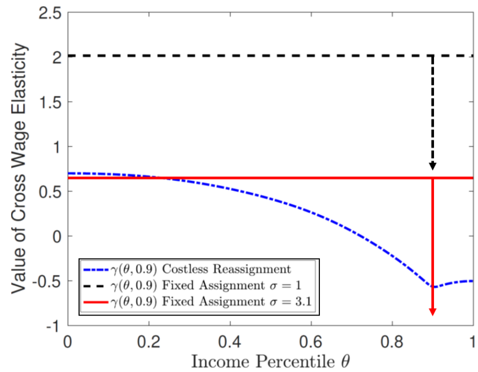


Figure 1: Cross wage elasticities in the model with endogenous costless reassignment of skills to tasks. Left panel: Elasticities $y \mapsto \gamma(y, y^*)$ with y^* in the 1^{st} (solid curve), 50^{th} (dashed curve), 90^{th} (dashed-dotted curve) deciles. Right panel: Comparison of the cross-wage elasticity (perturbation at the 90^{th} percentile) in the models with endogenous reassignment (dashed-dotted curve) and with exogenous assignment (CES production) for $\sigma = 1$ (dashed line) and $\sigma = 3.1$ (solid line).

2 Incidence of tax reforms

Consider a given initial, potentially suboptimal, tax schedule T, e.g., the U.S. tax code. In this section we derive closed-form formulas for the first-order effects of arbitrary local perturbations of this tax schedule ("tax reforms") on individual labor supplies, wages and utilities.

2.1 Effects on labor supply

As in the case of exogenous wages (Saez, 2001), analyzing the incidence of tax reforms relies crucially on solving for each individual's change in labor supply in terms of behavioral elasticities. This problem is, however, much more involved in general equilibrium. If wages are exogenous, a change in the tax rate of a given individual, say θ, induces only a change in the labor effort of that agent (measured by the elasticity (8)). In the general equilibrium setting, instead, this labor supply response of type θ affects the wage, and hence the labor supply, of every other skill $\theta' \neq \theta$. This in turn feeds back into the wage distribution, which further impacts labor supplies, and so on. Representing the total effect of this infinite sequence caused by arbitrarily non-linear tax reforms is thus a priori a complex task.[19]

[19]We could define, for each specific tax reform one might consider implementing, a "policy elasticity" (as in, e.g., Hendren (2015), Piketty and Saez (2013)), equal to each individual's total labor

The key step towards the general characterization of the economic incidence of taxes, and our first main theoretical contribution, consists of showing that this problem can be mathematically formulated as an integral equation (Lemma 1).[20] We can thus apply the tools and results of the theory of integral equations to solve in closed-form for the labor supply adjustments (Proposition 1). The incidence on wages and utilities is then straightforward to derive (Corollary 2).

Tax reforms and Gateaux derivatives. Formally, consider an arbitrary non-linear reform of the initial tax schedule $T(\cdot)$. This tax reform can be represented by a continuously differentiable function $\hat{T}(\cdot)$ on $\mathbb{R}_+$, so that the perturbed tax schedule is $T(\cdot) + \mu \hat{T}(\cdot)$, where $\mu \in \mathbb{R}$ parametrizes the size of the reform.[21] Our aim is to compute the first-order effect of this perturbation on individual labor supply (i.e., the solution to the first-order condition (1)), when the magnitude of the tax change is small, i.e., as $\mu \to 0$. This is formally given by the Gateaux derivative of the labor supply functional $T \mapsto l(\theta; T)$ in the direction $\hat{T}$, that is,[22]

$$\hat{l}(\theta) \equiv \lim_{\mu \to 0} \frac{1}{\mu} \left[l(\theta; T + \mu \hat{T}) - l(\theta; T) \right].$$

The variable $\hat{l}(\theta)$ gives the change in the labor supply of type θ in response to the tax reform $\hat{T}$, taking into account all the general equilibrium effects induced by the endogeneity of wages. We define analogously the changes in individual wages $\hat{w}(\theta)$, utilities $\hat{U}(\theta)$ and government revenue $\hat{\mathcal{R}}$.

Integral equation (IE). The following lemma provides an implicit characterization of the incidence of an arbitrary tax reform $\hat{T}$ on labor supplies.

supply response to the corresponding reform. The key challenge of the incidence problem consists of expressing this total labor supply response in terms of the structural elasticity parameters introduced in Section 1.2. Proposition 1 below gives the policy elasticity in closed-form in terms of these structural parameters.

[20] The general theory of linear integral equations is exposed in, e.g., Tricomi (1985), Kress (2014), and, as a concise introduction, in Zemyan (2012). Moreover, closed-form solutions can be derived in many cases (see Polyanin and Manzhirov (2008)). Finally, numerical techniques are widely available and can be easily implemented (see, e.g., Press (2007) and Section 2.6 in Zemyan (2012)), leading to straightforward quantitative evaluations of the incidence of arbitrary tax reforms (see Section 4).

[21] A tax reform that is a special example of this general definition consists of an increase in the marginal tax rate on a small interval (Piketty (1997); Saez (2001)). We formalize and analyze this class of perturbations in Section 3.1 below.

[22] The notation $\hat{l}(\theta)$ ignores for simplicity the dependence of this derivatives on the initial tax schedule T and on the tax reform $\hat{T}$.

Lemma 1. *The effect of a tax reform $\hat{T}$ of the initial tax schedule T on individual labor supplies, $\hat{l}(\cdot)$, is the solution to the functional equation:*

$$\frac{\hat{l}(\theta)}{l(\theta)} = -\varepsilon_r(\theta)\frac{\hat{T}'(y(\theta))}{1-T'(y(\theta))} + \varepsilon_w(\theta)\int_{\Theta}\gamma(\theta,\theta')\frac{\hat{l}(\theta')}{l(\theta')}d\theta' \tag{9}$$

for all $\theta \in \Theta$, where $\varepsilon_r(\theta)$ and $\varepsilon_w(\theta)$ are the elasticities of equilibrium labor of skill θ with respect to the retention rate and the wage, defined by

$$\frac{1}{\varepsilon_r(\theta)} \equiv \frac{1}{\varepsilon_r^S(\theta)} + \frac{1}{\varepsilon_w^D(\theta)} \quad \text{and} \quad \frac{1}{\varepsilon_w(\theta)} \equiv \frac{1}{\varepsilon_w^S(\theta)} + \frac{1}{\varepsilon_w^D(\theta)}.$$

Proof. See Appendix B.1.1. □

Formula (9) is a linear Fredholm integral equation of the second kind with kernel $\varepsilon_w(\theta)\gamma(\theta,\theta')$. Its unknown, which appears under the integral sign, is the function $\theta \mapsto \hat{l}(\theta)$. We start by providing the economic meaning of this equation.

Due to the reform, the retention rate $r(\theta) = 1 - T'(y(\theta))$ of individual θ changes, in percentage terms, by $\hat{r}(\theta) = -\frac{\hat{T}'(y(\theta))}{1-T'(y(\theta))}$. This tax reform induces a direct percentage change in labor effort $l(\theta)$ equal to $\varepsilon_r(\theta) \times \hat{r}(\theta)$, where $\varepsilon_r(\theta)$ is the elasticity of equilibrium labor on the market for skill θ. This is the expression one would obtain by considering the labor market θ in isolation and ignoring the cross-effects between markets. It resembles the expression one obtains assuming exogenous wages, with one difference: if the marginal product of labor is decreasing, then the initial labor supply adjustment (say, decrease) due to the tax reform causes an own-wage increase determined by $1/\varepsilon_w^D(\theta)$, which in turn raises labor supply and dampens the initial response. Thus the relevant elasticity satisfies $\varepsilon_r(\theta) \leq \varepsilon_r^S(\theta)$.

In general equilibrium, the labor supply of type θ is also impacted indirectly by the change in all other individuals' labor supplies, due to the skill complementarities in production. Specifically, the change in labor supply of each type θ', $\hat{l}(\theta')$, triggers a change in the wage of type θ equal to $\gamma(\theta,\theta')\hat{l}(\theta')$, and thus a further adjustment in her labor supply equal to $\varepsilon_w(\theta)\gamma(\theta,\theta')\hat{l}(\theta')$. Summing these effects over skills $\theta' \in \Theta$ leads to formula (9).

Solution to the IE and resolvent. We now characterize the solution to the integral equation (9).

Proposition 1. *Assume that the condition* $\int_{\Theta^2} |\varepsilon_w(\theta)\gamma(\theta,\theta')|^2 d\theta d\theta' < 1$ *holds.*[23] *The unique solution to the integral equation (9) is given by*

$$\frac{\hat{l}(\theta)}{l(\theta)} = -\varepsilon_r(\theta)\frac{\hat{T}'(y(\theta))}{1-T'(y(\theta))} - \varepsilon_w(\theta)\int_\Theta \Gamma(\theta,\theta')\varepsilon_r(\theta')\frac{\hat{T}'(y(\theta'))}{1-T'(y(\theta'))}d\theta', \quad (10)$$

where for all $(\theta,\theta') \in \Theta^2$, *the* resolvent $\Gamma(\theta,\theta')$ *is defined by*

$$\Gamma(\theta,\theta') \equiv \sum_{n=1}^{\infty}\Gamma_n(\theta,\theta'), \quad (11)$$

with $\Gamma_1(\theta,\theta') = \gamma(\theta,\theta')$ *and for all* $n \geq 2$,

$$\Gamma_n(\theta,\theta') = \int_\Theta \Gamma_{n-1}(\theta,\theta'')\varepsilon_w(\theta'')\gamma(\theta'',\theta')d\theta''.$$

Proof. See Appendix B.1.2. □

The mathematical representation (10) of the solution to the integral equation (9) has the following economic meaning. The first term on the right hand side of (10), $-\varepsilon_r(\theta)\frac{\hat{T}'(y(\theta))}{1-T'(y(\theta))}$, is the partial-equilibrium effect of the reform on labor supply $l(\theta)$, as already described in equation (9). The second (integral) term accounts for the cross-wage effects in general equilibrium. Note that this integral term has the same structure as the corresponding term in formula (9), except that: (i) the unknown labor supply changes $\hat{l}(\theta')$ are now replaced by their (known) partial-equilibrium counterparts $-\varepsilon_r(\theta')\frac{\hat{T}'(y(\theta'))}{1-T'(y(\theta'))}$; and (ii) the structural cross-wage elasticity $\gamma(\theta,\theta')$ is replaced by the "resolvent" cross-wage elasticity $\Gamma(\theta,\theta')$.

The elasticity variable $\Gamma(\theta,\theta')$, defined by the series (11), expresses the total effect of the labor supply of type θ' on the wage of type θ, i.e., it accounts for the infinite sequence of general equilibrium adjustments induced by the complementarities in production. The first iterated kernel ($n = 1$) in the series (11) is simply $\Gamma_1(\theta,\theta') = \gamma(\theta,\theta')$. It accounts for the impact of the labor supply of type θ' on the wage of type θ through direct cross-wage effects. The second iterated kernel ($n = 2$) in (11) accounts for the impact of the labor supply of θ' on the wage of θ, indirectly through

[23] This technical condition ensures that the infinite series (11) converges. We provide below sufficient conditions on primitives such that this condition holds. In more general cases it can be easily verified numerically. Finally, when it is not satisfied, we can more generally express the solution with a representation similar to (10) but with a more complex resolvent (see Section 2.4 in Zemyan (2012)).

the behavior of third parties θ''. This term reads

$$\Gamma_2(\theta,\theta') = \int_\Theta \gamma(\theta,\theta'')\,\varepsilon_w(\theta'')\,\gamma(\theta'',\theta')\,d\theta''. \tag{12}$$

For any θ', a percentage change in the labor supply of θ' induces a percentage change in the wage of any other type θ'' by $\gamma(\theta'',\theta')$, and hence a percentage change in the labor supply of θ'' given by $\varepsilon_w(\theta'')\,\gamma(\theta'',\theta')$. This in turn affects the wage of type θ by the amount $\gamma(\theta,\theta'')\,\varepsilon_w(\theta'')\,\gamma(\theta'',\theta')$. Summing over all intermediate types θ'' leads to expression (12). An inductive reasoning shows similarly that the terms $n \geq 3$ in the resolvent series (11) account for the impact of the labor supply of θ' on the wage of θ through n successive stages of cross-wage effects, e.g., for $n = 3$, $\theta' \to \theta'' \to \theta''' \to \theta$.

The case of separable cross-wage elasticities. A particularly tractable special case of Proposition 1 is obtained when the cross-wage elasticities are multiplicatively separable between skills. This occurs in particular when the production function is CES or, more generally, HSA. We then obtain the following corollary.

Corollary 1. *Suppose that the cross-wage elasticities $\gamma(\theta',\theta)$ are multiplicatively separable, i.e., there exist functions γ_1 and γ_2 such that for all (θ,θ'), $\gamma(\theta',\theta) = \gamma_1(\theta')\,\gamma_2(\theta)$. The resolvent cross-wage elasticities are then given by*

$$\Gamma(\theta',\theta) = \frac{\gamma(\theta',\theta)}{1-\int_\Theta \varepsilon_w(s)\,\gamma(s,s)\,ds}. \tag{13}$$

In particular, if the production function is CES, the integral in the denominator of (13) is equal to $\frac{1}{\sigma\mathbb{E}y}\mathbb{E}\left[y\varepsilon_w(y)\right]$.

Proof. See Appendix B.1.4. □

Equation (13) shows that, if the cross-wage elasticities are separable between θ and θ', the total impact $\Gamma(\theta,\theta')$ of a change in the labor supply of type θ' on the wage of type θ is proportional to the direct (structural) effect $\gamma(\theta,\theta')$. This is because each round of cross-wage general equilibrium effects, i.e., each term in the resolvent series (11), is a fraction of the first round. In particular, with a CES technology, the cross-wage elasticity $\gamma(\theta,\theta')$ depends only on θ' and is independent of θ, that is, a change in the aggregate labor supply of type θ' induces the same percentage adjustment in the wage of every skill $\theta \neq \theta'$. More generally, suppose that the cross-wage elasticities are the sum of multiplicatively separable functions: $\gamma(\theta', \theta) = \sum_{k=1}^{n} \gamma_1^k(\theta') \gamma_2^k(\theta)$, for some $n \geq 1$. This general separable functional form is useful because it can approximate arbitrarily closely any given map of (non-separable) elasticities $\gamma(\theta', \theta)$, and the solution to the Fredholm integral equation is continuous in its kernel. In Appendix A.4.3, we show that the solution (10) to the integral equation remains very tractable in this case: specifically, the resolvent cross-wage elasticities are given non-recursively by $\Gamma(\theta', \theta) = \sum_{1 \leq i,j \leq n} A_{ij} \gamma_1^i(\theta') \gamma_2^j(\theta)$ for some known constants A_{ij}.

Sufficient conditions on primitives ensuring convergence of the resolvent. Suppose that the production function is CES with parameter $\sigma > 0$, that the initial tax schedule is CRP with parameter $p < 1$, and that the disutility of labor is isoelastic with parameter $e > 0$. We show in Appendix B.1.4 that we have in this case $\frac{1}{\sigma \mathbb{E} y} \mathbb{E}[y \varepsilon_w(y)] < 1$ so that, by formula (13), $\Gamma(\theta, \theta') < \infty$. The convergence of the resolvent series (11) is thus satisfied.

One-to-one map between structural and resolvent cross-wage elasticities. For applied purposes, we can use both the structural parameters $\gamma(\theta, \theta')$ or the resolvent parameters $\Gamma(\theta, \theta')$ as primitive cross-wage elasticity variables: our tax incidence formulas can be expressed in terms of either of them. Some empirical studies may evaluate the structural parameters $\gamma(\theta, \theta')$ of the production function directly, while others may estimate the full general-equilibrium impact $\Gamma(\theta, \theta')$, including the spillovers generated by the initial shock. In the latter case, it may be useful to recover the structural elasticities $\gamma(\theta, \theta')$ as a function of the (observed) higher-order elasticities $\Gamma(\theta, \theta')$, e.g., for counterfactual analysis. We do so in Appendix B.1.3 by showing that $\gamma(\theta, \theta')$ can be expressed as the solution to an integral equation with a kernel determined by $\Gamma(\theta, \theta')$.

2.2 Effects on wages and utility

We can now easily obtain the incidence of an arbitrary tax reform $\hat{T}$ on individual wages and utilities.

Corollary 2. *The incidence of a tax reform $\hat{T}$ of the initial tax schedule T on individual wages, $\hat{w}(\cdot)$, is given by*

$$\frac{\hat{w}(\theta)}{w(\theta)} = \frac{1}{\varepsilon_w^S(\theta)} \left[\varepsilon_r^S(\theta) \frac{\hat{T}'(y(\theta))}{1 - T'(y(\theta))} + \frac{\hat{l}(\theta)}{l(\theta)} \right], \tag{14}$$

for all $\theta \in \Theta$, where the labor supply response $\hat{l}(\theta)$ is given by (10). The incidence on individual utilities, $\hat{U}(\cdot)$, is given by

$$\hat{U}(\theta) = -\hat{T}(y(\theta)) + (1 - T'(y(\theta)))\, y(\theta) \frac{\hat{w}(\theta)}{w(\theta)}. \tag{15}$$

Proof. See Appendix B.2. □

Corollary 2 gives the changes in individual wages due to the tax reform $\hat{T}$, as a function of the labor supply changes characterized by Proposition 1. Its meaning is straightforward. Multiplying both sides of (14) by $\varepsilon_w^S(\theta)$ simply gives the percentage adjustment of type-θ labor supply, $\frac{\hat{l}(\theta)}{l(\theta)}$, as the sum of its response in the case of exogenous wages, $-\varepsilon_r^S \frac{\hat{T}'}{1-T'}$, and the effect induced by the percentage wage change, $\varepsilon_w^S \times \frac{\hat{w}}{w}$.

Equation (15) gives the impact of the reform on individual welfare. The first term on the right hand side, $-\hat{T}(y(\theta))$, is due to the fact that a higher tax payment makes the individual poorer and hence reduces her utility. The second term accounts for the change in net income due to the wage adjustment $\hat{w}(\theta)$. If wages were exogenous, so that $\hat{w}(\theta) = 0$ in (15), the utility of agent θ would respond one-for-one with changes in her total tax payment $\hat{T}(y(\theta))$ and would not be affected by changes in her marginal tax rate $\hat{T}'(y(\theta))$ – this is a direct consequence of the envelope theorem. In general equilibrium, however, this is no longer true because marginal tax rates cause labor supply adjustments which in turn affect wages. We show in Appendix B.2 that if all pairs of types are Edgeworth complements and assignment of workers to tasks is exogenous, then a higher marginal tax rate at income $y(\theta)$ raises the utility of agents with skill θ and lowers that of all other agents.

3 Effects of tax reforms on government revenue

Having derived the change in equilibrium labor and wages in response to a tax reform $\hat{T}$ (Proposition 1 and Corollary 2), the impact on government revenue directly follows:

$$\hat{\mathcal{R}}(\hat{T}) = \int_\Theta \hat{T}(y(\theta)) f(\theta)\, d\theta + \int_\Theta T'(y(\theta)) \left[\frac{\hat{l}(\theta)}{l(\theta)} + \frac{\hat{w}(\theta)}{w(\theta)}\right] y(\theta) f(\theta)\, d\theta. \tag{16}$$

The first term on the right hand side of (16) is the statutory effect of the tax reform $\hat{T}(\cdot)$, i.e., the mechanical change in government revenue assuming that the individual labor supply and her wage remain constant. The second term is the behavioral effect of the reform. The labor supply and wage adjustments $\hat{l}(\theta)$ and $\hat{w}(\theta)$ both induce a change in government revenue proportional to the marginal tax rate $T'(y(\theta))$. Summing these effects over all individuals using the density $f(\cdot)$ leads to equation (16). The remainder of this section is devoted to deriving the economic implications of this formula. In Appendix C, we extend our analysis and characterize more generally the effects of tax reforms on social welfare.

3.1 Preliminaries

Elementary tax reforms. From now on, we focus without loss of generality on a specific class of "elementary" tax reforms, represented by the step function $\hat{T}(y) = (1 - F_Y(y^*))^{-1} \mathbb{I}_{\{y \geq y^*\}}$ for a given income level y^*.[24] That is, the total tax liability increases by the constant amount $(1 - F_Y(y^*))^{-1}$ for any income y above y^*, and the marginal tax rates are perturbed by the Dirac delta function at income y^*, i.e. $\hat{T}'(y) = (1 - F_Y(y^*))^{-1} \delta(y - y^*)$. Intuitively, this reform consists of raising the marginal tax rate at only one income level $y^* \in \mathbb{R}_+$, which implies a uniform increase in the total tax payment of agents with income $y > y^*$.[25] The normalization by $(1 - F_Y(y^*))^{-1}$ implies that the statutory increase in government revenue due to the reform (i.e., the first term on the r.h.s. of (16)) is equal to \$1. We denote by $\hat{\mathcal{R}}(y^*)$ the total effect (16) of this elementary tax reform on government revenue.[26]

Partial equilibrium benchmark. In the case of exogenous wages, the incidence on government revenue is given by expression (16) with $\hat{w}(\theta) = 0$ and $\hat{l}(\theta) = -\varepsilon_r^S(\theta) \frac{\hat{T}'(y(\theta))}{1-T'(y(\theta))}$. Applying this formula to the elementary tax reform at income

[24]Note that the function $\mathbb{I}_{\{y \geq y^*\}}$ is not differentiable. We show in Appendix C.1 that we can nevertheless use our theory to analyze this reform by applying (10) to a sequence of smooth perturbations $\{\hat{T}_n'(y)\}_{n \geq 1}$ that converges to the Dirac delta function $\delta(y - y^*)$. This notation simplifies the exposition and is made only for convenience. All our formulas can be easily written for any smooth tax reform $\hat{T}$ rather than step functions.

[25]Heuristically, consider a perturbation that raises the marginal tax rate by $\mathrm{d}T'$ on a small income interval $[y^* - \mathrm{d}y, y^*]$, so that the total tax payment above income y^* raises by the amount $\mathrm{d}T' \times \mathrm{d}y$ equal to, say, \$1. This class of tax reforms has been introduced by Saez (2001). Then shrink the size of the income interval on which the tax rate is increased, i.e. $\mathrm{d}y \to 0$, while keeping the increase in the tax payment above y^* fixed at \$1. The limit of the marginal tax rate increase $\mathrm{d}T'$ is the Dirac measure at y^*, and the change in the total tax bill converges to its c.d.f., the step function $\mathbb{I}_{\{y \geq y^*\}}$.

[26]Any tax reform $\hat{T}$ can be expressed as a linear combination of such income-specific elementary perturbations, so that is incidence on tax revenue is given by $\hat{\mathcal{R}}(\hat{T}) = \int \hat{\mathcal{R}}(y^*) (1 - F_Y(y^*)) \hat{T}'(y^*) \, dy^*$. See Golosov, Tsyvinski, and Werquin (2014) for details.

y^* easily leads to (see Saez (2001)):

$$\hat{\mathcal{R}}_{\text{ex}}(y^*) \;=\; 1 - T'(y^*)\,\frac{\varepsilon_r^S(y^*)}{1 - T'(y^*)}\,\frac{y^* f_Y(y^*)}{1 - F_Y(y^*)}. \tag{17}$$

Equation (17) expresses the impact of an increase in the marginal tax rate at income y^* as the sum of the statutory increase in government revenue, which is normalized to \$1 by construction, and the behavioral revenue loss equal to the product of: (i) the endogenous reduction in the labor income of agent y^*, $\frac{y^*}{1-T'(y^*)}\varepsilon_r^S(y^*)$; (ii) the share $T'(y^*)$ of this income change that accrues to the government; and (iii) the hazard rate of the income distribution, $\frac{f_Y(y^*)}{1-F_Y(y^*)}$. The hazard rate is a cost-benefit ratio that measures the fraction $f_Y(y^*)$ of agents whose labor supply is distorted by the reform, relative to the fraction $1 - F_Y(y^*)$ of agents whose tax bill increases lump-sum. Note that the second term in the right hand side of (17), $\varepsilon_r^S \frac{T'}{1-T'} \frac{y^* f_Y}{1-F_Y}$, captures how much revenue, per unit of mechanical increase in taxes, is lost through adjustments in behaviour. It is an expression for the marginal excess burden of a tax reform.

3.2 Effects on government revenue

We now derive and analyze the incidence of tax reforms on government revenue in general equilibrium and compare it to the expression (17) obtained assuming exogenous wages.

Proposition 2. *The incidence of the elementary tax reform at income y^* on government revenue is given by*

$$\begin{aligned} \hat{\mathcal{R}}(y^*) \;=\; & \hat{\mathcal{R}}_{\text{ex}}(y^*) + \frac{\varepsilon_r(y^*)}{1 - T'(y^*)} \\ & \times \int \left[T'(y^*)\left(1 + \varepsilon_w^S(y^*)\right) - T'(y)\left(1 + \varepsilon_w^S(y)\right)\right] \bar{\Gamma}(y, y^*)\,\frac{y\, f_Y(y)}{1 - F_Y(y^*)}\,dy. \end{aligned} \tag{18}$$

where $\bar{\Gamma}(y, y^*) \equiv \left(1 + \frac{\varepsilon_w^S(y)}{\varepsilon_w^D(y)}\right)^{-1} \Gamma(y, y^*)$ are normalized resolvent cross-wage elasticities.

Proof. See Appendix C.2. □

Sketch of proof. To understand formula (18), it is useful to first sketch its proof. The direct effect of the marginal tax rate increase at income y^* is to lower labor supply of these agents proportionally to $\varepsilon_r(y^*)$. This induces two additional effects in general equilibrium. First, complementarities in production imply that the wage of any agent with income $y \neq y^*$ changes (say, decreases), in percentage terms, by $\Gamma(y, y^*) \times \varepsilon_r(y^*)$, so that her income decreases by $\left(1 + \varepsilon_w^S(y)\right) y \times \Gamma(y, y^*) \varepsilon_r(y^*)$. A share $T'(y)$ of this income loss accrues to the government, leading to the second term in the square brackets of (18). Second, the non-constant marginal product of labor implies that the wage of agents with income y^* changes (say, increases), in percentage terms, by $\frac{1}{\varepsilon_w^D(y^*)} \times \varepsilon_r(y^*)$. Thus their income increases by $\left(1 + \varepsilon_w^S(y^*)\right) y^* \times \frac{1}{\varepsilon_w^D(y^*)} \varepsilon_r(y^*)$, a share $T'(y^*)$ of which accrues to the government.

The key step is to then sum over the whole population and apply Euler's homogeneous function theorem. Intuitively, constant returns to scale imply that the own-wage gains of agents with income y^* are exactly compensated by the aggregate cross-wage effects of the other incomes $y \neq y^*$.[27] This leads to an expression for the own-wage elasticity $\frac{1}{\varepsilon_w^D(y^*)}$ as a function of an integral of the cross-wage elasticities $\Gamma(y, y^*)$, thus leading to the first term in the square brackets of (18).

Linear tax schedule. Assume that the labor supply elasticities $\varepsilon_w^S(\cdot)$ are constant. Since the income changes of all agents cancel in the aggregate (by Euler's theorem) for fixed levels of labor supply, this assumption implies that the income changes of all agents also cancel once we account for the labor supply adjustments. That is, the reshuffling of wages due to the tax reform has distributional effects but keeps the economy's aggregate income constant. Now suppose in addition that the initial tax schedule is linear, so that every agent faces the same marginal tax rate. We then immediately obtain that the government's tax revenue gain coming from the higher income of agents y^* is exactly compensated by the tax revenue losses coming from the rest of the population (formally, the square bracket in formula (18) is zero). Therefore tax reforms have the same effect on tax revenue as in the environment with exogenous wages.

Corollary 3. *Suppose that the disutility of labor is isoelastic and the initial tax schedule is linear. Then the incidence of an arbitrary tax reform on government revenue*

[27] Euler's homogeneous function theorem in its most standard form is written in terms of the structural cross-wage elasticities $\gamma(y, y^*)$. This first round of wage changes then induces labor supply changes, which in turn lead to further rounds of own- and cross-wage effects in general equilibrium. Because Euler's theorem applies at every stage, the aggregate effect of all these wage adjustments is again equal to zero, so that the relationship can be expressed in terms of the resolvent cross-wage elasticities $\Gamma(y, y^*)$ (or, more rigorously, $\bar{\Gamma}(y, y^*)$). It is this formula that we use here; see Appendix A.1.2 for details.

is identical to that obtained assuming exogenous wages: $\hat{\mathcal{R}}(y^*) = \hat{\mathcal{R}}_{\text{ex}}(y^*)$ *for all* y^*.

Proof. See Appendix C.2. $\square$

Non-linear tax schedule. Suppose now, more generally, that the initial tax schedule is non-linear. In this case, as in the previous paragraph, aggregate income remains unchanged in response to a tax reform due to Euler's theorem. However, the distributional implications of the tax reform now lead to non-trivial effects on government revenue (formally, the square bracket in formula (18) is non-zero). Indeed, a zero-sum transfer of income from one agent to another is no longer neutral since these workers pay different tax rates to the government on their respective income gains and losses. To further characterize the general-equilibrium contribution to government revenue when the tax schedule is non-linear, assume again that the elasticities of labor supply $\varepsilon_w^S(\cdot)$ are constant (independent of y), which occurs if the disutility of labor is isoelastic and the initial tax schedule is CRP defined in (3). Moreover, assume that the elasticities of labor demand $\varepsilon_w^D(\cdot)$ are also constant, which occurs either when the production function is CES (see (4)), or in the microfoundation with endogenous and costless assignment of workers to tasks (in which case $1/\varepsilon_w^D(y) = 0$ for all y, see Sections 1.3 and A.3 for details). The general formula of Proposition 2 can then be simplified as follows.

Corollary 4. *Suppose that the disutility of labor is isoelastic, the initial tax schedule is CRP, and the labor demand elasticities are constant. We then have*

$$\begin{aligned} \hat{\mathcal{R}}(y^*) &= \hat{\mathcal{R}}_{\text{ex}}(y^*) + \frac{\varepsilon_r}{1-T'(y^*)}\frac{y^* f_Y(y^*)}{1-F_Y(y^*)}(1+\varepsilon_w^S) \\ &\quad \times\left\{\frac{1}{\varepsilon_w^D}\left(T'(y^*) - \mathbb{E}\left[T'(y)\right]\right) - \frac{1}{y^* f_Y(y^*)}\text{Cov}\left(T'(y)\,;\, y\bar{\Gamma}(y,y^*)\right)\right\}. \end{aligned} \tag{19}$$

(i) *If the production function is CES, then the covariance term on the right hand side of (19) is constant.*[28] *Letting* $\phi = \frac{1+\varepsilon_w^S}{\sigma+\varepsilon_w^S}$ *and* $\bar{T}' = \mathbb{E}\left[yT'(y)\right]/\mathbb{E}y$, *we then obtain*

$$\hat{\mathcal{R}}(y^*) = \hat{\mathcal{R}}_{\text{ex}}(y^*) + \phi\,\varepsilon_r^S\frac{T'(y^*) - \bar{T}'}{1-T'(y^*)}\frac{y^* f_Y(y^*)}{1-F_Y(y^*)}, \tag{20}$$

(ii) *If the production function is microfounded as in the assignment model of Section 1.3, then* $1/\varepsilon_w^D(y) = 0$ *for all* y *in formula (19).*

[28] This is because we then have $\bar{\Gamma}(y,y^*) = \gamma(y,y^*) = \frac{1}{\sigma\mathbb{E}[y]}y^* f_Y(y^*)$.

Proof. See Appendix C.3. □

Corollary 4 delivers novel and important insights. We first discuss both special cases of formula (19) in turn and then conclude on the economic implications of this result.

CES production. Consider first the case where the production function is CES (formula (20)). Suppose that the marginal tax rates are increasing in the initial economy, i.e., the rate of progressivity is $p > 0$. Consider a reform that raises the marginal tax rate at income y^*, so that the labor supply of agents with income y^* decreases, which in turn raises their own wage and lowers everyone else's wage. As explained above, by Euler's homogeneous function theorem and the fact that the labor supply elasticities are constant, the resulting income gain of agents with income y^* is exactly compensated in the aggregate by the income losses of the other agents $y \neq y^*$. Now suppose that agents with income y^* are high income earners, so that their marginal tax rate $T'(y^*)$ is larger than the (income-weighted) average marginal tax rate $\bar{T}'$ in the population. Then the government's revenue gain coming from the higher income of agents y^*, which is proportional to $T'(y^*)$, more than compensates the tax revenue loss coming from the rest of the population, which is proportional to $\bar{T}'$. We therefore obtain that $\hat{\mathcal{R}}(y^*) > \hat{\mathcal{R}}_{\text{ex}}(y^*)$. Therefore, the general-equilibrium contribution of the tax reform on government revenue is positive (resp., negative) if the marginal tax rate at y^* is larger (resp., smaller) than the income-weighted average marginal tax rate in the economy. Moreover, the larger the income y^* at which the marginal tax rate is increased, the larger the gain in government revenue relative to the exogenous-wage setting. That is, "trickle-down" forces imply higher benefits of *raising*, not lowering, the marginal tax rates on high incomes.[29]

Endogenous assignment. Consider next the case where the production function is microfounded as in Section 1.3, with endogenous and costless (re-)sorting of skills into tasks. In this case, the inverse labor demand elasticities $1/\varepsilon_w^D$ are equal to zero and equation (19) implies that the general-equilibrium contribution to the excess burden of the elementary tax reform is determined by the covariance between the initial marginal tax rates $T'(\cdot)$ and the production complementarities $\bar{\Gamma}(\cdot, y^*)$ with agent y^*.

[29] In Appendix C we extend this analysis to the case where the government's objective is to improve social welfare rather than tax revenue.

If this covariance is positive (resp., negative) at a given income y^*, then the general-equilibrium forces raise (resp., lower) the cost of increasing the marginal tax rate at income y^*, compared to the partial-equilibrium benchmark (17). Moreover, if this covariance is increasing with y^* (resp., decreasing), then the general-equilibrium forces raise (resp., lower) the cost of increasing the progressivity of the tax code. Section 4 evaluates this formula numerically for calibrated values of the cross-wage elasticities, but we can already anticipate the qualitative results. The left panel of Figure 1 clearly shows that the covariance between incomes and the cross-wage elasticities is positive for low values of y^* (solid curve) and negative for large values of y^* (dashed-dotted curve). Therefore, if the marginal tax rates are initially increasing with income, the covariance term $\text{Cov}(T'(y)\,; y\,\bar{\Gamma}(y, y^*))$ decreases with y^*. Consequently, the same qualitative insight as in the CES model holds: the general-equilibrium contribution to government revenue of a tax increase at income y^* increases with y^*. In other words, both terms in the curly brackets of formula (19) push in the same direction.

Conclusion: progressivity and trickle-down. The previous discussion implies that, starting from a progressive tax schedule, the standard partial-equilibrium formula (17) underestimates the revenue gains from raising the marginal tax rates at the top and lowering them at the bottom, i.e., from further raising the progressivity of the tax schedule.[30] Conversely, starting from a regressive tax schedule, the partial-equilibrium formula overestimates the gains (or underestimates the losses) from increasing marginal tax rates at the top. Thus, contrary to conventional wisdom that is based on optimal tax theory (see, e.g., Stiglitz (1982) and Section 6 below), the "trickle down" forces caused by the endogeneity of wages may either raise or lower the benefits of raising high-income tax rates, depending on the shape of marginal tax rates in the initial tax system. In particular, since the tax code in the U.S. is progressive (Heathcote, Storesletten, and Violante (2016)), the benefits of raising further its progressivity (i.e., increasing the marginal tax rates on high incomes) are larger than a model with fixed wages would predict. We therefore conclude that that one should be cautious, in practice, when applying the results of the theory of optimal taxation in general equilibrium (see Section 6) to partial reforms of a suboptimal tax code.

[30] In Appendix C.3, D.1.3 and F, we extend this result to the cases where the government has a social welfare objective that is no longer Rawlsian (i.e., revenue maximization) and agents' utility functions have income effects. Both of these extensions add a term that dampens the result of Corollary 4, but do not overturn it quantitatively for empirically reasonable values of the parameters.

4 Numerical simulations

In this section we calibrate our model to the U.S. economy and evaluate quantitatively the effects of the elementary tax reforms on government revenue. First, in Section 4.1, we show that the calibration of Saez (2001), which consists of inferring the wage distribution from the observed income distribution and the agents' preferences, can be extended to a general equilibrium setting. We illustrate numerically the result of formula (20) and show that the novel general-equilibrium effects are sizeable for plausible parameter calibrations. We further explore various extensions in the Appendix that show that these results are robust to relaxing the specific assumptions made in Corollary 4. Second, in Section 4.2, we build on the calibration of Ales, Kurnaz, and Sleet (2015) to evaluate the effects of tax reforms in the environment described in Sections 1.3 and A.3 where the assignment of workers to tasks is endogenous and costless.

4.1 Main specification

We assume that the disutility of labor $v(l)$ is isoelastic with parameter $e = 0.33$ (Chetty, 2012),[31] and that the U.S. tax schedule is CRP with parameters $p = 0.151$ and $\tau = -3$ (Heathcote, Storesletten, and Violante (2016)). To match the U.S. yearly earnings distribution, we assume that $f_Y(\cdot)$ is log-normal with mean 10 and variance 0.95 up to income $y = \$150,000$, above which we append a Pareto distribution with coefficient $\pi = 1.5$, i.e., $\lim_{\bar{y}\to\infty} \mathbb{E}[y|y \geq \bar{y}]/\bar{y} = \frac{\pi}{\pi-1} = 3$ (Diamond and Saez, 2011). As in Saez (2001), we infer the distribution of wages $w(\theta)$ from the earnings distribution and the individual first-order conditions (1). After choosing values for the elasticities of substitution, we obtain the remaining parameters of the production function. See Appendix F.1 for a more detailed description of the calibration procedure. We first study the case of a CES production function and then extend our results to a Translog production function, for which the elasticities of substitution between pairs of skills vary with the distance between them. The latter analysis is relegated to Appendix F.2.

In this section, we assume that the production function is CES and illustrate nu-

[31] In Appendix F.1.2, we discuss the connection between our model and the empirical literature that estimates the elasticity of taxable income (see, e.g., Saez, Slemrod, and Giertz (2012) for a survey). We show that the estimate for the taxable income elasticity at income $y(\theta)$ maps in our model to the equilibrium labor elasticity variable $\varepsilon_r(\theta)$ defined in Lemma 1.

merically the analytical result of Corollary 4 (formula (20)). We choose an elasticity of substitution $\sigma \in \{0.6\,;\,3.1\}$. The value $\sigma = 0.6$ is taken from Dustmann, Frattini, and Preston (2013) who study the impact of immigration along the U.K. wage distribution and, as in our framework, group workers according to their position in the wage distribution.[32,33] The value $\sigma = 3.1$ is taken from Heathcote, Storesletten, and Violante (2016), who structurally estimate this CES parameter for the U.S. by targeting cross-sectional moments of the joint equilibrium distribution of wages, hours, and consumption. There is no clear consensus in the empirical literature on how responsive relative wages are to changes in labor supply, and therefore on the appropriate value of σ;[34] our two values are on the lower and higher sides of the typical empirical estimates.

Our results for the CES specification are illustrated in Figure 2. We plot the impact on government revenue of elementary tax reforms at each income level in the model with exogenous wages (solid curve, equation (17)) and in general equilibrium (dashed curve, equation (20)), as a function of the income $y(\theta)$ where the marginal tax rate is perturbed. A value of 0.7, say, at a given income $y(\theta)$, means that for each additional dollar of tax revenue mechanically levied by the tax reform at $y(\theta)$, the government effectively gains 70 cents, while 30 cents are lost through the behavioral responses of individuals; that is, the marginal excess burden of this tax reform is 30 percent.

Consider first the solid curve: it has a U-shaped pattern which reflects the shape of $\frac{y^* f_Y(y^*)}{1-F_Y(y^*)}$ in (17). This is a well-known finding in the literature (Diamond, 1998; Saez, 2001). The difference between the dashed and solid curves captures the additional revenue effect due to the endogeneity of wages. In line with our analytical result of formula (20), we observe that this difference is positive for intermediate and high incomes (starting from about \$77,000, where the marginal tax rate equals its income-

[32]This literature is a useful benchmark because it studies the impact on relative wages of labor supply shocks of certain skills, which is exactly the channel we want to analyze in our tax setting (except that for us the labor supply shocks are caused by tax reforms rather than immigration inflows).

[33]Card (1990) and Borjas et al. (1997) also measure the skill type by the relative wage position when studying the impact of immigration on native wages. The setting of Dustmann, Frattini, and Preston (2013) fits our setting particularly well because they group workers into fine groups: 20 groups that contain 5% of the workforce respectively. In Appendix F.1.3, we formally show that the elasticity of substitution estimated in a framework with discrete earnings groups (e.g., percentiles or quartiles) can be used to calibrate a CES production function with a continuum of types.

[34]See, e.g., the debate on the impact of immigration on natives' wages (Peri and Yasenov, 2015; Borjas, 2015).

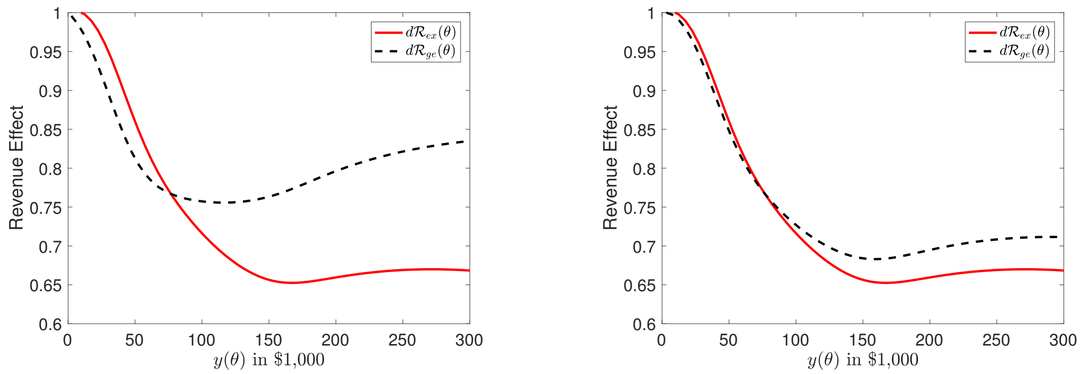


Figure 2: Revenue gains of elementary tax reforms at each income $y(\theta)$. Solid curves: exogenous wages (equation (17)). Dashed curves: CES technology with $\sigma = 0.6$ (left panel) and $\sigma = 3.1$ (right panel) (equation (20)).

weighted average). Raising the marginal tax rates for these income levels is more desirable, in terms of government revenue, when the general equilibrium effects are taken into account, while the opposite holds for low income levels. The magnitude of the difference is substantial: the marginal excess burden from increasing the marginal tax rate on income \$200,000 is equal to 0.22 cents (resp., 0.30 cents) per dollar if $\sigma = 0.6$ (resp., $\sigma = 3.1$) instead of 0.34 if $\sigma = \infty$, i.e., it is reduced by 35 percent (resp., 12 percent) due to the general equilibrium effects. Hence, the model with exogenous wages significantly underestimates the revenue gains from increasing the progressivity of the tax code.

We explore the robustness of these results in Appendix F.2. We first consider other specifications of the U.S. tax schedule, in particular, we account for the phasing-out of transfers, as estimated by Guner, Rauh, and Ventura (2017), which implies high marginal tax rates at the bottom of the income distribution. Our main insight regarding the additional benefit of raising progressivity in general equilibrium is mitigated but not reversed. Second, we compute the effects of the elementary tax reforms on social welfare. As discussed in Corollary 2, the general equilibrium forces imply an increase (resp., decrease) in wages and utilities for individuals whose marginal tax rate increases (resp., for everyone else). This channel reduces the benefits of raising the progressivity of the tax schedule. Nevertheless our main result still holds if the social marginal welfare weights fall sufficiently fast with income. Third, we consider Translog production functions with distance-dependent elasticities of substitution in Appendix F.1.4; again, our main insights continue to hold.

4.2 Endogenous assignment

We now investigate the effects of tax reforms on government revenue in the economy described in Sections 1.3 and A.3, with endogenous and costless reassignment of skills to tasks. We calibrate the technological parameters of a Cobb-Douglas production function over tasks as well as the productivity of each skill type for each task using the estimates of Ales, Kurnaz, and Sleet (2015) – we refer to their paper for details. We assume the same (CRP) initial tax schedule as in Section 4.1. We described the shape of the cross-wage elasticities in this environment in Section 1.3. We use these elasticities in our tax incidence formulas of Sections 2 and 3, and compare the resulting effects on government revenue with those obtained in Section 4.1 for a CES technology with fixed assignment.

Effects of tax reforms on government revenue. Figure 3 shows the government revenue impact of elementary tax reforms at each income level. Note that the calibration of Ales, Kurnaz, and Sleet (2015) assumes a bounded skill distribution, with a maximum income level (corresponding to the top percentile in the horizontal axis of the figure) below the Pareto tail. As a consequence, this calibration is unable to capture the large revenue gains from raising taxes on high incomes observed in Figure 2 above. Importantly, the boundedness of the domain also implies inversely U-shaped, rather than U-shaped, gains from tax increases, reflecting the inverse U-shaped optimum that they obtain in their quantitative analysis. Therefore, the shape and quantitative magnitudes of the revenue gains from tax reforms that we obtained in Figure 2, which account for the essential role played by the Pareto distribution of top incomes (Saez (2001)), are more realistic than those of Figure 3 below.

In both panels of Figure 3, the solid curve gives the revenue effects (17) in the model with exogenous wages, using the same wage distribution (with bounded support) as in Ales, Kurnaz, and Sleet (2015). The dashed curve is for the model with endogenous and costless reassignment (formula (19)), for the Cobb-Douglas production function over tasks estimated by Ales, Kurnaz, and Sleet (2015). The dashed-dotted curve is for the general equilibrium model of Section 4.1 with fixed assignment (formula (20)). As in our discussion in Section 1.3, we consider two calibrations for the latter model. First, in the left panel we assume a Cobb-Douglas production function ($\sigma = 1$), i.e., we shut down the reassignment channel in the calibration of Ales, Kurnaz, and Sleet (2015). Second, more relevant for our purposes, in the right panel

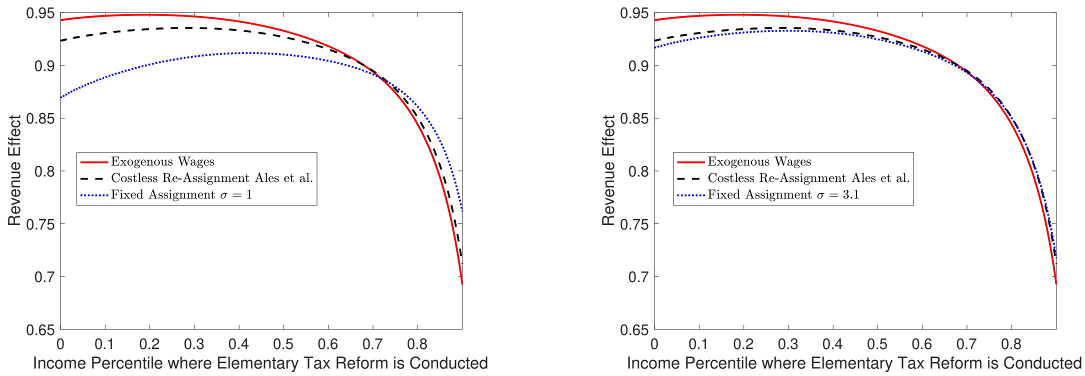


Figure 3: Revenue gains of elementary tax reforms at each income $y(\theta)$, using the calibration of Ales, Kurnaz, and Sleet (2015) for the wage distribution. Solid curves: exogenous wages (equation (17)). Dashed curves: Cobb-Douglas technology over tasks with endogenous costless reassignment of skills to tasks. Dotted curve: CES technology over labor supplies with $\sigma = 1$ (left panel) and $\sigma = 3.1$ (right panel) and fixed assignment.

we assume a CES production function with $\sigma = 3.1$, following the direct estimation of a technology over labor supplies of different skills by Heathcote, Storesletten, and Violante (2016) (see Section 4.1 above).

Qualitatively, as anticipated in Section 3.2, the fixed and endogenous assignment models deliver similar policy implications: the government revenue gains are higher (resp., lower) due to the endogeneity of wages if the marginal tax rates are raised on high (resp., low) incomes. Quantitatively, if we assume a Cobb-Douglas production function in the model with fixed assignment ($\sigma = 1$), we find that the endogenous reassignment of workers into new tasks mitigates the magnitude of the general-equilibrium effects on revenue: while still significant, they are around 30 percent[35] of those obtained with fixed assignment. However, if we use a value of σ that is directly estimated for a CES production function over skills ($\sigma = 3.1$ taken from Heathcote, Storesletten, and Violante (2016)), we obtain that the implications of tax reforms for government revenue are quantitatively very similar.

[35]Precisely, they are 40 percent at the 80$^{\text{th}}$ percentile, 30 percent at the 90$^{\text{th}}$ percentile and 20 percent at the 100$^{\text{th}}$ percentile.

5 Generalizations

In this section we extend our tax incidence techniques and results to more general and alternative environments.

Income effects. In Appendix D.1, we extend the model of Section 1 to a general utility function $U(c, l)$ over consumption and labor supply. This specification allows for arbitrary substitution and income effects. In addition to the elasticities defined in Section 1.2, we define the income effect parameter in response to a change in the agent's non-labor income. We show that the effect of an arbitrary tax reform $\hat{T}$ on individual labor supply is given by an integral equation analogous to (9), except that the partial-equilibrium response (first term on the r.h.s.) now also accounts for the income effect of the tax change. Its solution can then be straightforwardly derived as in Proposition 1. We also extend formula (20) to this setting.

Endogenous participation decisions. In Appendix D.2, we extend the model of Section 1 by letting agents choose their labor supply both on the intensive (hours) and extensive (participation) margins. Heterogeneity is now two-dimensional: individuals are indexed by their skill and their fixed cost of working. We define the elasticity of participation with respect to a change in the average tax rate and again show that the effect of an arbitrary tax reform $\hat{T}$ on the labor supply of a given skill is given by an integral equation analogous to (9). Its solution can then be straightforwardly derived as in Proposition 1.

Multiple sectors and Roy model. In Appendix D.3.2 and D.4, we analyze the incidence of tax reforms in settings with an alternative production structure. There are N sectors (or education groups, occupations, etc.). Heterogeneity is multi-dimensional: agents are indexed by a vector of sector-specific skills. They choose both the sector in which to work and their level of labor supply. Note that the wage distributions of different sectors overlap. The tax schedule depends only on income. This structure is the same as in Rothschild and Scheuer (2013).

Suppose first that the assignment of agents to sectors does not change in response to a small tax reform – e.g., there is a fixed switching cost. We then show that the incidence of an arbitrary tax reform $\hat{T}$ on the labor supplies of agents is given by a linear system of integral equations. Its solution is analogous to that of (9) (Proposition 1), the only difference being that the incidence now naturally depends on a larger number of cross-wage elasticities across both skills and sectors. We then apply these results to the so-called "canonical model" (Acemoglu and Autor, 2011), where individuals are grouped according to their level of education. We derive a result similar to that of Corollary 4 and show that our main insight on the progressivity of the tax code carries over to this setting. Suppose next that an infinitesimal tax change triggers a costless endogenous re-assignment of workers into different sectors. Analogous to our discussion of sufficient statistics in Section 1.3, we show that conditional on the cross-wage elasticities that we use as primitives, the tax incidence formulas are identical to those obtained in the fixed assignment model.

6 Optimal taxation

In this section we show that our tax incidence analysis delivers a characterization of the optimal (i.e., welfare-maximizing) tax schedule as a by-product. We first formally introduce the social welfare criterion. We then present simple extensions of two seminal formulas to the general equilibrium environment: the optimal marginal tax rate formula of Diamond (1998) and the optimal top tax rate formula of Saez (2001).[36] The general analysis and technical details are relegated to Appendix E. Appendix F.3 contains the numerical simulations.

6.1 Welfare function and welfare weights

The government evaluates social welfare by means of a concave function $G : \mathbb{R} \to \mathbb{R}$. Letting λ denote the marginal value of public funds, we thus define social welfare in monetary units by

$$\mathcal{G} = \frac{1}{\lambda} \int_{\Theta} G\left(U\left(\theta\right)\right) f\left(\theta\right) d\theta.$$

The optimal tax schedule maximizes social welfare $\mathcal{G}$ subject to the constraint that government revenue $\mathcal{R}$ is non-negative.

[36]Importantly, contrary to Saez (2001), this formula indeed characterizes the optimal top tax rate only if the *whole* tax schedule is set optimally. Our analysis of Section 3.2 shows that it is no longer valid if the initial tax schedule is suboptimal. We return to this point and discuss it further in Sections 6.3 and 6.4 below.

We denote by $g(\theta)$, or equivalently $g(y(\theta))$, the marginal social welfare weight[37] associated with individuals of type θ as

$$g(\theta) = \frac{1}{\lambda} G'(U(\theta)). \tag{21}$$

The weight $g(\theta)$ is the social value of giving one additional unit of consumption to individuals with type θ, relative to distributing it uniformly to the whole population.

6.2 Optimal tax schedule

We can easily extend our analysis to compute the effect $\hat{\mathcal{G}}(\hat{T})$ of arbitrary tax reforms $\hat{T}$ on social welfare (see Appendix C.2): we only need to add the effect on government revenue derived in Section 3.2 to the effects on individual utilities derived in Section 2.2, aggregating the latter and weighting them by the welfare weights $g(\theta)$. Moreover, a characterization of the optimum tax schedule can then be directly obtained from this incidence analysis, by imposing that the welfare effects of any tax reform of the initial tax schedule T are equal to zero. Corollary 7 in Appendix E.1.1 provides such a formula in our general environment. In this section, we focus on the special case of a CES production function. This implies a parsimonious generalization of the result of Stiglitz (1982) derived in a two-skill environment, and connects it to the formula of Diamond (1998) derived for exogenous wages.

Proposition 3. *Assume that the production function is CES with elasticity of substitution $\sigma > 0$. Then the optimal marginal tax rate at income y^* satisfies*

$$\frac{T'(y^*)}{1 - T'(y^*)} = \left[\frac{1}{\varepsilon_r^S(y^*)} + \frac{1}{\varepsilon_r^D(y^*)}\right](1 - \bar{g}(y^*)) \frac{1 - F_Y(y^*)}{y^* f_Y(y^*)} + \frac{g(y^*) - 1}{\sigma}, \tag{22}$$

where $\varepsilon_r^D(y^*) = \sigma$ and $\bar{g}(y^*) \equiv \mathbb{E}[g(y)\,|y \geq y^*]$ is the average marginal social welfare weight above income y^*.

Proof. See Appendix E.1.2. □

The first term on the right hand side of (22) shows that, analogous to the optimal tax formula obtained in the model with exogenous wages (Diamond (1998), Saez (2001)), the marginal tax rate at income y^* is is decreasing in the average social marginal welfare weight $\bar{g}(y^*)$, and increasing in the hazard rate of the income

[37] See, e.g., Saez and Stantcheva (2016).

distribution $\frac{1-F_Y(y^*)}{y^* f_Y(y^*)}$. However, the standard inverse elasticity rule is modified: the relevant parameter is now the sum of the inverse elasticity of labor supply and the inverse elasticity of labor demand. Since $\varepsilon_w^D(y^*) = \sigma > 0$, this novel force tends to raise optimal marginal tax rates. Intuitively, increasing the marginal tax rate at y^* leads these agents to lower their labor supply, which raises their own wage and thus mitigates their behavioral response.

The second term, $(g(y^*) - 1)/\sigma$ captures the fact that the wage and welfare of type θ^* increase due to a higher marginal tax rate $T'(y^*)$, at the expense of the other individuals whose wage and welfare decrease (see Section 2.2). Suppose that the government values the welfare of individuals θ^* less than average, i.e., $g(y^*) < 1$.[38] This negative externality induced by the behavior of θ^* implies that the cost of raising the marginal tax rate at y^* is higher than in partial equilibrium, and tends to lower the optimal tax rate. Conversely, the government gains by raising the optimal tax rates of individuals y^* whose welfare is valued more than average, i.e., $g(y^*) > 1$. This induces these agents to work less and earn a higher wage, which makes them strictly better off, at the expense of the other individuals in the economy, whose wage decreases. This term creates therefore a force for higher marginal tax rates at the bottom and lower marginal tax rates at the top if the government has a strictly concave social objective.

In Appendix F.3, we evaluate the optimality condition (22) numerically and find that the optimal U-shape of the marginal tax rates found by Diamond (1998) is more pronounced when general equilibrium effects are taken into account. We show moreover that the results remain quantitatively very similar in the case of a Translog production function with distance-dependent elasticities of substitution.

6.3 Optimal top tax rate

Assuming that the tax schedule is set optimally as in (22), we can now derive the implications for the asymptotic optimal marginal tax rate. Let $\Pi > 1$ denote the Pareto coefficient of the tail of the income distribution, that is, $1 - F_Y(y) \sim c\, y^{-\Pi}$ as $y \to \infty$ for some constant c. We can show that if the production function is CES and the top marginal tax rate that applies to these incomes is constant, then the tail of the income distribution has the same Pareto coefficient at the optimum as in the current data, even though the wage distribution is endogenous. In other words,

[38] Note that the average social marginal welfare weight in the economy is equal to 1.

shifting up or down the top tax rate modifies wages, but the tail parameter Π of the income distribution stays constant. We obtain the following Corollary.

Corollary 5. *Assume that the production function is CES with parameter $\sigma > 0$, that the disutility of labor is isoelastic with parameter e, and that incomes are Pareto distributed at the tail with coefficient $\Pi > 1$. Assume moreover that the social marginal welfare weights at the top converge to a constant $\bar{g}$. Then the top tax rate of the optimal tax schedule is given by*

$$\tau^* = \frac{1-\bar{g}}{1-\bar{g}+\Pi\,\varepsilon_r\,\zeta}, \quad \text{with} \quad \varepsilon_r = \frac{e}{1+\frac{e}{\sigma}} \quad \text{and} \quad \zeta = \frac{1}{1-\Pi\frac{\varepsilon_r}{\sigma}}. \tag{23}$$

In particular, τ^* is strictly smaller than the optimal top tax rate in the model with exogenous wages ($\sigma = \infty$).

Proof. See Appendix E.1.3. □

Formula (23) generalizes the familiar top tax rate result of Saez (2001) (in which $\varepsilon_r = \varepsilon_r^S$ and $\zeta = 1$) to a CES production function. There is one new sufficient statistic, the elasticity of substitution between skills in production σ, that is no longer restricted to being infinite. This proposition implies a strictly lower top marginal tax rate than if wages were exogenous. Immediate calculations of the optimal top tax rate illustrate this formula.[39] Suppose that $\bar{g} = 0$, $\Pi = 2$, $e = 0.5$, and $\sigma = 1.5$.[40] We immediately obtain that the optimal tax rate on top incomes is equal to $\tau^*_{\text{ex}} = 50$ percent in the model with exogenous wages, and falls to $\tau^* = 40$ percent once the general equilibrium effects are taken into account. Suppose instead that $\Pi = 1.5$ and $e = 0.33$, then we get $\tau^*_{\text{ex}} = 66$ percent and $\tau^* = 64$ percent. In this case the trickle-down forces barely affect the optimum tax rate quantitatively. We provide more comprehensive comparative statics in Appendix E.1.3.

Note that, at first sight, this result may seem at odds with those of Section 3.2. We just showed that even when the government's objective is to maximize revenue, the optimal top tax rate is always lower than when wages are exogenous. Instead, in Section 3.2 we argued that the general-equilibrium forces could lead to additional

[39] Again, it is important to keep in mind that equation (23) holds only if the whole tax schedule is set optimally; see our discussion below.

[40] These values are meant to be only illustrative but they are in the range of those estimated in the empirical literature. See the calibration in Section 4.

benefits of raising marginal tax rates on high incomes. The reason for this discrepancy is that the optimum tax code is U-shaped and thus has a form of regressivity – relatively high marginal tax rates at the bottom and relatively low marginal tax rates at the top. Instead in Section 3.2, we analyzed partial reforms of a *suboptimal*, namely progressive, tax code, with low marginal tax rates at the bottom and high rates at the top. In the latter environment, even though the overall gains of reforming the suboptimal schedule always point towards the optimal (U-shaped) tax code, the general-equilibrium contribution to these overall gains tends to mitigate (resp., reinforce) the partial-equilibrium contribution if the tax system being reformed is progressive (resp., regressive). We discuss this point further in the next section and Appendix E.1.4.

6.4 Further results and discussion

In Appendix E.1.4, we extend the result of Diamond (1998); Saez (2001) regarding the U-shape of the optimum tax schedule. After defining a relevant "exogenous-wage optimum" benchmark in our environment,[41] we show in Corollary 8 that the general-equilibrium correction to the optimal tax rates is itself U-shaped. This result echoes those of Proposition 2 and Corollary 4, according to which the general-equilibrium effects of tax reforms have a shape that is inherited from that of the initial tax schedule (which, here, is the hypothetical optimal tax schedule assuming exogenous wages). Intuitively, since a fraction $T'(y)$ of the endogenous wage changes accrues to the government, tax revenue (i.e., Rawlsian welfare) increases by a larger amount in an economy with initially progressive taxes, whereas the converse is true when the tax rates are high at the bottom and low at the top, as in the optimum tax system (3). These observations allow us to unify the insights of Section 3.2 (according to which the endogeneity of wages raises the benefits of increasing the top income marginal tax rates) and those of Section 6.3 (according to which the optimal top tax rate is lower than in partial equilibrium). The key take-away is therefore that insights about the optimum tax schedule may actually be reversed when considering partial reforms of the current, suboptimal tax code.

[41] We follow Rothschild and Scheuer (2013, 2014) and consider a self-confirming policy equilibrium.

7 Conclusion

We developed a variational approach for the study of nonlinear tax reforms in general equilibrium. Our methodology consisted of using the tools of the theory of integral equations to characterize the incidence of reforming a given tax schedule, e.g. the current U.S. tax code, as well as the optimal tax schedule. The formulas we derived are expressed in terms of sufficient statistics. The direct empirical estimation of these cross-wage elasticities is an important avenue for future research.

References

Acemoglu, D. and D. Autor (2011): "Skills, Tasks and Technologies: Implications for Employment and Earnings," in *Handbook of Labor Economics Vol. 4*, ed. by O. Ashenfelter and D. Card, Elsevier, 1043–1171.

Ales, L., A. A. Bellofatto, and J. J. Wang (2017): "Taxing Atlas: Executive compensation, firm size, and their impact on optimal top income tax rates," *Review of Economic Dynamics*, 26, 62–90.

Ales, L., M. Kurnaz, and C. Sleet (2015): "Technical change, wage inequality, and taxes," *The American Economic Review*, 105, 3061–3101.

Ales, L. and C. Sleet (2016): "Taxing Top CEO Incomes," *American Economic Review (forthcoming)*.

Antras, P., A. de Gortari, and O. Itskhoki (2016): "Globalization, Inequality and Welfare," *Working Paper*.

Bénabou, R. (2002): "Tax and Education Policy in a Heterogenous-Agent Economy: What of Levels of Redistribution Maximize Growth and Efficiency?" *Econometrica*, 70, 481–517.

Borjas, G. J. (2003): "The Labor Demand Curve Is Downward Sloping: Reexamining the Impact of Immigration on the Labor Market," *The Quarterly Journal of Economics*, 118, 1335–1374.

——— (2015): "The wage impact of the Marielitos: A reappraisal," *ILR Review*.

Borjas, G. J., R. B. Freeman, L. F. Katz, J. DiNardo, and J. M. Abowd (1997): "How much do immigration and trade affect labor market outcomes?" *Brookings papers on economic activity*, 1997, 1–90.

Card, D. (1990): "The impact of the Mariel boatlift on the Miami labor market," *ILR Review*, 43, 245–257.

——— (2009): "Immigration and Inequality," *American Economic Review*, 99, 1–21.

Card, D. and T. Lemieux (2001): "Can Falling Supply Explain the Rising Return to College for Younger Men? A Cohort-Based Analysis," *Quarterly Journal of Economics*, 116, 705–746.

Cesarini, D., E. Lindqvist, M. J. Notowidigdo, and R. Ostling (2017): "The Effect of Wealth on Individual and Household Labor Supply: Evidence from Swedish Lotteries," *American Economic Review (forthcoming)*.

Chen, L. and C. Rothschild (2015): "Screening with endogenous preferences," *Working Paper*.

Chetty, R. (2009a): "Sufficient Statistics for Welfare Analysis: A Bridge Between Structural and Reduced-Form Methods," *Annual Review of Economics*, 1, 451–488.

——— (2009b): "Sufficient Statistics for Welfare Analysis: A Bridge Between Structural and Reduced-Form Methods," *Annu. Rev. Econ.*, 1, 451–488.

——— (2012): "Bounds on elasticities with optimization frictions: A synthesis of micro and macro evidence on labor supply," *Econometrica*, 80, 969–1018.

Christensen, L. R., D. W. Jorgenson, and L. J. Lau (1973): "Transcendental logarithmic production frontiers," *The Review of Economics and Statistics*, 28–45.

——— (1975): "Transcendental logarithmic utility functions," *The American Economic Review*, 65, 367–383.

Costinot, A. and J. Vogel (2010): "Matching and inequality in the world economy," *Journal of Political Economy*, 118, 747–786.

Diamond, P. A. (1998): "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates," *American Economic Review*, 88, 83–95.

Diamond, P. A. and E. Saez (2011): "The Case for a Progressive Tax: From Basic Research to Policy Recommendations," *Journal of Economic Perspectives*, 25, 165–90.

Dustmann, C., T. Frattini, and I. P. Preston (2013): "The effect of immigration along the distribution of wages," *The Review of Economic Studies*, 80, 145–173.

Fullerton, D. and G. E. Metcalf (2002): "Tax incidence," *Handbook of public economics*, 4, 1787–1872.

Golosov, M., A. Tsyvinski, and N. Werquin (2014): "A Variational Approach to the Analysis of Tax Systems," *Working Paper*.

Green, R. K. and M. D. Phillips (2015): "Demand for'The 1%': Tax Incidence and Implications for Optimal Income Tax Rates," *Working Paper*.

Guner, N., R. Kaygusuz, and G. Ventura (2014): "Income taxation of US households: Facts and parametric estimates," *Review of Economic Dynamics*, 17, 559–581.

Guner, N., C. Rauh, and G. Ventura (2017): "Means-Tested Transfers in the US: Facts and Parametric Estimates," *Working Paper*.

Harberger, A. C. (1962): "The incidence of the corporation income tax," *The Journal of Political Economy*, 215–240.

Heathcote, J., K. Storesletten, and G. L. Violante (2016): "Optimal tax progressivity: An analytical framework," *Quarterly Journal of Economics (forthcoming)*.

Hendren, N. (2015): "The Policy Elasticity," in *Tax Policy and the Economy, Volume 30*, University of Chicago Press.

Hines, J. R. (2009): "Peter Mieszkowski and the General Equilibrium Revolution in Public Finance," in *Proceedings. Annual Conference on Taxation and Minutes of the Annual Meeting of the National Tax Association*, JSTOR, vol. 102, 213–216.

Imbens, G. W., D. B. Rubin, and B. I. Sacerdote (2001): "Estimating the effect of unearned income on labor earnings, savings, and consumption: Evidence from a survey of lottery players," *American Economic Review*, 778–794.

Itskhoki, O. (2008): "Optimal redistribution in an open economy," *Working Paper*.

Jacquet, L. and E. Lehmann (2017): "Optimal Income Taxation with Composition Effects," *IZA Discussion Paper No. 11019*.

Katz, L. F. and K. M. Murphy (1992): "Changes in Relative Wages, 1963-1987: Supply and Demand Factors," *The Quarterly Journal of Economics*, 107, 35–78.

Kim, H. Y. (1992): "The translog production function and variable returns to scale," *The Review of Economics and Statistics*, 546–552.

Kleven, H., C. Kreiner, and E. Saez (2009): "The Optimal Income Taxation of Couples," *Econometrica*, 77, 537–560.

Kotlikoff, L. J. and L. H. Summers (1987): "Tax incidence," *Handbook of public economics*, 2, 1043–1092.

Kress, R. (2014): *Linear integral equations (Third Edition)*, Springer.

Matsuyama, K. and P. Ushchev (2017): "Beyond CES: Three Alternative Cases of Flexible Homothetic Demand Systems," .

Mirrlees, J. A. (1971): "An Exploration in the Theory of Optimum Income Taxation," *The Review of Economic Studies*, 38, 175–208.

Musgrave, R. A. and T. Thin (1948): "Income tax progression, 1929-48," *Journal of political Economy*, 56, 498–514.

Peri, G. and V. Yasenov (2015): "The labor market effects of a refugee wave: Applying the synthetic control method to the Mariel boatlift," Tech. rep., National Bureau of Economic Research.

Piketty, T. (1997): "La redistribution fiscale face au chomage," *Revue française d'économie*, 12, 157–201.

Piketty, T. and E. Saez (2013): "A Theory of Optimal Inheritance Taxation," *Econometrica*, 81, 1851–1886.

Piketty, T., E. Saez, and S. Stantcheva (2014): "Optimal taxation of top labor incomes: A tale of three elasticities," *American Economic Journal: Economic Policy*, 6, 230–271.

Polyanin, A. D. and A. V. Manzhirov (2008): *Handbook of integral equations*, CRC press.

Press, W. H. (2007): *Numerical recipes 3rd edition: The art of scientific computing*, Cambridge university press.

Rothschild, C. and F. Scheuer (2013): "Redistributive Taxation in the Roy Model," *The Quarterly Journal of Economics*, 128, 623–668.

——— (2014): "A theory of income taxation under multidimensional skill heterogeneity," *NBER Working Paper No. 19822.*

——— (2016): "Optimal Taxation with Rent-Seeking," *Review of Economic Studies (forthcoming).*

Rothstein, J. (2010): "Is the EITC as good as an NIT? Conditional cash transfers and tax incidence," *American Economic Journal: Economic Policy*, 2, 177–208.

Roy, A. D. (1951): "Some thoughts on the distribution of earnings," *Oxford economic papers*, 3, 135–146.

Saez, E. (2001): "Using Elasticities to Derive Optimal Income Tax Rates," *Review of Economic Studies*, 68, 205–229.

——— (2002): "Optimal Income Transfer Programs: Intensive versus Extensive Labor Supply Responses," *Quarterly Journal of Economics*, 117, 1039–1073.

Saez, E., J. Slemrod, and S. H. Giertz (2012): "The elasticity of taxable income with respect to marginal tax rates: A critical review," *Journal of economic literature*, 50, 3–50.

Saez, E. and S. Stantcheva (2016): "Generalized Social Marginal Welfare Weights for Optimal Tax Theory," *American Economic Review*, 106, 24–45.

Sattinger, M. (1975): "Comparative advantage and the distributions of earnings and abilities," *Econometrica: Journal of the Econometric Society*, 455–468.

Scheuer, F. (2014): "Entrepreneurial Taxation with Endogenous Entry," *American Economic Journal: Economic Policy*, 6, 126–163.

Scheuer, F. and I. Werning (2016): "Mirrlees meets Diamond-Mirrlees," *NBER Working Paper No. 22076.*

——— (2017): "The taxation of superstars," *The Quarterly Journal of Economics*, 132, 211–270.

Seierstad, A. and K. Sydsaeter (1986): *Optimal control theory with economic applications*, Elsevier North-Holland, Inc.

Shoven, J. B. and J. Whalley (1984): "Applied general-equilibrium models of taxation and international trade: an introduction and survey," *Journal of Economic Literature*, 22, 1007–1051.

Stiglitz, J. E. (1982): "Self-selection and Pareto efficient taxation," *Journal of Public Economics*, 17, 213–240.

Teulings, C. N. (1995): "The wage distribution in a model of the assignment of skills to jobs," *Journal of Political Economy*, 280–315.

——— (2005): "Comparative advantage, relative wages, and the accumulation of human capital," *Journal of Political Economy*, 113, 425–461.

Tricomi, F. (1985): "Integral equations, Interscience, London, 1957, New edition," .

Zemyan, S. M. (2012): *The classical theory of integral equations: a concise treatment*, Springer Science & Business Media.

A Proofs of Section 1

A.1 Elasticities

A.1.1 Labor supply elasticities

Proof of equation (8).

Labor supply elasticity with respect to the retention rate along the linear budget constraint. The first-order condition (1) can be rewritten as $v'(l(\theta)) = r(\theta) w(\theta)$, where $r(\theta) = 1 - T'(w(\theta) l(\theta))$ is the retention rate of agent θ. We assume that this first-order condition has a unique solution $l(\theta)$. The first-order effect of perturbing the retention rate $r(\theta)$ by $dr(\theta)$ on the labor supply $l(\theta)$, keeping $w(\theta)$ constant, is obtained by a Taylor approximation of the first-order condition in the perturbed equilibrium,

$$v'(l(\theta) + dl(\theta)) = (r(\theta) + dr(\theta)) w(\theta),$$

around the initial equilibrium. Straightforward algebra shows that

$$\frac{dl(\theta)}{l(\theta)} = -\frac{v'(l(\theta))}{l(\theta) v''(l(\theta))} \frac{dr(\theta)}{r(\theta)},$$

which immediately leads the expression for the elasticity $e(\theta) = \frac{v'(l(\theta))}{l(\theta)v''(l(\theta))}$.

Labor supply elasticity with respect to the retention rate along the non-linear budget constraint. The perturbed individual first-order condition reads

$$v'(l(\theta) + dl(\theta)) = [1 - T'(w(\theta)(l(\theta) + dl(\theta))) + dr(\theta)] w(\theta).$$

A first-order Taylor expansion leads to

$$\frac{dl(\theta)}{l(\theta)} = \frac{e(\theta)}{1 + e(\theta)\frac{w(\theta)l(\theta)T''(w(\theta)l(\theta))}{1-T'(w(\theta)l(\theta))}} \frac{dr(\theta)}{1 - T'(w(\theta) l(\theta))},$$

which yields the first part of equation (8).

Labor supply elasticity with respect to the wage along the non-linear budget constraint. The perturbed individual first-order condition reads

$$v'(l(\theta) + dl(\theta)) = [1 - T'((w(\theta) + dw(\theta))(l(\theta) + dl(\theta)))] (w(\theta) + dw(\theta)).$$

A first-order Taylor expansion then implies

$$\frac{dl(\theta)}{l(\theta)} = \frac{\left(1 - \frac{w(\theta)l(\theta)T''(w(\theta)l(\theta))}{1-T'(w(\theta)l(\theta))}\right)\frac{v'(l(\theta))}{l(\theta)v''(l(\theta))}}{1 + \frac{v'(l(\theta))}{l(\theta)v''(l(\theta))}\frac{w(\theta)l(\theta)T''(w(\theta)l(\theta))}{1-T'(w(\theta)l(\theta))}} \frac{dw(\theta)}{w(\theta)},$$

which yields the second part of equation (8).

Assumptions on the elasticities. We assume throughout our analysis that these elasticities of labor supply are well-defined, which requires that

$$|p(y(\theta))\, e(\theta)| < 1$$

for all θ. The condition $p(y(\theta))\, e(\theta) > -1$ ensures that the second-order condition of the individual problem is satisfied, so that (1) characterizes a local maximum of their utility. The condition $p(y(\theta))\, e(\theta) < 1$ ensures the convergence of the labor supply response towards the fixed point that characterizes the elasticities along the nonlinear budget constraint in equation (8). We assume in addition that

$$\left|\varepsilon_w^S(\theta)/\varepsilon_w^D(\theta)\right| < 1$$

so that the equilibrium labor elasticities introduced in Lemma 1 are well defined. These conditions can be easily checked for a given tax schedule. The former is also required in the partial-equilibrium model with exogenous wages, while the latter is always satisfied in our microfoundation of Section 1.3.

□

A.1.2 Wage elasticities and Euler's theorem

We provide two versions of Euler's homogeneous function theorem in our economy.

Lemma 2. *The following relationship between the own-wage elasticity and the structural cross-wage elasticities is satisfied: for all y^*,*

$$-\frac{1}{\varepsilon_w^D(y^*)} y^* f_Y(y^*) + \int_{\mathbb{R}_+} \gamma(y, y^*) y f_Y(y)\, dy = 0. \tag{24}$$

Equivalently, this can be expressed as a relationship between the own-wage elasticity and the GE cross-wage elasticities: for all y^*,

$$-\frac{1}{\varepsilon_w^D(y^*)} y^* f_Y(y^*) + \int_{\mathbb{R}_+} \frac{\Gamma(y, y^*)}{1 + \varepsilon_w^S(y)/\varepsilon_w^D(y)} y f_Y(y)\, dy = 0. \tag{25}$$

Proof of Lemma 2.

Equation (24). Constant returns to scale imply that $\mathscr{F}(\{\lambda L(\theta)\}_{\theta\in\Theta}) = \lambda \mathscr{F}(\{\lambda L(\theta)\}_{\theta\in\Theta})$ for all λ. Differentiating both sides of this equation with respect to and evaluating at $\lambda = 1$ leads to $\mathscr{F}(\{\lambda L(\theta)\}_{\theta\in\Theta}) = \int_\Theta L(\theta)\, w(\theta)\, d\theta$. Differentiating both sides of this equation with respect to $L(\theta')$, using definitions (6) and (7) and rearranging terms leads to

$$-\frac{1}{\varepsilon_w^D(\theta')} y(\theta') f(\theta') + \int_\Theta \gamma(\theta, \theta') y(\theta) f(\theta)\, d\theta = 0. \tag{26}$$

Equivalently, changing variables from types θ to incomes $y(\theta)$ and defining

$$\gamma\left(y\left(\theta\right),y\left(\theta'\right)\right)\equiv\left(\frac{dy}{d\theta}\left(\theta'\right)\right)^{-1}\gamma\left(\theta,\theta'\right) \tag{27}$$

leads to (24).

Equation (25). Euler's theorem (24) implies that

$$\begin{aligned}\int_{\Theta}\frac{\hat{w}\left(\theta\right)}{w\left(\theta\right)}y\left(\theta\right)f\left(\theta\right)d\theta &= \int_{\Theta}\left[-\frac{1}{\varepsilon_{w}^{D}\left(\theta\right)}\hat{l}\left(\theta\right)+\int_{\Theta}\gamma\left(\theta,\theta'\right)\hat{l}\left(\theta'\right)d\theta'\right]y\left(\theta\right)f\left(\theta\right)d\theta\\ &=-\int_{\Theta}\left\{\frac{1}{\varepsilon_{w}^{D}\left(\theta\right)}y\left(\theta\right)f\left(\theta\right)+\int_{\Theta}\gamma\left(\theta',\theta\right)y\left(\theta'\right)f\left(\theta'\right)d\theta'\right\}\hat{l}\left(\theta\right)d\theta=0.\end{aligned}$$

Now, using formulas (10) and (14) applied to the elementary tax reform $\delta\left(y-y^{*}\right)$, we can rewrite the previous equality as

$$\begin{aligned}0&=\int_{\Theta}\frac{1}{\varepsilon_{w}^{S}\left(\theta\right)}\left[\varepsilon_{r}^{S}\left(\theta\right)\frac{\delta\left(y\left(\theta\right)-y^{*}\right)}{1-T'\left(y\left(\theta\right)\right)}+\hat{l}\left(\theta\right)\right]y\left(\theta\right)f\left(\theta\right)d\theta\\ &=\int_{\mathbb{R}_{+}}\frac{1}{\varepsilon_{w}^{S}\left(y\right)}\left[\varepsilon_{r}^{S}\left(y\right)\frac{\delta\left(y-y^{*}\right)}{1-T'\left(y\right)}-\varepsilon_{r}\left(y\right)\frac{\delta\left(y-y^{*}\right)}{1-T'\left(y\right)}-\varepsilon_{w}\left(y\right)\frac{\Gamma\left(y,y^{*}\right)\varepsilon_{r}\left(y^{*}\right)}{1-T'\left(y^{*}\right)}\right]yf\left(y\right)dy\\ &=\frac{1}{1-T'\left(y^{*}\right)}\varepsilon_{r}\left(y^{*}\right)\left[\frac{1}{\varepsilon_{w}^{D}(y^{*})}y^{*}f\left(y^{*}\right)-\int_{\mathbb{R}_{+}}\frac{\Gamma\left(y,y^{*}\right)}{1+\varepsilon_{w}^{S}\left(y\right)/\varepsilon_{w}^{D}(y^{*})}yf\left(y\right)dy\right],\end{aligned}$$

where the last equality follows from the fact that $\frac{\varepsilon_{r}^{S}(y^{*})-\varepsilon_{r}(y^{*})}{\varepsilon_{w}^{S}(y^{*})}=\frac{\varepsilon_{r}(y^{*})}{\varepsilon_{w}^{D}(y^{*})}$ and $\frac{\varepsilon_{w}(y)}{\varepsilon_{w}^{S}(y)}=\frac{1}{1+\varepsilon_{w}^{S}(y)/\varepsilon_{w}^{D}(y^{*})}$. This leads to formula (25).

□

A.2 Reduced-form production function

In this section, we consider the reduced-form production function $\mathcal{F}(\{L\left(\theta\right)\}_{\theta\in\Theta})$ introduced in Section 1.1. We show that itimplies a monotone mapping between wages and incomes, $y'(w)\geq 0$, or equivalently between types and incomes $y'(\theta)\geq 0$.

General technology. First, note that the individual first-order condition (1) implies that the elasticity of income with respect to the wage (i.e., the difference between the incomes y_1, y_2 of two agents with different wages w_1, w_2 in the cross-sectional distribution) is given by

$$\frac{w\left(\theta\right)}{y\left(w\left(\theta\right)\right)}y'(w\left(\theta\right))=1+\varepsilon_{w}^{S}\left(\theta\right).$$

This equation is the equivalent of Lemma 1 from Saez (2001) in our setting. It shows that $y'(w)>0$ if and only if $\varepsilon_{w}^{S}>-1$. We then show that $\varepsilon_{w}^{S}<-1$ would violate the Spence-Mirrlees condition. We have

$$\varepsilon_{w}^{S}\left(\theta\right)=\left(1-p\left(y\left(\theta\right)\right)\right)\frac{e\left(\theta\right)}{1+p\left(y\left(\theta\right)\right)e\left(\theta\right)}<-1\iff e\left(\theta\right)<-1.$$

Now, the Spence-Mirrlees condition requires that $v'\left(\frac{y}{w}\right)\frac{1}{w}$ is decreasing in w. This is equivalent to

$$-v''\left(\frac{y}{w}\right)\frac{y}{w^3}-v'\left(\frac{y}{w}\right)\frac{1}{w^2}<0 \iff -1<\frac{v'\left(\frac{y}{w}\right)}{\frac{y}{w}v''\left(\frac{y}{w}\right)}.$$

The right hand side of the last inequality is the labot supply elasticity e. This concludes the proof.

CES technology. We now show that if the production function is CES, the monotone mapping between types and wages, which is ensured for a given tax schedule by appropriately ordering the types, is preserved for any (possibly non-local) tax reform. This implies that the ordering of types does not change between the wage distribution calibrated using current data and the one implied by the optimal tax schedule. Without loss of generality we assume that types are uniformly distributed on the unit interval $\Theta = [0,1]$, so that $f(\theta) = 1$ for all θ. For a CES production function, recall that the wage of type θ is given by:

$$w(\theta) = a(\theta)\left(\frac{l(\theta)}{F(\mathscr{L})}\right)^{-\frac{1}{\sigma}}.$$

This implies

$$\frac{w'(\theta)}{w(\theta)} = \frac{a'(\theta)}{a(\theta)} - \frac{1}{\sigma}\frac{l'(\theta)}{l(\theta)} = \frac{a'(\theta)}{a(\theta)} - \frac{1}{\sigma}\frac{1}{l(\theta)}\frac{dl(\theta)}{dw(\theta)}w'(\theta),$$

and hence

$$\frac{w'(\theta)}{w(\theta)}\left(1+\frac{\varepsilon_w^S(\theta)}{\sigma}\right) = \frac{a'(\theta)}{a(\theta)}.$$

This shows that $w'(\theta)$ has the same sign as $a'(\theta)$ if we have

$$1+\frac{\varepsilon_w^S(\theta)}{\sigma} > 0.$$

Note that this is the condition that ensures that the equilibrium labor elasticities $\varepsilon_w(\theta)$ (introduced in Lemma 1) are well defined, which we assume throughout. Note that this condition always holds if the Spence-Mirrlees condition is fulfilled and $\sigma \geq 1$. It also holds for any $\sigma \geq 0$ if $\varepsilon_w^S(\theta) > 0$, which holds whenever the local rate of progressivity satisfies $p(y(\theta)) \leq 1$. Therefore the sign of $w'(\theta)$ is the same as that of $a'(\theta)$ independently of the tax system.

A.3 Microfoundation of the production function

In this section we describe the microfoundation of the production function $Y = \mathcal{F}(\{L(\theta)\}_{\theta\in\Theta})$. We extend the model of endogenous assignment of skills to tasks of Costinot and Vogel (2010) to incorporate endogenous labor supply choices by agents and nonlinear labor income taxes.

There is a continuum of mass one of agents indexed by their skill, $\theta \in \Theta = [\underline{\theta}, \bar{\theta}]$ and a continuum of tasks (e.g., manual, routine, abstract, etc.) indexed by their skill intensity, $\psi \in \Psi = [\underline{\psi}, \bar{\psi}]$. We denote by f the density of skills $\theta \in \Theta$ in the population. Let $A(\theta,\psi)$ be the product of a unit of labor of skill θ employed in task ψ. We assume that high-skill workers have a comparative advantage in tasks with high skill intensity, i.e., $A(\theta,\psi)$ is strictly log-supermodular:

$$A(\theta',\psi')\,A(\theta,\psi) \quad > \quad A(\theta,\psi')\,A(\theta',\psi)\,, \;\; \forall\theta'>\theta \;\text{ and }\; \psi'>\psi. \tag{28}$$

Individuals. An agent with skill θ earns wage $w(\theta)$ which he takes as given. The first-order condition for labor supply is given by

$$v'(l(\theta)) \quad = \quad [1-T'(w(\theta)\,l(\theta))]\,w(\theta)\,. \tag{29}$$

We assume that there is a unique solution $l(\theta)>0$ for all θ, and denote by $c(\theta)$ the agent's consumption of the final good.

Final good firm. The final good Y is produced using as inputs the output $Y(\psi)$ of each task $\psi\in\Psi$. For simplicity and as in Costinot and Vogel (2010), we assume that the production function is CES, so that the final good output is given by

$$Y \quad = \quad \left\{\int_{\underline{\psi}}^{\bar{\psi}} B(\psi)\,[Y(\psi)]^{\frac{\sigma-1}{\sigma}}\,d\psi\right\}^{\frac{\sigma}{\sigma-1}}.$$

The final good firm chooses the quantities of inputs $Y(\psi)$ of each type ψ to maximize its profit, i.e., it solves:

$$\max_{\{Y(\psi)\}_{\psi\in\Psi}} \quad Y-\int_{\Psi} p(\psi)\,Y(\psi)\,d\psi,$$

where $p(\psi)$ is the price of task ψ which the firm takes as given. The first-order conditions read: $\forall\psi\in\Psi$,

$$Y(\psi) \quad = \quad [p(\psi)]^{-\sigma}\,[B(\psi)]^{\sigma}\,Y. \tag{30}$$

Intermediate good firms. The output of task ψ is produced linearly by intermediate firms that hire the labor $L(\theta\mid\psi)$ of skills $\theta\in\Theta$ that they hire, so that

$$Y(\psi) \quad = \quad \int_{\Theta} A(\theta,\psi)\,L(\theta\mid\psi)\,d\theta.$$

The intermediate good firm of type ψ chooses its demand for labor $L(\theta)$ of each skill θ to maximize its profit taking the wage $w(\theta)$ as given, i.e., it solves:

$$\max_{\{L(\theta)\}_{\theta\in\Theta}} \quad p(\psi)\,Y(\psi)-\int_{\Theta} w(\theta)\,L(\theta\mid\psi)\,d\theta.$$

The first-order condition implies that this firm is willing to hire any quantity of labor that is supplied by the workers of type θ as long as their wage is given by

$$w(\theta) = p(\psi) A(\theta, \psi), \text{ if } L(\theta \mid \psi) > 0. \tag{31}$$

Moreover, the wage of any skill θ that is *not* employed in task ψ must satisfy

$$w(\theta) \geq p(\psi) A(\theta, \psi), \text{ if } L(\theta \mid \psi) = 0. \tag{32}$$

Market clearing. We first impose that the market for the final good market clears. This condition reads:

$$Y = \int_\Theta c(\theta) f(\theta) d\theta + \mathcal{R},$$

where

$$\mathcal{R} \equiv \int_\Theta T(w(\theta) l(\theta)) f(\theta) d\theta$$

is the government revenue in the initial equilibrium, that it uses to buy the final good. Using the agents' and the government budget constraints, this can be rewritten as:

$$Y = \int_\Theta w(\theta) l(\theta) f(\theta) d\theta. \tag{33}$$

Second, we impose that the market for each intermediate good $\psi \in \Psi$ clears. For simplicity, we assume at the outset that there is a one-to-one matching function $M : \Theta \to \Psi$ between skills and tasks – we show below that it is indeed the case in equilibrium. Letting $\psi = M(\theta)$ be the task assigned to skill θ, we must then have

$$\int_{\underline{\psi}}^{M(\theta)} Y(\psi) d\psi = \int_{\underline{\theta}}^{\theta} A(\theta', M(\theta')) L(\theta' \mid M(\theta')) d\theta',$$

or simply $Y(\psi) d\psi = A(\theta, M(\theta)) L(\theta \mid M(\theta)) d\theta$. This implies: $\forall \theta \in \Theta$,

$$Y(M(\theta)) M'(\theta) = A(\theta, M(\theta)) L(\theta \mid M(\theta)). \tag{34}$$

Formally, this condition is obtained by substituting for $L(\theta \mid \psi) = \delta_{\{\psi = M(\theta)\}}$ in the equation $Y(\psi) = \int_\Theta A(\theta, \psi) L(\theta \mid \psi) d\theta$, where δ is the dirac delta function, and changing variables from skills to tasks to compute the integral.

Third, we impose that the market for labor of each skill $\theta \in \Theta$ clears. These conditions read: $\forall \theta \in \Theta$,

$$l(\theta) f(\theta) = L(\theta \mid M(\theta)). \tag{35}$$

Competitive equilibrium. Given a tax function $T : \mathbb{R}_+ \to \mathbb{R}$, an equilibrium consists of a schedule of labor supplies $\{l(\theta)\}_{\theta\in\Theta}$, labor demands $\{L(\theta \mid \psi)\}_{\theta\in\Theta,\psi\in\Psi}$, intermediate goods $\{Y(\psi)\}_{\psi\in\Psi}$, final good Y, wages $\{w(\theta)\}_{\theta\in\Theta}$, prices $\{p(\psi)\}_{\psi\in\Psi}$, and a matching function $M : \Theta \to \Psi$ such that equations (29) to (35) hold.

Equilibrium assignment. The first part of the analysis consists of proving the existence of the continuous and strictly increasing one-to-one matching function $M : \Theta \to \Psi$ with $M(\underline{\theta}) = \underline{\psi}$ and $M(\bar{\theta}) = \bar{\psi}$. That is, there is positive assortative matching. The proof is identical to that in Costinot and Vogel (2010). The second part of the analysis consists of characterizing the matching function and the wage schedule. Specifically, we show that

$$M'(\theta) = \frac{A(\theta, M(\theta))\, l(\theta)\, f(\theta)}{[p(M(\theta))]^{-\sigma}\, [B(M(\theta))]^{\sigma}\, Y} \tag{36}$$

with $M(\underline{\theta}) = \underline{\psi}$ and $M(\bar{\theta}) = \bar{\psi}$, and where Y is given by (33) and $p(M(\theta))$ is given by (31).

$$\frac{w'(\theta)}{w(\theta)} = \frac{A_1'(\theta, M(\theta))}{A(\theta, M(\theta))}. \tag{37}$$

Proof of equations (36) and (37). Equation (36), which characterizes the equilibrium matching as the solution to a nonlinear differential equation, is a direct consequence of the market clearing equation (34), in which we use (30) to substitute for $Y(M(\theta))$. Equation (37), which characterizes the equilibrium wage schedule, is a consequence of the firms' profit maximization conditions (31). Specifically, we have

$$\begin{aligned} w(\theta) &= p(M(\theta))\, A(\theta, M(\theta)) \\ w(\theta + d\theta) &\geq p(M(\theta))\, A(\theta + d\theta, M(\theta)) \end{aligned}$$

and analogously,

$$\begin{aligned} w(\theta + d\theta) &= p(M(\theta + d\theta))\, A(\theta + d\theta, M(\theta + d\theta)) \\ w(\theta) &\geq p(M(\theta + d\theta))\, A(\theta, M(\theta + d\theta)). \end{aligned}$$

These easily imply bounds on $[w(\theta + d\theta) - w(\theta)]/d\theta$. Letting $d\theta \to 0$ yields the result.

□

This analysis implies the endogeneity of labor supply does not alter Costinot and Vogel (2010)'s results. Indeed, Costinot and Vogel (2010) allow for an arbitrary supply of skills $V(\theta)$, which we can interpret in our framework as the density of labor supply, $V(\theta) = l(\theta)\, f(\theta)$, where $l(\theta)$ is fixed at its equilibrium value determined by (29).

Reduced form production function. Equilibrium assignment of skills to taxes is endogenous to taxes – we denote by $M(\cdot \mid T) : \Theta \to \Psi$ the matching function with T as an explicit argument. The main result, for our purposes, is that *the tax schedule T affects the equilibrium assignment only* *through its effect on agents' labor supply choices* $\mathscr{L} \equiv \{l(\theta) f(\theta)\}_{\theta \in \Theta}$. Indeed, note that none of the equations (30)-(35), which define the equilibrium *for given labor supply levels* $\{l(\theta)\}_{\theta \in \Theta}$, depend directly on T. This implies that if two distinct tax schedules lead to the same equilibrium labor supply choices $\mathscr{L}$, they will also lead to the same assignment of skills to tasks M. Therefore, the matching function $M(\cdot \mid T)$ can be rewritten as $M(\cdot \mid \mathscr{L})$.

This result implies that the model can be summarized by a reduced-form production function $\mathcal{F}(\mathscr{L})$ over the labor supplies of different *skills* in the population. To see this, note that the production function (over tasks) of the final good can be written as

$$\begin{aligned} Y &= \left\{ \int_{\underline{\psi}}^{\bar{\psi}} B(\psi) \left[Y(\psi) \right]^{\frac{\sigma-1}{\sigma}} d\psi \right\}^{\frac{\sigma}{\sigma-1}} \\ &= \left\{ \int_{\underline{\theta}}^{\bar{\theta}} B(M(\theta)) \left[Y(M(\theta)) \right]^{\frac{\sigma-1}{\sigma}} M'(\theta)\, d\theta \right\}^{\frac{\sigma}{\sigma-1}} \\ &= \left\{ \int_{\underline{\theta}}^{\bar{\theta}} B(M(\theta)) \left[A(\theta, M(\theta))\, l(\theta) f(\theta) \right]^{\frac{\sigma-1}{\sigma}} \left[M'(\theta) \right]^{\frac{1}{\sigma}} d\theta \right\}^{\frac{\sigma}{\sigma-1}}, \end{aligned}$$

where the second equality follows from a change of variables from tasks to skills using the one-to-one map M between the two variables, and the third equality uses the market clearing conditions (34) and (35) to substitute for $Y(M(\theta))$. Now, rearrange the terms in this expression to obtain:

$$Y = \left\{ \int_{\underline{\theta}}^{\bar{\theta}} a(\theta, M) \left[l(\theta) f(\theta) \right]^{\frac{\sigma-1}{\sigma}} d\theta \right\}^{\frac{\sigma}{\sigma-1}}, \tag{38}$$

where we let

$$a(\theta, M) \equiv B(M(\theta)) \left[A(\theta, M(\theta)) \right]^{\frac{\sigma-1}{\sigma}} \left[M'(\theta) \right]^{\frac{1}{\sigma}}.$$

Note that, of course, this reduced-form production function is consistent with the wage schedule derived above. We find that $w(\theta) = B(M(\theta))\, A(\theta, M(\theta)) \left[\frac{Y}{Y(M(\theta))}\right]^{1/\sigma}$ by combining (31) and (30). Differentiating the reduced-form production function (38) with respect to $l(\theta) f(\theta)$ and using (34) leads to the same expression.

Equation (38) defines a production function over skills $\theta \in \Theta$ (rather than tasks). Interestingly, note that this production function inherits the CES structure of the original production function, except that the technological coefficients $a(\theta, M)$ are now endogenous to taxes, since they depend on the matching function M. We can write (38) as a function $\tilde{\mathcal{F}}(\{l(\theta) f(\theta)\}_{\theta \in \Theta}, M) \equiv \tilde{\mathcal{F}}(\mathscr{L}, M)$. Now, using the result proved above that the function $M \equiv M(\cdot \mid \mathscr{L})$ depends on taxes only through the equilibrium labor supplies $\mathscr{L}$, we finally obtain the following reduced form production function:

$$Y = \mathcal{F}(\mathscr{L}). \tag{39}$$

Cross-wage elasticities. Using the reduced-form production function (39), all of the results we have derived go through. We can still define wages as: for all θ,

$$w(\theta) \equiv \frac{\partial \mathcal{F}(\mathscr{L})}{\partial [l(\theta) f(\theta)]},$$

the cross-wage elasticities as: for all $\theta' \neq \theta$,

$$\gamma(\theta, \theta') \equiv \frac{\partial \ln w(\theta)}{\partial \ln [l(\theta') f(\theta')]},$$

and the own-wage elasticities $1/\varepsilon_w^D(\theta)$ as the jump, if there is one, in the cross-wage elasticities as $\theta' \approx \theta$. These cross-wage elasticities are defined as the impact of an exogenous shock to the supply of labor of type θ' (e.g., an immigration inflow) on the wage of type θ, keeping everyone's labor supply constant otherwise (since they are defined as a partial derivative), but allowing for the endogenous re-assignment of skills to tasks following this exogenous shock. Indeed, the reduced-form production function $\mathcal{F}$ accounts for the dependence of the matching function on agents' labor supplies.

A.4 Special cases of production functions

A.4.1 CES technology

Wages, cross-wage elasticities and Euler theorem. The CES technology is defined by (4). The wage schedule is then given by

$$w(\theta) = a(\theta) (L(\theta))^{-\frac{1}{\sigma}} \left[\int_\Theta a(x) (L(x))^{\frac{\sigma-1}{\sigma}} dx \right]^{\frac{1}{\sigma-1}}.$$

The cross-wage and own-wage elasticities are given by

$$\gamma(\theta, \theta') = \frac{1}{\sigma} \frac{a(\theta') (L(\theta'))^{\frac{\sigma-1}{\sigma}}}{\int_\Theta a(x) (L(x))^{\frac{\sigma-1}{\sigma}} dx} \quad \text{and} \quad \frac{1}{\varepsilon_w^D(\theta)} = \frac{1}{\sigma}. \tag{40}$$

This implies in particular, for all $\theta \in \Theta$,

$$\int_\Theta \gamma(\theta, \theta') d\theta' = \frac{1}{\sigma}.$$

Since $\gamma(\theta, \theta')$ does not depend on θ, Euler's homogeneous function theorem (26) can be rewritten as

$$-\frac{1}{\sigma} y' f_Y(y') (y'(\theta')) + \frac{1}{\sigma} \frac{a(\theta') (L(\theta'))^{\frac{\sigma-1}{\sigma}}}{\int_\Theta a(x) (L(x))^{\frac{\sigma-1}{\sigma}} dx} \int_{\mathbb{R}_+} y f_Y(y) dy = 0,$$

i.e.,

$$\frac{a(\theta') (L(\theta'))^{\frac{\sigma-1}{\sigma}}}{\int_\Theta a(x) (L(x))^{\frac{\sigma-1}{\sigma}} dx} = (y'(\theta')) \frac{y' f_Y(y')}{\int_{\mathbb{R}_+} x f_Y(x) dx}.$$

Substituting in expression (40) and changing variables, we thus obtain

$$\gamma\left(y,y'\right)=\frac{1}{\sigma}\frac{y'f_{Y}\left(y'\right)}{\int_{\mathbb{R}_{+}}xf_{Y}\left(x\right)dx}. \tag{41}$$

Labor supply elasticities with a CRP tax schedule. Next, assume in addition that the disutility of labor is isoelastic, i.e. $v\left(l\right)=\frac{l^{1+1/e}}{1+1/e}$, and that the initial tax schedule is CRP, as defined by (3). In particular, we have $1-T'\left(y\right)=\left(1-m\right)y^{-p}$ and $T''\left(y\right)=p\left(1-m\right)y^{-p-1}$. The labor supply elasticities (8) and the equilibrium labor elasticities (introduced in Lemma 1) are then all constant and given by

$$\begin{aligned}
\varepsilon_{r}^{S}\left(y\right)&=\frac{1-T'\left(y\right)}{1-T'\left(y\right)+eyT''\left(y\right)}e=\frac{e}{1+pe},\\
\varepsilon_{w}^{S}\left(y\right)&=\frac{1-T'\left(y\right)-yT''\left(y\right)}{1-T'\left(y\right)+eyT''\left(y\right)}e=\frac{\left(1-p\right)e}{1+pe},\\
\varepsilon_{r}\left(y\right)&=\frac{\varepsilon_{r}^{S}\left(\theta\right)}{1+\varepsilon_{w}^{S}\left(\theta\right)/\varepsilon_{w}^{D}\left(\theta\right)}=\frac{e}{1+pe+\left(1-p\right)\frac{e}{\sigma}},\\
\varepsilon_{w}\left(y\right)&=\frac{\varepsilon_{w}^{S}\left(\theta\right)}{1+\varepsilon_{w}^{S}\left(\theta\right)/\varepsilon_{w}^{D}\left(\theta\right)}=\frac{\left(1-p\right)e}{1+pe+\left(1-p\right)\frac{e}{\sigma}}.
\end{aligned} \tag{42}$$

Sufficient conditions ensuring the convergence of the resolvent (11). Suppose that the production function is CES with parameter σ, that the disutility of labor is isoelastic with parameter e, and that the initial tax schedule is CRP with parameter $p<1$. Corollary 1 implies that the resolvent series converges if

$$\frac{1}{\sigma\mathbb{E}y}\mathbb{E}\left[y\varepsilon_{w}\left(y\right)\right]=\frac{\left(1-p\right)e}{1+pe+\left(1-p\right)\frac{e}{\sigma}}<1.$$

Since $\left(1-p\right)e>0$, this condition is satisfied if $1+pe>0$. Recall that this condition is the second-order condition of the individual problem, which we assume is satisfied throughout the analysis. In particular, in the calibration to the U.S. economy, we have $p=0.15>0>-\frac{1}{e}\approx-3$ so this clearly holds.

A.4.2 Translog technology

The CES production function implies that, e.g., high-skill workers are equally substitutable with middle-skill workers as they are with low-skill workers. We now propose a more flexible parametrization of the production function that allows the elasticities of substitution to be distance-dependent, that is, closer skill types to be stronger substitutes (see Teulings (2005) and Section A.3).

Definition. The transcendental-logarithmic (Translog) production function is defined by

$$\begin{aligned}
&\ln\mathscr{F}(\{L\left(\theta\right)\}_{\theta\in\Theta})=a_{0}+\int_{\Theta}a\left(\theta\right)\ln L\left(\theta\right)d\theta+\dots\\
&\frac{1}{2}\int_{\Theta}\beta\left(\theta\right)\left(\ln L\left(\theta\right)\right)^{2}d\theta+\frac{1}{2}\int_{\Theta\times\Theta}\chi\left(\theta,\theta'\right)\left(\ln L\left(\theta\right)\right)\left(\ln L\left(\theta'\right)\right)d\theta d\theta',
\end{aligned} \tag{43}$$

where for all θ, θ', $\int_\Theta a(\theta')\, d\theta' = 1$, $\chi(\theta, \theta') = \chi(\theta', \theta)$, and $\beta(\theta) = -\int_\Theta \chi(\theta, \theta')\, d\theta'$. It is easy to check that these conditions ensure that the production function has constant returns to scale. When $\chi(\theta, \theta') = 0$ for all θ, θ', the production function is Cobb-Douglas. This specification can be used as a second-order local approximation to any production function (Christensen, Jorgenson, and Lau, 1973).

Wages and elasticities. The wage of type θ^* is given by

$$\begin{aligned} w(\theta^*) &= \lim_{\mu \to 0} \frac{1}{\mu} \left[\mathscr{F}(\mathscr{L} + \mu \delta(\theta - \theta^*)) - \mathscr{F}(\mathscr{L}) \right] \\ &= \frac{\mathscr{F}(\mathscr{L})}{L(\theta^*)} \{ a(\theta^*) + \beta(\theta^*) \ln L(\theta^*) + \int_\Theta \chi(\theta^*, \theta'') \ln(L(\theta''))\, d\theta'' \}. \end{aligned}$$

The cross-wage and own-wage elasticities are then given by

$$\gamma(\theta, \theta') = \left(\frac{w(\theta') L(\theta')}{\mathscr{F}(\mathscr{L})} \right) + \left(\frac{w(\theta) L(\theta)}{\mathscr{F}(\mathscr{L})} \right)^{-1} \chi(\theta, \theta') \text{ and } \frac{1}{\varepsilon_w^D(\theta)} = 1 - \left(\frac{w(\theta) L(\theta)}{\mathscr{F}(\mathscr{L})} \right)^{-1} \beta(\theta).$$

Finally, we have

$$\ln\left(\frac{w(\theta)}{w(\theta')} \right) = \ln\left(\frac{L(\theta')}{L(\theta)} \right) + \ln \frac{a(\theta) + \int_\Theta \chi(\theta, \theta'') \ln(L(\theta'')/L(\theta))\, d\theta''}{a(\theta') + \int_\Theta \chi(\theta', \theta'') \ln(L(\theta'')/L(\theta'))\, d\theta''},$$

so that the elasticities of substitution are given by

$$\frac{1}{\sigma(\theta, \theta')} = -\frac{\partial \ln(w(\theta)/w(\theta'))}{\partial \ln(L(\theta)/L(\theta'))} = 1 + \left[\left(\frac{w(\theta) L(\theta)}{\mathscr{F}(\mathscr{L})} \right)^{-1} + \left(\frac{w(\theta') L(\theta')}{\mathscr{F}(\mathscr{L})} \right)^{-1} \right] \chi(\theta, \theta').$$

Define the elasticities of substitution between two income levels as $\sigma(y, y') = -\frac{\partial \ln(w(y)/w(y'))}{\partial \ln(L(y)/L(y'))}$. For $y = y(\theta)$, let $a(y) \equiv a(\theta)$, $\beta(y) = \beta(\theta)$, and $\chi(y, y') = \left(\frac{dy(\theta')}{d\theta} \right)^{-1} \chi(\theta, \theta')$. We then have

$$\frac{1}{\sigma(y, y')} = 1 + \left[\left(\frac{w(y) L(y)}{\mathscr{F}(\mathscr{L})} \right)^{-1} + \left(\frac{w(y') L(y')}{\mathscr{F}(\mathscr{L})} \right)^{-1} \right] \frac{dy(\theta')}{d\theta} \chi(y, y').$$

This change of variables ensures that the (arbitrary) choice of the underlying set of types Θ does not impact the elasticity of substitution between two agents. We propose a calibration of these elasticities in Section F.1.4 below.

A.4.3 HSA technology and separable kernels

We finally propose a class of production functions with variable elasticities of substitution for which the incidence of tax reforms is as simple as for the CES production.

Separable kernels: special case

Suppose that the kernel $\varepsilon_w(\theta)\gamma(\theta,\theta')$ of the integral equation (9) is multiplicatively separable between θ and θ', i.e., the cross-wage elasticities have the following form: for all θ,θ',

$$\gamma(\theta,\theta') = \gamma_1(\theta)\times\gamma_2(\theta'). \tag{44}$$

The kernel (44) can be interpreted as follows. First, there is a "fixed effect" $\gamma_2(\theta')$ for skill θ' that determines how much the labor effort of these agents affects the wages of other workers. E.g., high-skill workers may have a stronger impact on the wage distribution than low- or middle-skill workers. Second, there is a fixed effect $\gamma_1(\theta)$ for skill θ that determines how sensitive the wage of agents with skill θ is to the labor effort of other workers. E.g., the wage of middle-skill workers may react more strongly to overall changes in labor effort. The total impact of the labor effort of skill θ' on the wage of skill θ is the product of these two fixed effects.

A special case of (44) arises when the production function is CES, because then the cross-wage elasticities $\gamma(\theta,\theta')$ do not depend on θ. Specifically, we have in this case

$$\gamma(\theta,\theta') = \frac{1}{\sigma}a(\theta')\left(L(\theta')/\mathscr{F}(\mathscr{L})\right)^{\frac{\sigma-1}{\sigma}}.$$

Thus, in particular, a one-percent increase in the labor effort of skill θ' raises the wage of every agent $\theta\neq\theta'$ by the same percentage amount. The CES functional form implies moreover that the elasticities of subsitution between any two pairs of skills are constant, i.e. $\sigma(\theta,\theta')=\sigma$. The separability restriction (44), however, is consistent with a larger class of production functions with variable elasticities of substitution, as we now show.

HSA production functions. We now study a class of production functions that generates a multiplicatively separable kernel (44). The "homothetic demand systems with a single aggregator " (HSA), defined in (5), has been introduced and analyzed by Matsuyama and Ushchev (2017). We refer to their paper for the technical results and proofs. Proposition 1 in Matsuyama and Ushchev (2017) provides necessary and sufficient restrictions on the functions $s(\cdot;\theta)$ ensuring that there exists a well-defined production function $\mathscr{F}$ generating the demand system (5). Their Proposition 1 shows that the labor share mappings $s(\cdot;\theta)$ can be treated as a primitive of the model.

Examples. The CES production function is a special case of the HSA class. In this case we know the production function $\mathscr{F}(\mathscr{L}) = [\int_\Theta a(\theta)(L(\theta))^{\frac{\sigma-1}{\sigma}}d\theta]^{\frac{\sigma}{\sigma-1}}$ in closed-form, and it is straightforward to verify that the property (5) holds for $s(x;\theta)=(a(\theta))^\sigma x^{1-\sigma}$ and

$$\mathcal{A} = [\int (a(\theta))^\sigma (w(\theta))^{1-\sigma} d\theta]^{\frac{1}{1-\sigma}}.$$

Another example is the production function generated by a separable Translog cost function (Christensen et al. (1973, 1975)). The Translog cost function is often used in empirical estimations of production functions (see, e.g., Kim (1992)), and is a useful alternative to the Translog production function introduced in Section A.4.2. This cost function is defined by

$$\begin{aligned}\ln \mathscr{C}(\{w(\theta)\}_{\theta\in\Theta};Y) &= a_0 + a_1 \ln Y + a_2 (\ln Y)^2 + \int_\Theta a(\theta) \ln w(\theta)\, d\theta + \int_\Theta a_Y(\theta) \ln w(\theta) \ln Y d\theta \\ &\quad + \frac{1}{2}\int_\Theta \beta(\theta) (\ln w(\theta))^2 d\theta + \frac{1}{2}\int_{\Theta\times\Theta} \chi(\theta,\theta') (\ln w(\theta)) (\ln w(\theta'))\, d\theta d\theta',\end{aligned}$$

where we assume that the coefficients $\chi(\theta,\theta') = \zeta\chi(\theta)\chi(\theta')$ are multiplicatively separable, with $\int_\Theta \chi(\theta)\, d\theta = 1$ and $\beta(\theta) = -\int_\Theta \chi(\theta,\theta')\, d\theta'$. Using Shephard's lemma, we obtain that the labor share of skill θ is given by

$$\frac{w(\theta) L(\theta)}{\mathscr{F}\left(\{L(\theta)\}_{\theta\in\Theta}\right)} = a(\theta) + a_Y(\theta) \ln Y + \beta(\theta) \ln w(\theta) + \zeta \int_\Theta \chi(\theta)\chi(\theta') \ln w(\theta')\, d\theta',$$

which can be easily shown to be of the form (5). This example shows in particular that it is not necessary to know in closed-form the production function itself to verify that the wages and labor inputs it implies are in the HSA class.

Matsuyama and Ushchev (2017) provide several other special cases of HSA production structures. We refer to their paper (Examples 3b to 5) for further examples. Interestingly, note that the HSA class of production functions is defined non-parametrically, which makes it particularly flexible.

Cross-wage elasticities. The cross-wage elasticities $\gamma(\theta,\theta')$ implied by HSA production functions are multiplicatively separable (i.e., of the form (44)). Specifically, we show that the cross-wage elasticities induced by an HSA production function are given by

$$\gamma(\theta,\theta') = \left[1 - \frac{\frac{w(\theta)}{\mathcal{A}(\boldsymbol{w})} s'\left(\frac{w(\theta)}{\mathcal{A}(\boldsymbol{w})};\theta\right)}{s\left(\frac{w(\theta)}{\mathcal{A}(\boldsymbol{w})};\theta\right)}\right]^{-1} \left[\frac{w(\theta') L(\theta')}{\mathscr{F}(\mathscr{L})}\right]. \tag{45}$$

Proof of equation (45). To derive the cross-wage elasticities $\gamma(\theta,\theta')$, we differentiate the HSA property (5) with respect to $w(\theta)$ and $L(\theta')$, keeping the aggregator $\mathcal{A}(\{w(\theta)\}_{\theta\in\Theta})$ constant. We obtain

$$\begin{aligned}&\frac{w(\theta) L(\theta)}{\mathscr{F}(\{L(\theta)\}_{\theta\in\Theta})} \frac{dw(\theta)}{w(\theta)} - \frac{w(\theta) L(\theta)}{\mathscr{F}(\{L(\theta)\}_{\theta\in\Theta})} \frac{L(\theta') \mathscr{F}'_{\theta'}(\{L(\theta)\}_{\theta\in\Theta})}{\mathscr{F}(\{L(\theta)\}_{\theta\in\Theta})} \frac{dL(\theta')}{L(\theta')} \\ = \;& s'\left(\frac{w(\theta)}{\mathcal{A}(\{w(\theta)\}_{\theta\in\Theta})};\theta\right) \frac{w(\theta)}{\mathcal{A}(\{w(\theta)\}_{\theta\in\Theta})} \frac{dw(\theta)}{w(\theta)}.\end{aligned}$$

Hence, using the relationship $\frac{w(\theta)L(\theta)}{\mathscr{F}(\mathscr{L})} = s\left(\frac{w(\theta)}{\mathcal{A}(\boldsymbol{w})};\theta\right)$ and rearranging terms, we obtain

$$\left[1 - \frac{s'\left(\frac{w(\theta)}{\mathcal{A}(\boldsymbol{w})};\theta\right) \frac{w(\theta)}{\mathcal{A}(\boldsymbol{w})}}{s\left(\frac{w(\theta)}{\mathcal{A}(\boldsymbol{w})};\theta\right)}\right] \frac{dw(\theta)}{w(\theta)} = \frac{w(\theta') L(\theta')}{\mathscr{F}(\mathscr{L})} \frac{dL(\theta')}{L(\theta')}.$$

This easily leads to (45).

□

The first term in square brackets is a function of θ only and is determined by the elasticity of the labor share function $s(\cdot,\theta)$. This term is equal to 1 if the production function is CES. The second term in square brackets depends on θ' only and is given by the labor share of skill θ'. Note that, except for the special CES case, the elasticities of substitution implied by the HSA production functions are not constant.

Alternative (dual) formulation. Following Remark 3 in Matsuyama and Ushchev (2017), we can define alternatively another class of production functions by

$$\frac{w(\theta)L(\theta)}{\mathscr{F}\left(\{L(\theta)\}_{\theta\in\Theta}\right)} = \tilde{s}\left(\frac{L(\theta)}{\mathcal{B}\left(\{L(\theta)\}_{\theta\in\Theta}\right)};\theta\right).$$

It is straightforward to show in particular that the Translog production function that we studied in the paper (as opposed to the Translog cost function described above) with $\chi(\theta,\theta') = \chi(\theta)\chi(\theta')$ satisfies this restriction. In this case, the cross-wage elasticities $\gamma(\theta,\theta')$ are not multiplicatively separable as in (44), but the sum of two multiplicatively separable terms:

$$\gamma(\theta,\theta') = \frac{w(\theta')L(\theta')}{\mathscr{F}(\mathscr{L})} - \frac{\frac{L(\theta)}{\mathcal{B}(\mathscr{L})}\tilde{s}'\left(\frac{L(\theta)}{\mathcal{B}(\mathscr{L})};\theta\right)}{\tilde{s}\left(\frac{L(\theta)}{\mathcal{B}(\mathscr{L})};\theta\right)} \times \frac{L(\theta')\mathcal{B}'_{\theta'}(\mathscr{L})}{\mathcal{B}(\mathscr{L})}.$$

This kernel is the sum of two multiplicatively separable kernels. We now study the properties of this general class of kernels.

<u>Separable kernels: general case</u>

Suppose more generally that the cross-wage elasticities are a sum of multiplicatively separable functions:

$$\gamma(\theta,\theta') = \sum_{k=1}^{n}\gamma_1^k(\theta)\gamma_2^k(\theta'). \tag{46}$$

In this case, we can show that there exists a matrix $A = (A_{ij})_{1\leq i,j\leq n}$ such that the resolvent of the integral equation is equal to

$$\Gamma(\theta,\theta') = \sum_{1\leq i,j\leq n} A_{ij}\gamma_1^i(\theta)\gamma_2^j(\theta'). \tag{47}$$

Specifically, A is given by (see Section 4.9.20 in Polyanin and Manzhirov (2008)):

$$A = [\mathrm{Id}_n - B]^{-1},$$

assuming this matrix exists, where $B = (B_{mp})_{1 \leq m,p \leq n}$ is the matrix defined by

$$B_{mp} = \int_{\Theta} \gamma_2^m(s)\, \gamma_1^p(s)\, ds.$$

This result is useful, as the most important results of the theory of integral equations (the Fredholm theorems, which lead in particular to our Proposition 1), are derived (when the convergence conditions stated in our Proposition do not necessarily hold) by showing that a general kernel can be approximated arbitrarily closely by such separable kernels. This construction is described formally in Section 2.4 of Zemyan (2012). Specifically, by the Weierstrass approximation theorem, we can approximate the kernel $\gamma(\theta, \theta')$ by the separable polynomial in θ and θ' of the form (46).

Theorem of uniform approximation. Moreover, the Theorem of Uniform Approximation (see, e.g., Section 2.6.1 in Zemyan (2012)) shows that if the kernels of two Fredholm integral equations are close, then their solutions are close as well – that is, the solution to an integral equation is continuous in its kernel. Formally, suppose that for some constants $\delta_1, \delta_2 > 0$, we have

$$\begin{aligned} \max_{0 \leq \theta \leq \bar{\theta}} & \quad |g(\theta) - \bar{g}(\theta)| < \delta_1 \\ \max_{0 \leq \theta, \theta' \leq \bar{\theta}} & \quad |\gamma(\theta, \theta') - \bar{\gamma}(\theta, \theta')| < \delta_2. \end{aligned}$$

Suppose moreover that the resolvent series $\Gamma(\theta, \theta')$ and $\bar{\Gamma}(\theta, \theta')$ of the Fredholm integral equations

$$\begin{aligned} f(\theta) &= g(\theta) + \int_{\Theta} \gamma(\theta, \theta') f(\theta')\, d\theta' \\ \bar{f}(\theta) &= \bar{g}(\theta) + \int_{\Theta} \bar{\gamma}(\theta, \theta') \bar{f}(\theta')\, d\theta' \end{aligned}$$

converge. Then there exist constants β_1, β_2 such that

$$\max_{0 \leq \theta \leq \bar{\theta}} \left| f(\theta) - \bar{f}(\theta) \right| \leq \beta_1 \delta_1 + \beta_2 \delta_2.$$

Combined with the previous result that any kernel can be approximated arbitrarily closely by a separable kernel, we obtain that the solution to the integral equation can be approximated arbitrarily closely by the corresponding resolvent (47).

A.4.4 Relationship with Scheuer and Werning (2016) and Scheuer and Werning (2017)

Scheuer and Werning (2016, 2017) analyze a general equilibrium extension of Mirrlees (1971) and prove a neutrality result: in their model, the optimal tax formula is the same as in partial equilibrium, even though they consider a more general production function than Mirrlees (1971).[42] The key

[42] The policy implications can nevertheless be different. For instance, in Scheuer and Werning (2017), the relevant earnings elasticity in the formula written in terms of the observed income

modeling difference between our framework and theirs is the following. In theirs, all the agents produce the same input with different productivities θ. Denoting by $\lambda(\theta) = \theta l(\theta)$ the agent's production of that input (i.e., her efficiency units of labor), the aggregate production function then maps the distribution of λ into output. In equilibrium, a nonlinear price (earnings) schedule $p(\cdot)$ emerges such that an agent who produces λ units earns income $p(\lambda)$, irrespective of her underlying productivity θ. Hence, when an (atomistic) individual θ provides more effort $l(\theta)$, her income moves along the non-linear schedule $l \mapsto p(\theta \times l)$; e.g., in their superstars model with a convex equilibrium earnings schedule, her income increases faster than linearly. By contrast, in our framework, different values of θ index different inputs in the aggregate production function; for each of these inputs, there is one specific price (wage) $w(\theta)$, and hence a linear earnings schedule $l \mapsto w(\theta) \times l$. Therefore, when an individual θ provides more effort $l(\theta)$, her income increases linearly, as her wage remains constant (since her sector θ doesn't change). In their framework, Scheuer and Werning (2017, 2016) show that the general equilibrium effects exactly cancel out at the optimum tax schedule, even though they would of course be non-zero in the characterization of the incidence effects of tax reforms around a suboptimal tax code. In our framework, as in those of Stiglitz (1982); Rothschild and Scheuer (2014); Ales, Kurnaz, and Sleet (2015), these general equilibrium forces are also present at the optimum.

B Proofs of Section 2

B.1 Incidence of tax reforms on labor supply

B.1.1 Variational approach: derivation of the integral equation

Proof of Lemma 1. Denote the perturbed tax function by $\tilde{T}(y) = T(y) + \mu \hat{T}(y)$. Denote by $\hat{l}(\theta)$ the Gateaux derivative of the labor supply of type θ in response to this perturbation, and let $\hat{L}(\theta) = \hat{l}(\theta) f(\theta)$. The labor supply response of type θ is given by the solution to the perturbed first-order condition

$$\begin{aligned} 0 =& v'\left(l(\theta) + \mu \hat{l}(\theta)\right) \\ &- \left\{1 - T'\left[\tilde{w}(\theta) \times \left(l(\theta) + \mu \hat{l}(\theta)\right)\right] - \mu \hat{T}'\left[\tilde{w}(\theta) \times \left(l(\theta) + \mu \hat{l}(\theta)\right)\right]\right\} \tilde{w}(\theta), \end{aligned} \tag{48}$$

where $\tilde{w}(\theta)$ is the perturbed wage schedule, which satisfie

$$\begin{aligned} \frac{\tilde{w}(\theta) - w(\theta)}{\mu} &= \frac{1}{\mu}\left\{\mathscr{F}'_\theta(\{(l(\theta') + \mu \hat{l}(\theta')) f(\theta')\}_{\theta' \in \Theta}) - \mathscr{F}'_\theta(\{l(\theta') f(\theta')\}_{\theta' \in \Theta})\right\} \\ &\underset{\mu \to 0}{=} \mathscr{F}'_\theta \int_\Theta \frac{L(\theta') \mathscr{F}''_{\theta,\theta'}}{\mathscr{F}'_\theta} \frac{\hat{l}(\theta')}{l(\theta')} d\theta' \\ &= w(\theta)\left[-\frac{1}{\varepsilon_w^D(\theta)} \frac{\hat{l}(\theta)}{l(\theta)} + \int_\Theta \gamma(\theta, \theta') \frac{\hat{l}(\theta')}{l(\theta')} d\theta'\right]. \end{aligned} \tag{49}$$

distribution is higher due to the superstar effects.

Taking a first-order Taylor expansion of the perturbed first-order conditions (48) around the baseline allocation, using (49) to substitute for $\tilde{w}(\theta) - w(\theta)$, and solving for $\hat{l}(\theta)$ yields

$$\left\{1 + \frac{1 - T'(y(\theta)) - y(\theta) T''(y(\theta))}{1 - T'(y(\theta)) + \frac{v'(l(\theta))}{l(\theta)v''(l(\theta))} y(\theta) T''(y(\theta))} \frac{v'(l(\theta))}{l(\theta) v''(l(\theta))} \frac{1}{\varepsilon_w^D(\theta)}\right\} \frac{\hat{l}(\theta)}{l(\theta)}$$
$$= \frac{1 - T'(y(\theta)) - y(\theta) T''(y(\theta))}{1 - T'(y(\theta)) + \frac{v'(l(\theta))}{l(\theta)v''(l(\theta))} y(\theta) T''(y(\theta))} \frac{v'(l(\theta))}{l(\theta) v''(l(\theta))} \int_\Theta \gamma(\theta, \theta') \frac{\hat{l}(\theta')}{l(\theta')} d\theta'$$
$$- \frac{1}{1 - T'(y(\theta)) + \frac{v'(l(\theta))}{l(\theta)v''(l(\theta))} y(\theta) T''(y(\theta))} \frac{v'(l(\theta))}{l(\theta) v''(l(\theta))} \hat{T}'(y(\theta)),$$

which leads to equation (9).

□

B.1.2 Solution to the integral equation

Proof of Proposition 1. Assume that the condition $\int_{\Theta^2} |\varepsilon_w(\theta) \gamma(\theta, \theta')|^2 d\theta d\theta' < 1$ holds. Substituting for $dl(\theta')$ in the integral of equation (9), using the r.h.s. of the integral equation (9), yield

$$\frac{\hat{l}(\theta)}{l(\theta)} = -\varepsilon_r(\theta) \frac{\hat{T}'(y(\theta))}{1 - T'(y(\theta))} - \int_\Theta \varepsilon_w(\theta) \gamma(\theta, \theta') \varepsilon_r(\theta') \frac{\hat{T}'(y(\theta'))}{1 - T'(y(\theta'))} d\theta'$$
$$+ \int_\Theta \left\{ \int_\Theta \varepsilon_w(\theta) \gamma(\theta, \theta') \varepsilon_w(\theta') \gamma(\theta', \theta'') \frac{\hat{l}(\theta'')}{l(\theta)} d\theta'' \right\} d\theta'.$$

Applying Fubini's theorem yields

$$\frac{\hat{l}(\theta)}{l(\theta)} = -\varepsilon_r(\theta) \frac{\hat{T}'(y(\theta))}{1 - T'(y(\theta))}$$
$$- \int_\Theta \varepsilon_w(\theta) \gamma(\theta, \theta') \varepsilon_r(\theta') \frac{\hat{T}'(y(\theta'))}{1 - T'(y(\theta'))} d\theta' + \int_\Theta \varepsilon_w(\theta) \Gamma_2(\theta, \theta') \frac{\hat{l}(\theta')}{l(\theta)} d\theta',$$

where $\Gamma_2(\theta, \theta') = \int_\Theta \gamma(\theta, \theta'') \varepsilon_w(\theta'') \gamma(\theta'', \theta') d\theta''$. Following analogous steps, repeated substitutions lead to: for all $n \geq 1$,

$$\frac{\hat{l}(\theta)}{l(\theta)} = -\varepsilon_r(\theta) \frac{\hat{T}'(y(\theta))}{1 - T'(y(\theta))} - \sum_{i=1}^{n} \int_\Theta \varepsilon_w(\theta) \Gamma_i(\theta, \theta') \varepsilon_r(\theta') \frac{\hat{T}'(y(\theta'))}{1 - T'(y(\theta'))} d\theta'$$
$$+ \int_\Theta \varepsilon_w(\theta) \Gamma_{n+1}(\theta, \theta') \frac{\hat{l}(\theta')}{l(\theta)} d\theta',$$

where, for all $i \geq 3$, $\Gamma_i(\theta, \theta') = \int_\Theta \Gamma_{i-1}(\theta, \theta'') \varepsilon_w(\theta'') \gamma(\theta'', \theta') d\theta''$. We now show that

$$\int_\Theta \Gamma_{n+1}(\theta, \theta') \frac{\hat{l}(\theta')}{l(\theta)} d\theta'$$

converges to zero as $n \to \infty$. Applying the Cauchy-Schwartz inequality to the iterated kernel yields

$$\left|\Gamma_{n+1}(\theta, \theta')\right|^2 \leq \left(\int_\Theta \left|\Gamma_n(\theta, \theta'')\right|^2 d\theta''\right)\left(\int_\Theta \left|\varepsilon_w(\theta'')\gamma(\theta'', \theta')\right|^2 d\theta''\right).$$

Integrating this inequality with respect to θ' implies

$$\begin{aligned}\int_\Theta \left|\Gamma_{n+1}(\theta, \theta')\right|^2 d\theta' &\leq \left(\int_\Theta \left|\Gamma_n(\theta, \theta'')\right|^2 d\theta''\right)\left(\int_\Theta\int_\Theta \left|\varepsilon_w(\theta'')\gamma(\theta'', \theta')\right|^2 d\theta'' d\theta'\right) \\ &= \|\varepsilon_w\gamma\|_2^2 \times \int_\Theta \left|\Gamma_n(\theta, \theta'')\right|^2 d\theta''.\end{aligned}$$

By induction, we obtain

$$\int_\Theta \left|\Gamma_{n+1}(\theta, \theta')\right|^2 d\theta' \leq \|\varepsilon_w\gamma\|_2^{2n} \times \int_\Theta \left|\Gamma_1(\theta, \theta'')\right|^2 d\theta''.$$

We thus have, using the Cauchy-Schwartz inequality again,

$$\begin{aligned}&\left|\int_\Theta \Gamma_{n+1}(\theta, \theta')\,\varepsilon_r(\theta')\,\frac{\hat{T}'(y(\theta'))}{1-T'(y(\theta'))}d\theta'\right|^2 \\ &\leq \left(\int_\Theta \left|\Gamma_{n+1}(\theta, \theta'')\right|^2 d\theta''\right)\left(\int_\Theta \left|\varepsilon_r(\theta')\,\frac{\hat{T}'(y(\theta'))}{1-T'(y(\theta'))}\right|^2 d\theta''\right) \\ &\leq \|\varepsilon_w\gamma\|_2^{2n} \times \left(\int_\Theta \left|\gamma(\theta, \theta'')\right|^2 d\theta''\right) \times \left\|\varepsilon_r\frac{\hat{T}'}{1-T'}\right\|_2^2\end{aligned}$$

Thus, for all $\theta \in \Theta$, for all $i \geq 1$,

$$\left|\int_\Theta \Gamma_i(\theta, \theta')\,\varepsilon_r(\theta')\,\frac{\hat{T}'(y(\theta'))}{1-T'(y(\theta'))}d\theta'\right| \leq \left[\left\|\varepsilon_r\frac{\hat{T}'}{1-T'}\right\|_2 \sqrt{\int_\Theta \left|\gamma(\theta, \theta'')\right|^2 d\theta''}\right] \times \|\varepsilon_w\gamma\|_2^i.$$

Since $\|\varepsilon_w\gamma\|_2 < 1$, the previous arguments imply that the sequence $\{\kappa_n(\theta)\}_{n\geq 1}$ defined by

$$\kappa_n(\theta) \equiv \int_\Theta \sum_{i=1}^n \Gamma_i(\theta, \theta')\,\varepsilon_r(\theta')\,\frac{\hat{T}'(y(\theta'))}{1-T'(y(\theta'))}d\theta'$$

is dominated by a convergent geometric series of positive terms, and therefore it converges absolutely and uniformly to a unique limit $\kappa(\theta)$ on Θ. Similarly, we have

$$\lim_{n\to\infty}\left|\int_\Theta \Gamma_{n+1}(\theta, \theta')\,\frac{\hat{l}(\theta')}{l(\theta')}d\theta'\right| = 0.$$

Therefore, we can write

$$\frac{\hat{l}(\theta)}{l(\theta)} = -\,\varepsilon_r(\theta)\,\frac{\hat{T}'(y(\theta))}{1-T'(y(\theta))} - \varepsilon_w(\theta)\int_\Theta \sum_{i=1}^\infty \Gamma_i(\theta, \theta')\,\varepsilon_r(\theta')\,\frac{\hat{T}'(y(\theta'))}{1-T'(y(\theta'))}d\theta',$$

which proves equation (10). To show the uniqueness of the solution, suppose that $\hat{l}_1(\theta)$ and $\hat{l}_2(\theta)$ are two solutions to (10). Then $\Delta\hat{l}(\theta) \equiv \hat{l}_2(\theta) - \hat{l}_1(\theta)$ satisfies the homogeneous integral equation

$$\Delta\frac{\hat{l}(\theta)}{l(\theta)} = \int_\Theta \varepsilon_w(\theta)\gamma(\theta,\theta')\Delta\frac{\hat{l}(\theta')}{l(\theta')}d\theta'.$$

The Cauchy-Schwartz inequality implies

$$\left|\Delta\frac{\hat{l}(\theta)}{l(\theta)}\right|^2 \leq \left(\int_\Theta |\varepsilon_w(\theta)\gamma(\theta,\theta')|^2 d\theta'\right)\left(\int_\Theta \left|\Delta\frac{\hat{l}(\theta')}{l(\theta')}\right|^2 d\theta'\right).$$

Integrating with respect to θ yields

$$\int_\Theta \left|\Delta\frac{\hat{l}(\theta)}{l(\theta)}\right|^2 d\theta \leq \|\varepsilon_w\gamma\|_2^2 \int_\Theta \left|\Delta\frac{\hat{l}(\theta')}{l(\theta')}\right|^2 d\theta',$$

Assumption $\|\varepsilon_w\gamma\|_2 < 1$ then implies $\int_\Theta \left|\Delta\frac{\hat{l}(\theta)}{l(\theta)}\right|^2 d\theta = 0$, i.e., $\Delta\frac{\hat{l}(\theta)}{l(\theta)} = 0$ for all $\theta \in \Theta$.

□

B.1.3 Inverting the integral equation

We now show how to back out the structural elasticity parameters $\gamma(\theta,\theta')$ from the knowledge of the resolvent elasticities $\Gamma(\theta,\theta')$. It is straightforward to show that

$$\gamma(\theta,\theta') = \Gamma(\theta,\theta') - \int_\Theta \Gamma(\theta,\theta'')\varepsilon_w(\theta'')\gamma(\theta'',\theta')d\theta'' \quad .$$

Now fix θ' and denote $\beta(\theta) = \gamma(\theta,\theta')$ and $\zeta(\theta,\theta'') = \Gamma(\theta,\theta'')\varepsilon_w(\theta'')$. The previous equation can be rewritten, for a fixed θ', as:

$$\beta(\theta) = \Gamma(\theta,\theta') - \int_\Theta \zeta(\theta,\theta'')\beta(\theta'')d\theta''$$

This is an integral equation with solution $\beta(\theta)$, which can be solved using the standard techniques.

B.1.4 Resolvent in the case of a CES production

Proof of equation (13).

Derivation of the resolvent. Suppose that the cross-wage elasticities are multiplicatively separable, i.e., of the form $\gamma(\theta',\theta) = \gamma_1(\theta')\gamma_2(\theta)$. The integral equation (9) then reads

$$\hat{l}(\theta) = -\varepsilon_r(\theta)\frac{\hat{T}'(y(\theta))}{1-T'(y(\theta))} + \varepsilon_w(\theta)\gamma_1(\theta)\int_\Theta \gamma_2(\theta')\hat{l}(\theta')d\theta'$$

and can be easily solved as follows. Multiplying by $\gamma_2(\theta')$ both sides of the integral equation evaluated at θ' and integrating with respect to θ' leads to

$$\int_{\Theta} \gamma_2(\theta') \hat{l}(\theta') d\theta' = -\int_{\Theta} \gamma_2(\theta') \varepsilon_r(\theta') \frac{\hat{T}'(y(\theta'))}{1-T'(y(\theta'))} d\theta' + \left(\int_{\Theta} \varepsilon_w(\theta') \gamma_1(\theta') \gamma_2(\theta') d\theta'\right) \left(\int_{\Theta} \gamma_2(\theta') \hat{l}(\theta') d\theta'\right),$$

i.e.,

$$\int_{\Theta} \gamma_2(\theta') \hat{l}(\theta') d\theta' = -\frac{\int_{\Theta} \gamma_2(\theta') \varepsilon_r(\theta') \frac{\hat{T}'(y(\theta'))}{1-T'(y(\theta'))} d\theta'}{1-\int_{\Theta} \varepsilon_w(\theta') \gamma_1(\theta') \gamma_2(\theta') d\theta'}.$$

Substituting into the right hand side of the integral equation (9) leads to

$$\hat{l}(\theta) = -\varepsilon_r(\theta) \frac{\hat{T}'(y(\theta))}{1-T'(y(\theta))} - \varepsilon_w(\theta) \int_{\Theta} \frac{\gamma_1(\theta) \gamma_2(\theta')}{1-\int_{\Theta} \varepsilon_w(\theta') \gamma_1(\theta') \gamma_2(\theta') d\theta'} \varepsilon_r(\theta') \frac{\hat{T}'(y(\theta'))}{1-T'(y(\theta'))} d\theta'.$$

Next, suppose in particular that the production function is CES, so that $\gamma_1(\theta) \equiv 1$. We saw in equation (27) that we then have

$$\gamma_2(\theta') = \frac{1}{\sigma} \frac{y(\theta') f_Y(y(\theta'))}{\int x f_Y(x) dx} \left[\frac{dy}{d\theta}(\theta')\right].$$

Changing variables from types θ' to incomes $y' \equiv y(\theta')$ in the integral $\int_{\Theta} \varepsilon_w(\theta') \gamma_2(\theta') d\theta'$ in the denominator of the previous equation, we can rewrite this integral as

$$\frac{1}{\sigma} \int_{\mathbb{R}_+} \varepsilon_w(y') \frac{y' f_Y(y')}{\int x f_Y(x) dx} dy'.$$

This concludes the proof.

Sufficient conditions ensuring convergence of the resolvent. Note that the solution to the integral equation in the CES case is well defined if $1-\frac{1}{\sigma}\int_{\mathbb{R}_+} \varepsilon_w(y') \frac{y' f_Y(y')}{\int x f_Y(x) dx} dy' > 0$. Suppose that the the initial tax schedule is CRP with parameter p, and that the disutility of labor is isoelastic with parameter e. In this case, we saw that $\varepsilon_w(y') = \frac{(1-p)e}{1+pe+(1-p)\frac{e}{\sigma}}$ is constant. Therefore we obtain

$$1-\frac{1}{\sigma}\int_{\mathbb{R}_+} \varepsilon_w(y') \frac{y' f_Y(y')}{\int x f_Y(x) dx} dy' = 1-\frac{\varepsilon_w}{\sigma} = \frac{1+pe}{1+pe+(1-p)\frac{e}{\sigma}} > 0.$$

Therefore the convergence of the resolvent is always ensured in this case.

□

B.2 Incidence of tax reforms on wages and utilities

Proof of Corollary 2.

Equation (14). By equation (49), the Gateaux derivative of the wage functional is given by

$$\frac{\hat{w}(\theta)}{w(\theta)} = -\frac{1}{\varepsilon_w^D(\theta)}\frac{\hat{l}(\theta)}{l(\theta)} + \int_\Theta \gamma(\theta,\theta')\frac{\hat{l}(\theta')}{l(\theta')}d\theta'.$$

Substituting in the integral equation (9) implies

$$\frac{\hat{l}(\theta)}{l(\theta)} = -\varepsilon_r(\theta)\frac{\hat{T}'(y(\theta))}{1-T'(y(\theta))} + \varepsilon_w(\theta)\left[\frac{\hat{w}(\theta)}{w(\theta)} + \frac{1}{\varepsilon_w^D(\theta)}\frac{\hat{l}(\theta)}{l(\theta)}\right],$$

which leads to formula (14) by noting that $\frac{1-\varepsilon_w(\theta)/\varepsilon_w^D(\theta)}{\varepsilon_w(\theta)} = \frac{1}{\varepsilon_w^S(\theta)}$ and that $\frac{\varepsilon_r(\theta)}{\varepsilon_w(\theta)} = \frac{\varepsilon_r^S(\theta)}{\varepsilon_w^S(\theta)}$.

Equation (15). The first-order effects of a tax reform $\hat{T}$ on individual welfare are given by

$$\begin{aligned}\hat{U}(\theta) &= (1-T'(y(\theta)))\,y(\theta)\left(\frac{\hat{w}(\theta)}{w(\theta)} + \frac{\hat{l}(\theta)}{l(\theta)}\right) - l(\theta)\,v'(l(\theta))\frac{\hat{l}(\theta)}{l(\theta)} - \hat{T}(y(\theta))\\ &= (1-T'(y(\theta)))\,y(\theta)\frac{\hat{w}(\theta)}{w(\theta)} - \hat{T}(y(\theta)),\end{aligned}$$

where the last equality uses the first order condition (1). We obtain formula (15).

□

Corollary 6. *Suppose that the cross-wage elasticities satisfy $\gamma(\theta,\theta') \geq 0$ for all θ,θ'. Then, given a total (average) tax change $\hat{T}(y(\theta))$ at income $y(\theta)$, a higher marginal tax rate $\hat{T}'(y(\theta)) > 0$ raises the utility of agents with type θ and lowers that of all other agents. That is, $\hat{U}(\theta) > 0$, and $\hat{U}(\theta') < 0$ for all $\theta' \neq \theta$.*

Proof of Corollary 6. Suppose that $\gamma(\theta',\theta) > 0$ for all θ,θ', which implies that $\Gamma(\theta',\theta) > 0$ for all θ,θ'. We then have, using equations (10) and (14), for any $\theta' \in \Theta$

$$\frac{\hat{w}(\theta)}{w(\theta)} = \frac{\varepsilon_r(\theta')}{\varepsilon_w^D(\theta')}\frac{\hat{T}'(y(\theta'))}{1-T'(y(\theta'))} - \frac{\varepsilon_w(\theta')}{\varepsilon_w^S(\theta')}\int_\Theta \Gamma(\theta',\theta)\,\varepsilon_r(\theta)\frac{\hat{T}'(y(\theta))}{1-T'(y(\theta))}d\theta.$$

Since $\frac{\varepsilon_w(\theta')}{\varepsilon_w^S(\theta')}\Gamma(\theta',\theta)\,\varepsilon_r(\theta) > 0$, a higher marginal tax rate $\hat{T}'(y(\theta)) > 0$ at income $y(\theta)$ lowers the wage, and hence lowers the utility (conditional on the total tax change $\hat{T}(y(\theta'))$ at income $y(\theta')$), of type $\theta' \neq \theta$. This is because the higher tax rate lowers the labor supply of type θ and the labor of type θ' is complementary to that of type θ in production. Moreover, since $\frac{\varepsilon_r(\theta')}{\varepsilon_w^D(\theta')} > 0$, a higher marginal tax rate $\hat{T}'(y(\theta')) > 0$ at income $y(\theta')$ raises the wage, and hence raises the utility (conditional on the total tax change $\hat{T}(y(\theta'))$ at income $y(\theta')$), of type θ'. The easiest way to show this is to consider an elemtary tax reform at income $y(\theta')$, as defined in Section 3.1. We then have

$$\frac{\hat{w}(\theta)}{w(\theta)} = \frac{\varepsilon_r(\theta')}{\varepsilon_w^D(\theta')}\delta(0) - \frac{\varepsilon_w(\theta')}{\varepsilon_w^S(\theta')}\Gamma(\theta',\theta')\frac{\varepsilon_r(\theta')}{1-T'(y(\theta'))},$$

which is positive. Note that an increase in the marginal tax rate at income $y(\theta)$ implies that individuals with skill $\theta' > \theta$ are made worse off for two separate reasons: (i) their total tax bill is now mechanically higher, since the marginal tax rate on income $y(\theta)$ has increased; (ii) their wage is lower, since the labor supply of agents θ is distorted downward.

$\square$

C Proofs of Section 3

C.1 Preliminaries

Proof of Section 3.1.

Equation (16). The first-order effects of a tax reform $\hat{T}$ on individual θ's tax payment are given by:

$$dT(w(\theta)l(\theta)) = \hat{T}(y(\theta)) + \left[\frac{\hat{w}(\theta)}{w(\theta)} + \frac{\hat{l}(\theta)}{l(\theta)}\right] w(\theta)l(\theta)T'(y(\theta))$$

so that the first-order effects of the tax reform $\hat{T}$ on government revenue are given by (changing variables from types θ to incomes $y \equiv y(\theta)$)

$$\hat{\mathcal{R}} = \int \hat{T}(y) f_Y(y)\,dy + \int T'(y)\left[\frac{\varepsilon_r^S(y)}{\varepsilon_w^S(y)}\frac{\hat{T}'(y)}{1-T'(y)} + \left(1+\frac{1}{\varepsilon_w^S(y)}\right)\frac{\hat{l}(y)}{l(y)}\right] y f_Y(y)\,dy, \tag{50}$$

where $\hat{l}(y)$ is the change in labor supply of agents with income initially equal to y. Moreover we have

$$\hat{\mathcal{G}} \equiv d\int \frac{G[U(\theta)]}{\lambda} f(\theta)\,d\theta = \int (1-T'(y))\, y\frac{\hat{w}(y)}{w(y)} g(y) f_Y(y)\,dy - \int \hat{T}(y) g(y) f_Y(y)\,dy,$$

where $g(y) = \frac{G'(U(\theta))}{\lambda}$ denotes the marginal social welfare weight at income y, and where $\hat{w}(y)$ is the change in labor supply of agents with income initially equal to y. The first-order effects of the tax reform $\hat{T}$ on social welfare are then given by

$$\begin{aligned}\hat{\mathcal{W}} = \hat{\mathcal{R}} + \hat{\mathcal{G}} = &\int (1-g(y))\hat{T}(y) f_Y(y)\,dy - \int \frac{T'(y)}{1-T'(y)}\varepsilon_r^S(y)\hat{T}'(y)\, y f_Y(y)\,dy \\ &+ \int \left[\left(1+\varepsilon_w^S(y)\right)T'(y) + (1-T'(y))\,g(y)\right]\frac{\hat{w}(y)}{w(y)} y f_Y(y)\,dy.\end{aligned} \tag{51}$$

where we used equation (14).

Elementary tax reforms. Suppose that the tax reform T is the step function $T(y) = \mathbb{I}_{\{y \geq y^*\}}$, so that $T'(y) = \delta(y - y^*)$ is the Dirac delta function (hence marginal tax rates are perturbed at income y^* only). To apply formula (10) to this non-differentiable perturbation, construct a sequence of smooth funtions $\varphi_{y^*,\epsilon}(y)$ such that

$$\delta(y-y^*) = \lim_{\epsilon\to 0} \varphi_{y^*,\epsilon}(y),$$

in the sense that for all continuous functions ψ with compact support,

$$\lim_{\epsilon\to 0}\int_{\mathbb{R}}\varphi_{y^*,\epsilon}(y)\,\psi(y)\,dy=\psi(y^*),$$

i.e., changing variables in the integral,

$$\lim_{\epsilon\to 0}\int_{\Theta}\varphi_{y^*,\epsilon}(y(\theta'))\left\{\psi(y(\theta'))\frac{dy(\theta')}{d\theta}\right\}d\theta'=\psi(y^*).$$

This can be obtained by defining an absolutely integrable and smooth function $\varphi_{y^*}(y)$ with compact support and $\int_{\mathbb{R}}\varphi_{y^*}(y)\,dy=1$, and letting $\varphi_{y^*,\epsilon}(y)=\epsilon^{-1}\varphi_{y^*}(\frac{y}{\epsilon})$. Letting $\Phi_{y^*,\epsilon}$ be such that $\Phi'_{y^*,\epsilon}=\varphi_{y^*,\epsilon}$, we then have, for all $\epsilon>0$, the following labor supply incidence formula:

$$\hat{l}(\theta,\Phi_{y^*,\epsilon})=-\,\varepsilon_r(\theta)\frac{\varphi_{y^*,\epsilon}(y(\theta))}{1-T'(y(\theta))}-\varepsilon_w(\theta)\int_{\Theta}\Gamma(\theta,\theta')\,\varepsilon_r(\theta')\frac{\varphi_{y^*,\epsilon}(y(\theta'))}{1-T'(y(\theta'))}d\theta'.$$

Letting $\epsilon\to 0$, we obtain the incidence of the elementary tax reform at y^*:

$$\begin{aligned}\hat{l}(\theta)&=-\,\varepsilon_r(\theta)\frac{\delta_{y^*}(y(\theta))}{1-T'(y(\theta))}-\varepsilon_w(\theta)\frac{\Gamma(\theta,\theta^*)}{y'(\theta^*)}\varepsilon_r(\theta^*)\frac{1}{1-T'(y(\theta^*))}\\&=-\,\varepsilon_r(y)\frac{\delta_{y^*}(y)}{1-T'(y)}-\varepsilon_w(y)\,\Gamma(y,y^*)\,\varepsilon_r(y^*)\frac{1}{1-T'(y^*)},\end{aligned}\tag{52}$$

where in the last equality we let $y=y(\theta)$ and $y^*=y(\theta^*)$, and we use the change of variables $\Gamma(y,y^*)=\frac{\Gamma(\theta,\theta^*)}{y'(\theta^*)}$.

□

C.2 Aggregate tax incidence: general case

<u>**Proof of Proposition 2 and Corollary 3**</u>.

Incidence on government revenue. Applying formula (10) to express the incidence on labor supply of the elementary tax reform at income y^*, formula (50) implies that the incidence on government revenue is given by

$$\begin{aligned}\hat{\mathcal{R}}(y^*)\;&=\;1+\frac{T'(y^*)}{1-T'(y^*)}\frac{\varepsilon_r^S(y^*)}{\varepsilon_w^S(y^*)}\frac{y^*f_Y(y^*)}{1-F_Y(y^*)}+\int_{\mathbb{R}_+}T'(y)\left(1+\frac{1}{\varepsilon_w^S(y)}\right)\dots\\&\quad\times\left[-\varepsilon_r(y)\frac{\delta(y-y^*)}{1-T'(y)}-\frac{1}{1-T'(y^*)}\varepsilon_w(y)\,\Gamma(y,y^*)\,\varepsilon_r(y^*)\right]\frac{yf_Y(y)}{1-F_Y(y^*)}dy\\&=\;\hat{\mathcal{R}}_{\text{ex}}(y^*)+\frac{T'(y^*)}{1-T'(y^*)}\varepsilon_r(y^*)\frac{y^*f_Y(y^*)}{1-F_Y(y^*)}\left(1+\varepsilon_w^S(y^*)\right)\frac{1}{\varepsilon_w^D(y^*)}\\&\quad-\frac{\varepsilon_r(y^*)}{1-T'(y^*)}\int_{\mathbb{R}_+}T'(y)\left(1+\varepsilon_w^S(y)\right)\frac{\Gamma(y,y^*)}{1+\varepsilon_w^S(y)/\varepsilon_w^D(y)}\frac{yf_Y(y)}{1-F_Y(y^*)}dy.\end{aligned}\tag{53}$$

Using Euler's theorem (25) easily leads to equation (18).

Linear initial tax schedule. Suppose in particular that the disutility of labor is constant and the initial tax schedule is linear, so that the marginal tax rate $T'(y)$ and the elasticity $\varepsilon_w^S(y)$ are constant. Applying equation (18) immediately implies that $\hat{\mathcal{R}}(y^*) = \hat{\mathcal{R}}_{\text{ex}}(y^*)$.

□

C.3 Aggregate tax incidence with constant elasticities

Proof of Corollary 4.

Proof of formula (19). Suppose that the disutility of labor is isoelastic, the initial tax schedule is CRP, and the labor demand elasticities are constant. We have shown above (equation (53)) that

$$\begin{aligned}\hat{\mathcal{R}}(y^*) \quad = \quad & \hat{\mathcal{R}}_{\text{ex}}(y^*) + \frac{\varepsilon_r\left(1+\varepsilon_w^S\right)}{1-T'(y^*)}\frac{y^* f_Y(y^*)}{1-F_Y(y^*)}\frac{1}{\varepsilon_w^D}T'(y^*) \\ & -\frac{\varepsilon_r\left(1+\varepsilon_w^S\right)}{1-T'(y^*)}\frac{y^* f_Y(y^*)}{1-F_Y(y^*)}\frac{1}{1+\frac{\varepsilon_w^S}{\varepsilon_w^D}}\mathbb{E}\left[T'(y)\frac{y\Gamma(y,y^*)}{y^* f_Y(y^*)}\right].\end{aligned}$$

The expectation in the second line can be rewritten as

$$\mathbb{E}\Big[T'(y)\frac{y\Gamma(y,y^*)}{y^* f_Y(y^*)}\Big] = \text{Cov}\Big(T'(y); \frac{y\Gamma(y,y^*)}{y^* f_Y(y^*)}\Big) + \frac{1}{y^* f_Y(y^*)}\mathbb{E}\left[T'(y)\right]\mathbb{E}\left[y\Gamma(y,y^*)\right].$$

But by Euler's theorem (equation (25)), we have

$$\frac{1}{1+\frac{\varepsilon_w^S}{\varepsilon_w^D}}\mathbb{E}\left[y\Gamma(y,y^*)\right] = \frac{1}{\varepsilon_w^D(y^*)}y^* f_Y(y^*).$$

Substituting into the previous expression easily leads to (19).

Proof of formula (20). Suppose that the production function is CES with parameter σ, so that $\Gamma(y,y^*)$ is given by formula (13) with $\gamma(y,y^*) = \frac{1}{\sigma\mathbb{E}y}y^* f_Y(y^*)$. Suppose moreover that the initial tax schedule is CRP. Equations (42) then show that the elasticities $\varepsilon_r(y)$ and $\varepsilon_w(y)$ are constant (independent of y). Consider the elementary tax reform at income y^*, i.e. $\hat{T}(y) = \mathbb{I}_{\{y\geq y^*\}}$ and $\hat{T}'(y) = \delta(y-y^*)$. Expression (50) implies

$$\begin{aligned}\hat{\mathcal{R}}(y^*) =& 1 + \int\frac{T'(y)}{1-T'(y)}\left[\frac{\varepsilon_r^S}{\varepsilon_w^S} - \left(1+\frac{1}{\varepsilon_w^S}\right)\varepsilon_r\right]\frac{yf_Y(y)}{1-F_Y(y^*)}\delta(y-y^*)\,dy \\ & -\int\frac{T'(y)}{1-T'(y^*)}\left(1+\frac{1}{\varepsilon_w^S}\right)\frac{\varepsilon_r\varepsilon_w}{1-\frac{\varepsilon_w}{\sigma}}\gamma(y,y^*)\frac{yf_Y(y)}{1-F_Y(y^*)}dy \\ =& \hat{\mathcal{R}}_{\text{ex}}(y^*) + \varepsilon_r\left(1+\varepsilon_w^S\right)\left[\frac{T'(y^*)}{1-T'(y^*)}\frac{1}{\sigma}\frac{y^* f_Y(y^*)}{1-F_Y(y^*)} - \int_{\mathbb{R}_+}\frac{T'(y)}{1-T'(y^*)}\gamma(y,y^*)\frac{ydF_Y(y)}{1-F_Y(y^*)}\right].\end{aligned}$$

Suppose first that $p=0$, i.e., the initial tax schedule is linear. In this case, we have $T'(y^*) = T'(y)$ for all y, so that the term in the square brackets is equal to 0 by Euler's homogeneous function theorem. More generally, with a nonlinear tax schedule, we can use expression (41) for $\gamma(y,y^*)$ to rewrite the term in square brackets as

$$\frac{1}{1-T'(y^*)}\frac{1}{\sigma}\frac{y^* f_Y(y^*)}{1-F_Y(y^*)}\left[T'(y^*)-\int_{\mathbb{R}_+} T'(y)\frac{y}{\mathbb{E}y}f_Y(y)\,dy\right].$$

Using the fact that $\left(1+\varepsilon_w^S\right)\frac{\varepsilon_r}{\sigma}=\frac{1+\varepsilon_w^S}{\sigma+\varepsilon_w^S}\varepsilon_r^S$ leads to equation (20). Note that we could also have derived this result from equation (19): we have

$$\Gamma(y,y^*)=\frac{\gamma(y,y^*)}{1-\frac{\varepsilon_w}{\sigma}}=\frac{\frac{1}{\sigma\mathbb{E}y}y^* f_Y(y^*)}{1-\frac{1}{\sigma}\frac{\varepsilon_w^S}{1+\varepsilon_w^S/\sigma}}=\frac{1}{\sigma\mathbb{E}y}\left(1+\frac{\varepsilon_w^S}{\sigma}\right)y^* f_Y(y^*).$$

Substituting for $\Gamma(y,y^*)$ into the covariance $\operatorname{Cov}(T'(y);y\Gamma(y,y^*))$ and using $\frac{1}{\mathbb{E}y}\operatorname{Cov}(T'(y);y)=\frac{1}{\mathbb{E}y}\mathbb{E}[yT'(y)]-\mathbb{E}[T'(y)]$ easily leads to (20).

□

Finally, for completeness, we characterize the effects of elementary tax reforms on social welfare.

Incidence on social welfare. The first-order effects of the reform on social welfare are given by

$$\begin{aligned}\hat{\mathcal{G}}(y^*)=&-\int_{y^*}^{\infty} g(y)\frac{f_Y(y)}{1-F_Y(y^*)}dy+\int g(y)\left[\frac{\varepsilon_r^S}{\varepsilon_w^S}-\frac{1-T'(y)}{1-T'(y^*)}\frac{\varepsilon_r}{\varepsilon_w^S}\right]\frac{yf_Y(y)}{1-F_Y(y^*)}\delta(y-y^*)\,dy\\&-\int g(y)\frac{1-T'(y)}{1-T'(y^*)}\frac{\varepsilon_r}{1-\frac{\varepsilon_w}{\sigma}}\frac{\varepsilon_w}{\varepsilon_w^S}\gamma(y,y^*)\frac{yf_Y(y)}{1-F_Y(y^*)}dy.\\=&-\int_{y^*}^{\infty} g(y)\frac{f_Y(y)}{1-F_Y(y^*)}dy+\frac{1}{1-T'(y^*)}\frac{\varepsilon_r}{\sigma}\left[g(y^*)(1-T'(y^*))\right.\\&\left.-\int_{\mathbb{R}_+} g(y)(1-T'(y))\frac{y}{\mathbb{E}y}f_Y(y)\,dy\right]\frac{y^* f_Y(y^*)}{1-F_Y(y^*)},\end{aligned}$$

where the second equality uses the expression (41) for $\gamma(y,y^*)$. We thus obtain the incidence of the elementary tax reform on social welfare $\hat{\mathcal{W}}=\hat{\mathcal{R}}+\hat{\mathcal{G}}$ as

$$\begin{aligned}\hat{\mathcal{W}}(y^*)=&\int_{y^*}^{\infty}(1-g(y))\frac{f_Y(y)}{1-F_Y(y^*)}dy-\varepsilon_r^S\frac{T'(y^*)}{1-T'(y^*)}\frac{y^* f_Y(y^*)}{1-F_Y(y^*)}\\&+\frac{\varepsilon_r/\sigma}{1-T'(y^*)}\left[\psi(y^*)-\int_{\mathbb{R}_+}\psi(y)\frac{y}{\mathbb{E}y}f_Y(y)\,dy\right]\frac{y^* f_Y(y^*)}{1-F_Y(y^*)},\end{aligned}\tag{54}$$

where $\psi(y)$ is defined by

$$\psi(y)=\left(1+\varepsilon_w^S(y)\right)T'(y)+g(y)(1-T'(y)).\tag{55}$$

Thus, the variable $T'(y)\left(1+\varepsilon_w^S(y)\right)$ in equation (18), which measures the total impact of a wage adjustment $\hat{w}(y)$ on the government budget, is now replaced by the more general expression $\psi(y)$. Its second term comes from the fact that the share $1-T'(y)$ of the income gain due to the wage adjustment $\hat{w}(y)$ is kept by the individual; this in turn raises social welfare in proportion to the welfare weight $g(y)$.

□

D Generalizations of the baseline model

D.1 Income effects

D.1.1 Elasticity concepts

In this section we extend the model of Section 1 to a general utility function over consumption and labor supply $U(c,l)$, where $U_c, U_{cc} > 0$ and $U_l, U_{ll} < 0$. This specification allows for arbitrary substitution and income effects. The utility of agent θ has the general form $U(\theta) \equiv u[w(\theta)l(\theta) - T(w(\theta)l(\theta)), l(\theta)]$. The first-order condition of the agent writes

$$[1 - T'(w(\theta)l(\theta))]\, w(\theta)\, u_c(\theta) + u_l(\theta) = 0.$$

Differentiating this equation allows us to define the compensated (Hicksian) elasticity of labor supply with respect to the retention rate $\varepsilon_r^S(\theta)$ and the income effect $\varepsilon_R^S(\theta)$ as follows (see, e.g., p. 208 in Saez (2001)):

$$e_r^c(\theta) \equiv \left.\frac{\partial \ln l(\theta)}{\partial \ln r(\theta)}\right|_{u \text{ cst}} = \frac{U_l(\theta)/l(\theta)}{U_{ll}(\theta) + \left(\frac{U_l(\theta)}{U_c(\theta)}\right)^2 U_{cc}(\theta) - 2\left(\frac{U_l(\theta)}{U_c(\theta)}\right) U_{cl}(\theta)}, \tag{56}$$

and

$$e_R(\theta) \equiv r(\theta)\, w(\theta) \frac{\partial l(\theta)}{\partial R} = \frac{-\left(\frac{U_l(\theta)}{U_c(\theta)}\right)^2 U_{cc}(\theta) + \left(\frac{U_l(\theta)}{U_c(\theta)}\right) U_{cl}(\theta)}{U_{ll}(\theta) + \left(\frac{U_l(\theta)}{U_c(\theta)}\right)^2 U_{cc}(\theta) - 2\left(\frac{U_l(\theta)}{U_c(\theta)}\right) U_{cl}(\theta)}. \tag{57}$$

We define moreover the elasticities along the nonlinear budget constraint as

$$\begin{aligned}
\varepsilon_r^{c,S}(\theta) &= \frac{e_r^c(\theta)}{1 + p(y(\theta))\, e_r^c(\theta)}, \\
\varepsilon_R^S(\theta) &= \frac{e_R(\theta)}{1 + p(y(\theta))\, e_r^c(\theta)}, \\
\varepsilon_w^S(\theta) &= \frac{(1 - p(y(\theta)))\, e_r^c(\theta) + e_R(\theta)}{1 + p(y(\theta))\, e_r^c(\theta)},
\end{aligned}$$

We also define the elasticities of equilibrium labor as

$$\begin{aligned}
\varepsilon_r^c(\theta) &= \frac{\varepsilon_r^{c,S}}{1 + \varepsilon_w^S(\theta)/\varepsilon_w^D(\theta)}, \\
\varepsilon_R(\theta) &= \frac{\varepsilon_R^S(\theta)}{1 + \varepsilon_w^S(\theta)/\varepsilon_w^D(\theta)}, \\
\varepsilon_w(\theta) &= \frac{\varepsilon_w^S(\theta)}{1 + \varepsilon_w^S(\theta)/\varepsilon_w^D(\theta)},
\end{aligned}$$

where the structural cross-wage elasticity $\gamma(\theta,\theta')$ and the own-wage elasticity $1/\varepsilon_w^D(\theta)$ are defined as in (6) and (7). Finally, the resolvent cross-wage elasticity $\Gamma(\theta,\theta')$ is defined by $\Gamma(\theta,\theta') \equiv \sum_{n=1}^{\infty} \Gamma_n(\theta,\theta')$ with $\Gamma_1(\theta,\theta') = \gamma(\theta,\theta')$ and for all $n \geq 2$,

$$\Gamma_n(\theta,\theta') = \int_{\Theta} \Gamma_{n-1}(\theta,\theta'')\, \varepsilon_w(\theta'')\, \gamma(\theta'',\theta')\, d\theta'',$$

where in this expression $\varepsilon_w(\theta'')$ is given by the previous equation (rather than by its expression in the quasilinear environment).

D.1.2 Tax incidence formula

With general preferences, the incidence of an arbitrary tax reform $\hat{T}$ on individual labor supply is given by the following formula, which generalizes (10):

$$\hat{l}(\theta) = \hat{l}_{\text{pe}}(\theta) + \varepsilon_w(\theta) \int_{\Theta} \Gamma(\theta,\theta')\, \hat{l}_{\text{pe}}(\theta')\, d\theta', \tag{58}$$

where $\varepsilon_w(\theta)$, and $\Gamma(\theta,\theta')$ are given by their generalized definitions above, and where

$$\hat{l}_{\text{pe}}(\theta) \equiv -\varepsilon_r(\theta) \frac{\hat{T}'(y(\theta))}{1 - T'(y(\theta))} + \varepsilon_R(\theta) \frac{\hat{T}(y(\theta))}{(1 - T'(y(\theta)))\, y(\theta)}.$$

The interpretation of this formula is identical to that of (10), except that the partial-equilibrium impact of the reform $\hat{l}_{\text{pe}}(\theta)$ is modified: in addition to the substitution effect already described in the quasilinear model, labor supply now also rises by an amount proportional to $\varepsilon_R(\theta)$ due to an income effect induced by the higher total tax payment $\hat{T}(y(\theta))$ of agent θ. Note that the partial-equilibrium formula for $\hat{l}_{\text{pe}}(\theta)$ is identical to that derived in models with exogenous wages by Saez (2001) and Golosov, Tsyvinski, and Werquin (2014), except that that now the elasticities $\varepsilon_r(\theta)$ and $\varepsilon_R(\theta)$ take into account the own-wage effects $\alpha(\theta)$. The (closed-form) incidence on wages, utilities and government revenue are then derived identically to the corresponding formulas in Section 2.2.

Proof of equation (58). In the model with exogenous wages, the incidence of a tax reform $\hat{T}$ on labor supply is given by (see Golosov, Tsyvinski, and Werquin (2014))

$$\hat{l}_{\text{pe}}(\theta) \equiv -e_r^c(\theta) \frac{\hat{T}'(y(\theta))}{1 - T'(y(\theta))} + e_R(\theta) \frac{\hat{T}(y(\theta))}{(1 - T'(y(\theta)))\, y(\theta)}.$$

Now, in the general-equilibrium model, consider a tax reform $\hat{T}$. The perturbed first order condition reads (letting $w_\theta = w(\theta)$, etc. for conciseness):

$$\begin{aligned} 0 = & \left[1 - T'\left((w_\theta + \mu\hat{w}_\theta)\left(l_\theta + \mu\hat{l}_\theta\right)\right) - \mu\hat{T}'(w_\theta l_\theta)\right](w_\theta + \mu\hat{w}_\theta)\ldots \\ & \times U_c\left[(w_\theta + \mu\hat{w}_\theta)\left(l_\theta + \mu\hat{l}_\theta\right) - T\left((w_\theta + \mu\hat{w}_\theta)\left(l_\theta + \mu\hat{l}_\theta\right)\right) - \mu\hat{T}(w_\theta l_\theta)\ ,\ l_\theta + \mu\hat{l}_\theta\right] \\ & + U_l\left[(w_\theta + \mu\hat{w}_\theta)\left(l_\theta + \mu\hat{l}_\theta\right) - T\left((w_\theta + \mu\hat{w}_\theta)\left(l_\theta + \mu\hat{l}_\theta\right)\right) - \mu\hat{T}(w_\theta l_\theta)\ ,\ l_\theta + \mu\hat{l}_\theta\right]. \end{aligned}$$

Tedious but straightforward algebra leads to the following first-order Taylor expansion:

$$
\begin{aligned}
0 = \quad & \Big[(1-T'(y_\theta))^2 w_\theta y_\theta U_{cc}(\theta) + (1-T'(y_\theta)) y_\theta U_{cl}(\theta) + \dots \\
& \quad (1-T'(y_\theta)) w_\theta U_c(\theta) - w_\theta y_\theta T''(y_\theta) U_c(\theta)\Big] \frac{\hat{w}_\theta}{w_\theta} \\
& + \Big[(1-T'(y_\theta))^2 w_\theta^2 U_{cc}(\theta) + (1-T'(y_\theta)) w_\theta U_{cl}(\theta) + \dots \\
& \quad (1-T'(y_\theta)) w_\theta U_{cl}(\theta) + U_{ll}(\theta) - w_\theta^2 T''(y_\theta) U_c(\theta)\Big] \hat{l}_\theta \\
& - w_\theta U_c(\theta) \hat{T}'(y_\theta) - [(1-T'(y_\theta)) w_\theta U_{cc}(\theta) + U_{cl}(\theta)] \hat{T}(y_\theta).
\end{aligned}
$$

Solving for $\hat{l}_\theta$ yields

$$
\begin{aligned}
\frac{\hat{l}_\theta}{l_\theta} = & \frac{e_R(\theta) + (1-p(y_\theta)) e_r^c(\theta)}{1 + p(y_\theta) e_r^c(\theta)} \frac{\hat{w}_\theta}{w_\theta} \\
& - \frac{e_r^c(\theta)}{1 + p(y_\theta) e_r^c(\theta)} \frac{\hat{T}'(y_\theta)}{1 - T'(y_\theta)} - \frac{e_R(\theta)}{1 + p(y_\theta) e_r^c(\theta)} \frac{\hat{T}(y_\theta)}{(1 - T'(y_\theta)) y_\theta}.
\end{aligned}
$$

Now, identical calculations as in the quasilinear model (see equation (49)) implies that in response to the tax reform $\hat{T}$, the first-order change in the wage $w(\theta) = \frac{\partial \mathscr{F}}{\partial L(\theta)}$ is given by

$$
\frac{\hat{w}_\theta}{w_\theta} = -\frac{1}{\varepsilon_w^D(\theta)} \frac{\hat{l}_\theta}{l_\theta} + \int_\Theta \gamma(\theta, \theta') \frac{\hat{l}_{\theta'}}{l_{\theta'}} d\theta'.
$$

We therefore obtain the following integral equation:

$$
\frac{\hat{l}_\theta}{l_\theta} = -\varepsilon_r^c(\theta) \frac{\hat{T}'(y_\theta)}{1 - T'(y_\theta)} - \varepsilon_R(\theta) \frac{\hat{T}(y_\theta)}{(1 - T'(y_\theta)) y_\theta} + \varepsilon_w(\theta) \int_\Theta \gamma(\theta, \theta') \frac{\hat{l}_{\theta'}}{l_{\theta'}} d\theta'.
$$

Following the same steps as in Proposition 1, the solution to this integral equation is given by

$$
\hat{l}(\theta) = \hat{l}_{\text{pe}}(\theta) + \varepsilon_w(\theta) \int_\Theta \Gamma(\theta, \theta') \hat{l}_{\text{pe}}(\theta') d\theta'.
$$

□

D.1.3 Generalization of Corollary 4

We now generalize formula (20) characterizing the incidence of tax reforms on government revenue when the production function is CES and the tax schedule is CRP. Suppose in addition that the utility function has the form $U(c,l) = \frac{c^{1-\eta}}{1-\eta} - \frac{l^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}$. The revenue effect of the elementary tax reform at income y^* is then given by

$$\hat{\mathcal{R}}\left(y^*\right) = \hat{\mathcal{R}}_{\text{ex}}\left(y^*\right) + \phi\,\varepsilon_r^S \frac{T'\left(y^*\right) - \bar{T}'}{1 - T'\left(y^*\right)} \frac{y^* f_Y\left(y^*\right)}{1 - F_Y\left(y^*\right)} - \phi\,\varepsilon_r^S\,\eta\,\mathbb{E}\left[\frac{T'\left(y\right) - \bar{T}'}{1 - T'\left(y\right)}\,|y > y^*\right], \tag{59}$$

where $\bar{T}' = \mathbb{E}\left[yT'\left(y\right)\right]/\mathbb{E}y$ is the income-weighted average marginal tax rate in the economy and where ϕ is as defined in Corollary 4. Note that for $\eta = 0$, this formula reduces to equation (20). If $\eta > 0$ and the baseline tax schedule is progressive, then the first and second general-equilibrium contributions have opposite signs. If top incomes are Pareto distributed and the baseline tax schedule is CRP, we derive below a necessary and sufficient condition on the progressivity parameter p, the Pareto coefficient and the curvature of the utility function η such that the first general-equilibrium term in (59) is larger than the second as $y \to \infty$, so that the theoretical insights of Section 3.2 remain valid with income effects. For empirically plausible values of the income effect parameter, the magnitude of the general-equilibrium contribution to government revenue incidence obtained in Section is reduced by a third (in particular, it keeps the same direction).

Proof of equation (59). Assume that the utility function has the form $\frac{c^{1-\eta}}{1-\eta} - \frac{l^{1+\frac{1}{e}}}{1+\frac{1}{e}}$, that the production function is CES and the initial tax schedule is CRP. First, we show that the labor supply elasticities $\varepsilon_r^{c,S}\left(\theta\right)$, $\varepsilon_w^S\left(\theta\right)$ and $\varepsilon_R^S\left(\theta\right)$ take a particularly simple form given these assumptions. We easily get

$$\begin{aligned}
\varepsilon_r^{c,S} =& \frac{e}{\eta e(1-p) + pe + 1} \\
\varepsilon_R^S =& -(1-p)\eta\varepsilon_r^{c,S}\left(\theta\right) \\
\varepsilon_w^S =& (1-p)\varepsilon_r^{c,S} + \varepsilon_R^S = \left(1-p\right)\left(1-\eta\right)\varepsilon_r^{c,S}.
\end{aligned}$$

Next, we turn to to solution to the integral equation given the assumption of a CES production function. As in the case of a quasilinear utility function, the kernel of the integral equation is multiplicatively separable, and its solution (given an elementary tax reform at income $y(\theta^*)$) is given by

$$\begin{aligned}
\hat{l}\left(\theta\right) =& -\frac{\varepsilon_r(\theta^*)}{1 - T'(y(\theta^*))}\delta\left(y\left(\theta\right) - y\left(\theta^*\right)\right) + \frac{\varepsilon_R(\theta)}{(1 - T'(y(\theta))y(\theta)}\mathbb{I}_{\{\theta > \theta^*\}} + \frac{\varepsilon_w(\theta)}{1 - \int_\Theta \varepsilon_w(\theta')\gamma(\theta,\theta')d\theta'} \\
&\times\left[-\gamma(\theta,\theta^*)\frac{\varepsilon_r(\theta^*)}{1 - T'(y(\theta^*))} + \int_{\theta^*}^{\overline{\theta}} \gamma(\theta,\theta')\frac{\varepsilon_R(\theta')}{(1 - T'(y(\theta'))y(\theta')}d\theta'\right].
\end{aligned} \tag{60}$$

Next, the revenue effect of a tax reform $\hat{T}$ is given by:

$$\begin{aligned}
\hat{\mathcal{R}} =& \int \hat{T}(y(\theta))dF(\theta) + \int T'(y(\theta))y(\theta)\left[\hat{l}(\theta) + \hat{w}(\theta)\right]dF\left(\theta\right) \\
=& \int \hat{T}(y(\theta))dF(\theta) + \int_\theta T'(y(\theta))y(\theta)\left[\hat{l}(\theta)\left(1 + \frac{1}{\varepsilon_w^S(\theta)}\right)\right. \\
&\left. + \frac{\varepsilon_r^S(\theta)}{\varepsilon_w^S(\theta)}\frac{\hat{T}'(y(\theta))}{1 - T'(y(\theta))} - \frac{\varepsilon_R^S(\theta)}{\varepsilon_w^S(\theta)}\frac{\hat{T}(y(\theta))}{(1 - T'(y(\theta))y(\theta)}\right]dF\left(\theta\right).
\end{aligned}$$

Focusing on the elementary tax reforms at $y(\theta^*)$ and substituting for $\hat{l}(\theta)$ using (60) leads to:

$$\begin{aligned}\hat{\mathcal{R}}(y(\theta^*)) = \hat{\mathcal{R}}_{\text{ex}}(y(\theta^*)) + \frac{\varepsilon_r^S(\theta^*)}{1-T(y(\theta^*))}\Bigg(&\frac{1}{\varepsilon_w^D(\theta^*)}\frac{1+\varepsilon_w^S(\theta^*)}{1+\frac{\varepsilon_w^S(\theta^*)}{\varepsilon_w^D(\theta^*)}}T'(y(\theta^*))y(\theta^*)f(\theta^*)\ldots \\ &-\int T'(y(\theta))y(\theta)\frac{\gamma(\theta,\theta^*)\frac{1+\varepsilon_w^S(\theta)}{1+\varepsilon_w^S(\theta)/\varepsilon_w^D(\theta)}\frac{1}{1+\varepsilon_w^S(\theta^*)/\varepsilon_w^D(\theta^*)}}{1-\int\varepsilon_w(x)\gamma(x,x)dx}dF(\theta)\Bigg) \\ +\int_{\theta^*}^{\bar{\theta}}\frac{\varepsilon_R^S(\theta')}{(1-T'(y(\theta'))y(\theta')}\Bigg(&\frac{1}{\varepsilon_w^D(\theta')}\frac{1+\varepsilon_w^S(\theta')}{1+\frac{\varepsilon_w^S(\theta')}{\varepsilon_w^D(\theta')}}T'(y(\theta'))y(\theta')f(\theta') \\ &-\int T'(y(\theta))y(\theta)\frac{\gamma(\theta,\theta')\frac{1+\varepsilon_w^S(\theta)}{1+\varepsilon_w^S(\theta)/\varepsilon_w^D(\theta)}\frac{1}{1+\varepsilon_w^S(\theta')/\varepsilon_w^D(\theta')}}{1-\int\varepsilon_w(x)\gamma(x,x)dx}dF(\theta)\Bigg)d\theta'\end{aligned}$$

where we used $\varepsilon_R\left(1+1/\varepsilon_w^S\right)-\varepsilon_R^S/\varepsilon_w^S=\varepsilon_R^S+\frac{-\varepsilon_R^S/\varepsilon_w^D-\varepsilon_R^S\varepsilon_w^S/\varepsilon_w^D}{1+\varepsilon_w^S/\varepsilon_w^D}$ and the analogous relationship between ε_r^S and ε_r. Using the CES and the CRP functional forms, which imply constant elasticities, as well as the relationship $\varepsilon_R^S=-(1-p)\eta\varepsilon_r^{c,S}$, we obtain

$$\begin{aligned}\frac{\hat{\mathcal{R}}(y(\theta^*))}{1-F(\theta^*)} &= \frac{\hat{\mathcal{R}}_{\text{ex}}(y(\theta^*))}{1-F(\theta^*)} \\ &+\frac{1}{\varepsilon_w^D}\frac{1+\varepsilon_w^S}{1+\frac{\varepsilon_w^S}{\varepsilon_w^D}}\varepsilon_r^S\left(\frac{y(\theta^*)f(\theta^*)}{1-F(\theta^*)}\left(\frac{T'(y(\theta^*))-\bar{T}'}{1-T'(y(\theta^*))}\right)-\eta\,(1-p)\,\mathbb{E}\left[\frac{T'(y(\theta))-\bar{T}'}{1-T'(y(\theta))}|\theta>\theta^*\right]\right)\end{aligned} \tag{61}$$

where $\bar{T}'$ is the income-weighted average of the marginal tax rate. But since the tax schedule is CRP we have

$$\bar{T}'=\frac{\int_\Theta[1-(1-m)\,y(\theta)^{-p}]y(\theta)dF(\theta)}{\bar{y}}=1-(1-m)\mathbb{E}[y(\theta)^{1-p}]\frac{1}{\bar{y}}.$$

Equation (61) can thus be rewritten as

$$\begin{aligned}\frac{\hat{\mathcal{R}}(y(\theta^*))}{1-F(\theta^*)} = \frac{\hat{\mathcal{R}}_{\text{ex}}(y(\theta^*))}{1-F(\theta^*)}+\frac{1}{\varepsilon_w^D}\frac{1+\varepsilon_w^S}{1+\frac{\varepsilon_w^S}{\varepsilon_w^D}}\varepsilon_r^S\Bigg(&\frac{y(\theta^*)f(\theta^*)}{1-F(\theta^*)}\left(\left(\frac{y(\theta^*)^p\mathbb{E}[y(\theta)^{1-p}]}{\bar{y}}-1\right)\right) \\ &-\eta(1-p)\left(\frac{\mathbb{E}[y(\theta')^p|\theta'>\theta^*]\mathbb{E}[y(\theta)^{1-p}]}{\bar{y}}-1\right)\Bigg)\end{aligned}$$

Now assume that incomes above θ^* are Pareto distributed with tail parameter Π. Then we have $\mathbb{E}[y^p|y>y^*]=\frac{\Pi}{\Pi-p}y^{*p}$. Moreover, using $\frac{y(\theta^*)f(\theta^*)}{1-F(\theta^*)}\to\Pi$ and $\mathbb{E}(y^{1-p})/\mathbb{E}(y)=\frac{1-\bar{T}'}{1-m}$, we get

$$\begin{aligned}\frac{\hat{\mathcal{R}}(y(\theta^*))}{1-F(\theta^*)} =&\frac{\hat{\mathcal{R}}_{\text{ex}}(y(\theta^*))}{1-F(\theta^*)}+\frac{1}{\varepsilon_w^D}\frac{1+\varepsilon_w^S}{1+\alpha\varepsilon_w^S}\varepsilon_r^S \\ &\times\left[\Pi\left(\frac{1-\bar{T}'}{1-T'(y(\theta^*))}-1\right)-\eta(1-p)\left(\frac{\Pi}{\Pi-p}\frac{1-\bar{T}'}{1-T'(y(\theta^*))}-1\right)\right].\end{aligned} \tag{62}$$

The term in brackets becomes positive for y large enough if and only if $\Pi>\eta(1-p)\frac{\Pi}{\Pi-p}$ (i.e.,

$\Pi > \eta + p(1-\eta))$. This is because $T'(y) \to 1$ as $y \to \infty$.

Equation (62) leads to simple calculations of the additional general equilibrium effect on government revenue. To illustrate this, we consider a parameterization that is based on the empirical literature that estimates the impact of lottery wins on labor supply (Imbens, Rubin, and Sacerdote (2001), Cesarini et al. (2017)). Using these wealth shocks they find that a one dollar increase in wealth leads to a decrease in life-cycle labor income (in net present value) of 10-11 cents. Thus, we calibrate our (static) model such that an increase in unearned income of 1 dollar implies a decrease in earnings of 10-11 cents. Further, we set $\varepsilon_r^{c,S}(\theta) = 0.33$ Chetty (2012). As in our benchmark calibration in the main body, we assume that $p = 0.15$. To target the value of the lottery papers, we set $\varepsilon_R^S(\theta) = -0.08$, which captures approximatey a 10-11 cents decrease in gross income if the marginal tax rate is around 25%. The relationship $\varepsilon_R^S(\theta) = -(1-p)\eta\varepsilon_r^{c,S}$ then yields a value of $\eta \approx 0.29$. Finally, the value for e that is consistent with $\varepsilon_r^{c,S} = 0.33$ is $e = 0,38$.

Evaluating the second term on the right hand side of (62) for these numbers reveals that it becomes positive for income levels where the marginal tax rate is above 27,6%, a number that is slightly higher than the income-weighted average marginal tax rate, which is equal to 26%. The income levels that correspond to these tax rates are approximately \$85,000 and \$77,000.

A last simple exercise is then to evaluate general equilibrium revenue effect at a higher income level and compare it to the value that is obtained in the absence of income effects. We do this comparison for the income level of \$200,000 and find that the additional revenue effect coming from the endogeneity of wages is reduced by 28% (32% respectively) if the elasticity of substitution is $\sigma = 0.66$ ($\sigma = 3.1$ respectively).

□

D.2 Intensive and extensive margins

D.2.1 Formal model

We now extend the model of Section 1 to an environment where individuals choose their labor supply both on the intensive margin (hours l conditional on participating in the labor force) and on the extensive margin (participation decision).

There are two dimensions of heterogeneity: individuals are indexed by their skill $\theta \in [\underline{\theta}, \bar{\theta}] \equiv \Theta$ and by their fixed cost of working $\kappa \in \mathbb{R}_+$. The utility function is given by

$$U(c,l) = u[c - v(l) - \kappa \mathbb{I}_{\{l>0\}}],$$

where $\mathbb{I}_{\{l>0\}}$ is an indicator function equal to 1 if the agent is employed (i.e., $l > 0$).

An individual of type (θ, κ) chooses both whether to participate in the labor force at wage $w(\theta)$, and if so, how much effort to provide. If he decides to stay non-employed, his labor supply and income are equal to zero and he consumes the government-provided transfer $-T(0)$. Thus agent (θ, κ) solves the maximization problem

$$U(\theta,\kappa) \equiv \max \left\{ \sup_{l>0} u\left[w(\theta) l - T(w(\theta) l) - v(l) - \kappa\right] ; u(-T(0)) \right\}.$$

Due to the lack of income effects, the labor supply $l(\theta)$ that an agent (θ, κ) chooses conditional on participation is independent of κ, and it is the solution to the first order condition

$$v'(l(\theta)) = [1 - T'(w(\theta) l(\theta))] w(\theta).$$

Moreover, an agent with skill θ decides to participate if and only if his fixed cost of work κ is smaller than a threshold $\bar{\kappa}(\theta)$, given by

$$\kappa^*(\theta) = w(\theta) l(\theta) - T(w(\theta) l(\theta)) - v(l(\theta)) + T(0). \tag{63}$$

Note that both $l(\theta)$ and $\kappa^*(\theta)$ are endogenous to the tax schedule: the intensive margin choice of labor effort $l(\theta)$ depends on the marginal tax rate $T'(y(\theta))$, while the extensive margin choice of participation depends on the average tax rate relative to transfers, $T(y(\theta)) - T(0)$. Denote by $f(\theta, \kappa)$ the density of κ conditional on skill θ, by

$$\pi(\theta) = \frac{\int_0^{\kappa^*(\theta)} f(\theta, \kappa) \, d\kappa}{\int_0^{\infty} f(\theta, \kappa) \, d\kappa}$$

the employment rate within the population of skill θ, and by

$$L(\theta) = l(\theta) \int_0^{\kappa^*(\theta)} f(\theta, \kappa) \, d\kappa$$

the total amount of labor supplied by workers of skill θ. The rest of the environment is identical to that of Section 1.

D.2.2 Elasticity concepts

We define the participation elasticity $\eta_T^S(\theta)$ of the population with skill θ with respect to their average tax rate as

$$\eta_T^S(\theta) \equiv \frac{\partial \ln \pi(\theta)}{\partial \ln [y(\theta) - T(y(\theta)) + T(0)]} = [y(\theta) - T(y(\theta)) + T(0)] \frac{f(\theta, \kappa^*(\theta))}{\pi(\theta) \int_0^{\infty} f(\theta, \kappa) \, d\kappa}. \tag{64}$$

This elasticity is determined by the reservation density $f(\theta, \kappa^*(\theta))$ of agents with skill θ who are close to indifference between participation and non-participation in the baseline tax system. We also define the participation elasticity $\eta_w^S(\theta)$ with respect to the wage as

$$\eta_w^S(\theta) \equiv \frac{\partial \ln \pi(\theta)}{\partial \ln w(\theta)} = (1 - T'(y(\theta))) y(\theta) \frac{f(\theta, \kappa^*(\theta))}{\pi(\theta) \int_0^{\infty} f(\theta, \kappa) \, d\kappa}. \tag{65}$$

Note that these elasticities are partial equilibrium concepts: they ignore the feedback impact of these initial adjustments in participation on individual wages and, in turn, labor supply. We then define the partial equilibrium elasticities as

$$\begin{aligned}\varepsilon_r(\theta) &= \frac{\varepsilon_r^S(\theta)}{1+\left(\varepsilon_w^S(\theta)+\eta_w^S(\theta)\right)\alpha(\theta)},\\ \varepsilon_w(\theta) &= \frac{\varepsilon_w^S(\theta)}{1+\left(\varepsilon_w^S(\theta)+\eta_w^S(\theta)\right)\alpha(\theta)},\\ \eta_T(\theta) &= \frac{\eta_T^S(\theta)}{1+\left(\varepsilon_w^S(\theta)+\eta_w^S(\theta)\right)\alpha(\theta)},\\ \eta_w(\theta) &= \frac{\eta_w^S(\theta)}{1+\left(\varepsilon_w^S(\theta)+\eta_w^S(\theta)\right)\alpha(\theta)}.\end{aligned} \tag{66}$$

We finally define the GE cross-wage elasticities by $\Gamma(\theta,\theta') = \sum_{n\geq 1}\Gamma_n(\theta,\theta')$, where $\Gamma_1(\theta,\theta') = \gamma(\theta,\theta')$ and for all $n \geq 2$,

$$\Gamma_n(\theta,\theta') = \int_\Theta \Gamma_{n-1}(\theta,\theta'')\left(\varepsilon_w(\theta'')+\eta_w(\theta'')\right)\gamma(\theta'',\theta')\,d\theta''.$$

Compared to its expression (11) in the quasilinear environment, $\Gamma(\theta,\theta')$ now takes into account that the labor supply changes along both the intensive and the extensive (participation) margins of type θ'' impact their wage, through the respective elasticities $\varepsilon_w(\theta'')$ and $\eta_w(\theta'')$. Let

$$\rho(\theta) \equiv \frac{f(\theta,\kappa^*(\theta))}{\int_0^{\kappa^*(\theta)} f(\theta,\kappa)\,d\kappa}$$

denote the density of employed agents on the verge of non-participation.

D.2.3 Tax incidence formula

The incidence of an arbitrary tax reform $\hat{T}$ on the total labor supply $L(\theta)$ of agents of skill θ is given by the following formula, which generalizes Proposition 1:

$$\hat{L}(\theta) = \hat{L}_{\text{pe}}(\theta) + \left(\varepsilon_w(\theta)+\eta_w(\theta)\right)\int_\Theta \Gamma(\theta,\theta')\,\hat{L}_{\text{pe}}(\theta')\,d\theta', \tag{67}$$

where $\varepsilon_w(\theta)$, and $\Gamma(\theta,\theta')$ are replaced by their generalized definitions, and where

$$\hat{L}_{\text{pe}}(\theta) \equiv -\varepsilon_r(\theta)\frac{\hat{T}'(y(\theta))}{1-T'(y(\theta))} - \eta_T(\theta)\frac{\hat{T}(y(\theta))}{y(\theta)-T(y(\theta))+T(0)}.$$

The interpretation of this formula is identical to that of (10), with two differences. First, the partial-equilibrium impact ($\hat{L}^{\text{pe}}(\theta)$) is modified: in addition to the substitution effect already described in the quasilinear model, the tax reform now raises the tax payment of agents with skill θ by $\hat{T}(y(\theta))$, which lowers the total labor supply of that skill by an amount proportional to $\eta_T(\theta)$, by inducing those with a large fixed cost of working to drop out of the labor force. Second, the change in the wage of type θ induces a decrease in total hours (from both intensive and extensive margin responses) given by $(\varepsilon_w(\theta)+\eta_w(\theta))$ rather than simply $\varepsilon_w(\theta)$. From this formula, it is straightforward to obtain the incidence of any tax reform on individual labor supplies, wages, participation thresholds, participation rates, utilities, and government revenue.

Proof of formula (67). We first derive the incidence of a tax reform $\hat{T}$ on the wage $w(\theta)$, participation threshold $\kappa^*(\theta)$, individual labor supply $l(\theta)$, and aggregate labor supply $L(\theta)$ of agents of skill θ. We have

$$\begin{aligned}\hat{w}(\theta) = \lim_{\mu\to 0}\frac{1}{\mu}\Bigg[&\mathscr{F}'_\theta(\{\left[l(\theta') + \mu\hat{l}(\theta')\right]\int_0^{\kappa^*(\theta')+\mu\hat{\kappa}^*(\theta')} f(\theta',\kappa)\,d\kappa\}_{\theta'\in\Theta})\\ &- \mathscr{F}'_\theta(\{l(\theta')\int_0^{\kappa^*(\theta')} f(\theta',\kappa)\,d\kappa\}_{\theta'\in\Theta})\Bigg]\\ =& w(\theta)\left[-\frac{1}{\varepsilon_w^D(\theta)}\left\{\frac{\hat{l}(\theta)}{l(\theta)} + \rho(\theta)\hat{\kappa}^*(\theta)\right\} + \int_\Theta \gamma(\theta,\theta')\left\{\frac{\hat{l}(\theta')}{l(\theta')} + \rho(\theta')\hat{\kappa}^*(\theta')\right\}d\theta'\right].\end{aligned}$$

Moreover, we have

$$\begin{aligned}\hat{\kappa}^*(\theta) = \lim_{\mu\to 0}\frac{1}{\mu}\Big\{&[w(\theta)+\mu\hat{w}(\theta)]\left[l(\theta)+\mu\hat{l}(\theta)\right] - w(\theta)l(\theta)\\ &- T\left([w(\theta)+\mu\hat{w}(\theta)]\left[l(\theta)+\mu\hat{l}(\theta)\right]\right) + T(w(\theta)l(\theta))\\ &-\mu\hat{T}\left([w(\theta)+\mu\hat{w}(\theta)]\left[l(\theta)+\mu\hat{l}(\theta)\right]\right) - v\left(\left[l(\theta)+\mu\hat{l}(\theta)\right]\right) + v(l(\theta))\Big\}\\ =& [1-T'(y(\theta))]\,y(\theta)\,\hat{w}(\theta) - \hat{T}(y(\theta)).\end{aligned}$$

Next, a Taylor expansion of the agent's first-order condition

$$\begin{aligned}0 =& \left\{1 - T'\left([w(\theta)+\mu\hat{w}(\theta)]\left[l(\theta)+\mu\hat{l}(\theta)\right]\right) - \mu\hat{T}'\left([w(\theta)+\mu\hat{w}(\theta)]\left[l(\theta)+\mu\hat{l}(\theta)\right]\right)\right\}\\ &\times[w(\theta)+\mu\hat{w}(\theta)] - [1-T'(w(\theta)l(\theta))]\,w(\theta) - v'\left(\left[l(\theta)+\mu\hat{l}(\theta)\right]\right) + v'(l(\theta))\end{aligned}$$

leads to

$$\frac{\hat{l}(\theta)}{l(\theta)} = -\,\varepsilon_r^S(\theta)\frac{\hat{T}'(w(\theta)l(\theta))}{1-T'(y(\theta))} + \varepsilon_w^S(\theta)\,\hat{w}(\theta).$$

We finally have

$$\begin{aligned}\hat{L}(\theta) = \lim_{\mu\to 0}\frac{1}{\mu}&\left\{\left[l(\theta)+\mu\hat{l}(\theta)\right]\int_0^{\kappa^*(\theta)+\mu\hat{\kappa}^*(\theta)} f(\theta,\kappa)\,d\kappa - l(\theta)\int_0^{\kappa^*(\theta)} f(\theta,\kappa)\,d\kappa\right\}\\ =& L(\theta)\left\{\frac{\hat{l}(\theta)}{l(\theta)} + \rho(\theta)\hat{\kappa}^*(\theta)\right\}.\end{aligned}$$

These equations lead to

$$\begin{aligned}\frac{\hat{L}(\theta)}{L(\theta)} =& -\,\varepsilon_r(\theta)\frac{\hat{T}'(y(\theta))}{1-T'(y(\theta))} - \eta_T(\theta)\frac{\hat{T}(y(\theta))}{y(\theta)-T(y(\theta))+T(0)}\\ &+ (\varepsilon_w(\theta)+\eta_w(\theta))\int_\Theta \gamma(\theta,\theta')\frac{\hat{L}(\theta')}{L(\theta')}d\theta'.\end{aligned}$$

This is an integral equation. Its solution can be found following identical steps as in the pure intensive margin environment, and is given by (67). The incidence on wages is then given by

$$\hat{w}(\theta) = -\frac{1}{\varepsilon_w^D(\theta)}\hat{L}(\theta) + \left[\frac{\hat{L}(\theta) - \hat{L}_{\text{pe}}(\theta)}{\varepsilon_w(\theta) + \eta_w(\theta)}\right].$$

The incidence on participation and individual labor supplies is then easily obtained from the expressions for $\hat{\kappa}^*(\theta)$ and $\hat{l}(\theta)$ derived above.

□

D.3 Multiple sectors

In this section we consider a multi-sector model where the sorting of agents across sectors is not affected by tax reforms. In section D.4 below, we consider a Roy model in which the assignment is endogenous to taxes and responds to tax reforms.

D.3.1 Model and elasticity concepts

The aggregate production function $\mathscr{F}$ we used in Section 1 takes as inputs the labor supply of each one-dimensional skill type $\theta \in \Theta$. In this framework, the skill θ of an agent can be interpreted as her percentile in the wage distribution $\{w(\theta)\}_{\theta\in\Theta}$. Suppose now that the population is divided into N groups (e.g., sectors, education levels, etc.). Each group i is composed of a continuum of agents indexed by their skill $\theta \in \Theta$ who earn wage $w_i(\theta)$. The assignment of each individual to a given group i is exogenous. Note that the wage distributions $\{w_i(\theta)\}_{\theta\in\Theta}$ and $\{w_j(\theta)\}_{\theta\in\Theta}$ of different groups $i \neq j$ overlap. The aggregate production function is now defined by

$$\mathscr{F}(\{L_i(\theta)\}_{(\theta,i)\in\Theta\times\{1,\ldots,N\}}), \tag{68}$$

where $L_i(\theta)$ is the aggregate labor supply of the agents of type θ who work in sector i.

We define the wage, labor supply, and income of type θ in sector i by $w_i(\theta)$, $l_i(\theta)$, and $y_i(\theta)$ respectively, and let $f_i(\theta)$ the density of types in that sector. The first order condition of this agent writes

$$v'(l_{\theta,i}) = [1 - T'(w_{\theta,i}l_{\theta,i})]\, w_{\theta,i}.$$

Define this agent's labor supply elasticitiy with respect to the retention rate as

$$\varepsilon_r^S(\theta,i) = \frac{e(\theta,i)}{1 + e(\theta,i)\, p(y_{\theta,i})}, \text{ where } e(\theta,i) = \frac{v'(l_{\theta,i})}{l_{\theta,i}v''(l_{\theta,i})}$$

and with respect to the wage as

$$\varepsilon_w^S(\theta,i) = (1 - p(y_{\theta,i}))\, \varepsilon_r^S(\theta,i).$$

We also define the cross-wage elasticity of the wage of agent θ in sector i, with respect to the labor supply of agent θ' in sector j, as

$$\gamma\left(\left(\theta,i\right),\left(\theta',j\right)\right)=\frac{\partial\ln w_{\theta,i}}{\partial\ln L_{\theta',j}}=\frac{L_{\theta',j}\mathscr{F}''_{(\theta,i),(\theta',j)}}{\mathscr{F}'_{(\theta,i)}}, \tag{69}$$

and the own-wage elasticity of agent θ in sector i as

$$\frac{1}{\varepsilon_w^D\left(\theta,i\right)}=\frac{\partial\ln w_{\theta,i}}{\partial\ln L_{\theta,i}}-\lim_{\theta'\to\theta}\frac{\partial\ln w_{\theta',i}}{\partial\ln L_{\theta,i}}.$$

We finally define the elasticities of equilibrium labor as

$$\varepsilon_r\left(\theta,i\right)=\frac{\varepsilon_r^S\left(\theta,i\right)}{1+\varepsilon_w^S\left(\theta,i\right)/\varepsilon_w^D\left(\theta,i\right)},\ \varepsilon_w\left(\theta,i\right)=\frac{\varepsilon_s^S\left(\theta,i\right)}{1+\varepsilon_w^S\left(\theta,i\right)/\varepsilon_w^D\left(\theta,i\right)}.$$

A change of variables then allows us to define, for each income-group pair (y,i), the wage $w_{y,i}$ of the agents who earn income y in group i, and the $N\times 1$ vector $\boldsymbol{w}_y=\left(w_{y,i}\right)_{i=1,\ldots,N}$. We define analogously the vectors $\boldsymbol{l}_y$, $\hat{\boldsymbol{l}}_y/\boldsymbol{l}_y$ (where the "/" sign denotes here an element-by-element division), $\boldsymbol{\varepsilon}_r\left(y\right)$, $\boldsymbol{\varepsilon}_w\left(y\right)$, and the $N\times N$ matrices $\boldsymbol{\gamma}\left(y,y'\right)$ and $\boldsymbol{\Gamma}\left(y,y'\right)$.

D.3.2 Tax incidence formula

We now show that the result of Lemma 1 is replaced by a *system* of linear integral equations, which can be solved using analogous steps as those leading to Proposition 1. We obtain that the incidence of an arbitrary tax reform $\hat{T}$ on individual labor supplies is given in closed-form by

$$\frac{\hat{\boldsymbol{l}}_y}{\boldsymbol{l}_y}=-\boldsymbol{\varepsilon}_r\left(y\right)\frac{\hat{T}'\left(y\right)}{1-T'\left(y\right)}-\int_{\mathbb{R}_+}\text{Diag}\left(\boldsymbol{\varepsilon}_w\left(y\right)\right)\boldsymbol{\Gamma}\left(y,y'\right)\boldsymbol{\varepsilon}_r\left(y'\right)\frac{\hat{T}'\left(y'\right)}{1-T'\left(y'\right)}dy'. \tag{70}$$

The interpretation of this formula is identical to that of (10), with the only difference that the incidence of tax reforms now naturally depends on a larger number of (cross-sector) elasticities.

Proof of formula (70). The first-order condition in the perturbed equilibrium, after a tax reform $\hat{T}$, reads

$$0=v'\left(l_{\theta,i}+\hat{l}_{\theta,i}\right)-\left[1-T'\left(\left(w_{\theta,i}+\hat{w}_{\theta,i}\right)\left(l_{\theta,i}+\hat{l}_{\theta,i}\right)\right)-\hat{T}'\left(w_{\theta,i}l_{\theta,i}\right)\right]\left(w_{\theta,i}+\hat{w}_{\theta,i}\right).$$

Taking a first-order Taylor expansion and solving for $\hat{l}_{\theta,i}$ leads to

$$\hat{l}_{\theta,i}=l_{\theta,i}\left[-\varepsilon_r^S\left(\theta,i\right)\frac{\hat{T}'\left(y_{\theta,i}\right)}{1-T'\left(y_{\theta,i}\right)}+\varepsilon_w^S\left(\theta,i\right)\frac{\hat{w}_{\theta,i}}{w_{\theta,i}}\right].$$

Now, the perturbed wage equation writes

$$0=\left(w_{\theta,i}+\hat{w}_{\theta,i}\right)-\left[\mathscr{F}'_{(\theta,i)}+\sum_{j=1}^{N}\int_{\Theta}\mathscr{F}''_{(\theta,i),(\theta',j)}\hat{l}_{\theta',j}f_j\left(\theta'\right)d\theta'\right]$$

so that

$$\frac{\hat{w}_{\theta,i}}{w_{\theta,i}} = \sum_{j=1}^{N} \int_{\Theta} \gamma\left(\left(\theta,i\right),\left(\theta',j\right)\right) \frac{\hat{l}_{\theta',j}}{l_{\theta',j}} d\theta' + \frac{1}{\varepsilon_w^D\left(\theta,i\right)} \frac{\hat{l}_{\theta,i}}{l_{\theta,i}}.$$

We therefore obtain the following system of integral equations for labor supply: for all θ, i,

$$\frac{\hat{l}_{\theta,i}}{l_{\theta,i}} = -\,\varepsilon_r\left(\theta,i\right) \frac{\hat{T}'\left(y_{\theta,i}\right)}{1-T'\left(y_{\theta,i}\right)} + \varepsilon_w\left(\theta,i\right) \sum_{j=1}^{N} \int_{\Theta} \gamma\left(\left(\theta,i\right),\left(\theta',j\right)\right) \frac{\hat{l}_{\theta',j}}{l_{\theta',j}} d\theta'.$$

Define $\gamma_{i,j}\left(y\left(\theta\right),y\left(\theta'\right)\right) = \left(\frac{dy}{d\theta}\left(\theta'\right)\right)^{-1} \gamma_{i,j}\left(\theta,\theta'\right)$. Changing variables from types θ to incomes y in each sector i implies

$$\frac{\hat{l}_{y,i}}{l_{y,i}} = -\,\varepsilon_r\left(y,i\right) \frac{\hat{T}'\left(y\right)}{1-T'\left(y\right)} + \varepsilon_w\left(y,i\right) \sum_{j=1}^{N} \int_{\Theta} \gamma\left(\left(y,i\right),\left(y',j\right)\right) \frac{\hat{l}_{y',j}}{l_{y',j}} dy'.$$

Now define, for each income y, the $N \times 1$ vectors

$$\boldsymbol{l}_y = \left(l_{y,i}\right)_{i=1,\ldots,N},$$
$$\hat{\boldsymbol{l}}_y = \left(\hat{l}_{y,i}\right)_{i=1,\ldots,N},$$

and

$$\boldsymbol{\varepsilon}_r\left(y\right) = \left(\varepsilon_r\left(y,i\right)\right)_{i=1,\ldots,N}$$
$$\boldsymbol{\varepsilon}_w\left(y\right) = \left(\varepsilon_w\left(y,i\right)\right)_{i=1,\ldots,N},$$

and for each (y,y'), the $N \times N$ matrix

$$\boldsymbol{\gamma}\left(y,y'\right) = \left(\gamma\left(\left(y,i\right),\left(y',j\right)\right)\right)_{i,j=1,\ldots,N}.$$

We can then rewrite the previous system of integral equations in matrix form: for all y,

$$(\hat{\boldsymbol{l}}_y/\boldsymbol{l}_y) = -\,\boldsymbol{\varepsilon}_r\left(y\right) \frac{\hat{T}'\left(y\right)}{1-T'\left(y\right)} + \int_0^{\infty} \mathrm{Diag}\left(\boldsymbol{\varepsilon}_w\left(y\right)\right) \boldsymbol{\gamma}\left(y,y'\right) (\hat{\boldsymbol{l}}_{y'}/\boldsymbol{l}_{y'}) dy',$$

where $\mathrm{Diag}\left(\boldsymbol{\varepsilon}_w\left(y\right)\right)$ is the $N \times N$ diagonal matrix with elements $\left(\varepsilon_w\left(y,i\right)\right)_{i=1,\ldots,N}$ and the operator "/" denotes an element-by-element division. Now this integral equation can be easily solved in closed form. Define the matrices $\left(\boldsymbol{\Gamma}_n\left(y,y'\right)\right)_{n\geq 1}$ by $\boldsymbol{\Gamma}_1\left(y,y'\right) = \boldsymbol{\gamma}\left(y,y'\right)$ and for all $n \geq 2$,

$$\boldsymbol{\Gamma}_n\left(y,y''\right) = \int_0^{\infty} \boldsymbol{\Gamma}_{n-1}\left(y,y''\right) \mathrm{Diag}\left(\boldsymbol{\varepsilon}_w\left(y''\right)\right) \boldsymbol{\gamma}\left(y'',y'\right) dy'',$$

and let $\boldsymbol{\Gamma}\left(y,y'\right) = \sum_{n=1}^{\infty} \boldsymbol{\Gamma}_n\left(y,y'\right)$. Following the same steps as those leading to Proposition 1, we easily obtain

$$(\hat{\boldsymbol{l}}_y/\boldsymbol{l}_y) = -\,\boldsymbol{\varepsilon}_r\left(y\right) \frac{\hat{T}'\left(y\right)}{1-T'\left(y\right)} - \int_0^{\infty} \mathrm{Diag}\left(\boldsymbol{\varepsilon}_w\left(y\right)\right) \boldsymbol{\Gamma}\left(y,y'\right) \boldsymbol{\varepsilon}_r\left(y'\right) \frac{\hat{T}'\left(y'\right)}{1-T'\left(y'\right)} dy'.$$

This formula is the direct analogue of (10). It naturally depends on a larger number of elasticities, namely, the labor supply elasticities at each income level in each sector, and the cross-wage elasticities between any two types in any two sectors.

$\square$

D.3.3 Canonical model: two education groups

A special case of the general production function (68) is the so-called canonical model (Acemoglu and Autor, 2011), where individuals are categorized according to their level of education (high school vs. college). This model has been studied empirically by Katz and Murphy (1992) and Card and Lemieux (2001).

Formal model

Consider the following production function (for now we only assume constant returns to scale):

$$\mathscr{F}(\mathscr{L}_H, \mathscr{L}_C)$$

where

$$\mathscr{L}_C = \int_\Theta l_C(\theta)\theta dG_C(\theta) \quad \text{and} \quad \mathscr{L}_H = \int_\Theta l_H(\theta)\theta dG_H(\theta)$$

are aggregate college labor and aggregate high school labor, respectively. Wages are

$$w_j(\theta) = \frac{\partial \mathscr{F}}{\partial L_j(\theta)} = w_j \times \theta,$$

where $w_j \equiv \frac{\partial \mathscr{F}}{\partial \mathscr{L}_j}$ for $j = H, C$. In particular, we have $\frac{w_i(\theta')}{w_i(\theta)} = \frac{\theta'}{\theta}$ for any two types (θ, θ'), so that the relative wages within each group i are given by the ratio of the corresponding exogenous skills.

An individual of type θ in sector j solves

$$\max \quad w_j\theta l - T(w_j\theta l) - v(l).$$

The optimal labor supply depends only the product $w_j\theta$ (rather than on the sector j and on the type θ independently), so that individuals (θ', C) and (θ'', H) earn the same income if $w_C\theta' = w_H\theta''$. Denote by ω the product of ability and wage. Its density is

$$f(\omega) = g_C\left(\frac{\omega}{w_C}\right) + g_H\left(\frac{\omega}{w_H}\right).$$

Further, define the density of incomes by $f_Y(y(\omega)) = f(\omega)\frac{1}{y'(\omega)}$.

Now define the cross- and own-wage elasticities:

$$\gamma_{i,j} = \frac{dw_i}{d\mathscr{L}_j}\frac{\mathscr{L}_j}{w_i}.$$

Euler's homeogeous function theorem reads

$$\gamma_{ii} + \gamma_{ji} \frac{\mathscr{L}_j w_j}{\mathscr{L}_i w_i} = 0. \tag{71}$$

Finally, we also define aggregate income in both sectors

$$Y_i = w_i \mathscr{L}_i,$$

as well as the aggregate sector shares

$$s_i = \frac{Y_i}{Y_1 + Y_2}.$$

Generalization of formula (20)

Suppose now in addition that the aggregate production function is given by a CES aggregator of $\mathscr{L}_H$ and $\mathscr{L}_C$, i.e.,

$$\mathscr{F} = \left[\mathscr{L}_H^{\frac{\sigma-1}{\sigma}} + \mathscr{L}_C^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}.$$

In this model, there is an infinite elasticity of substitution between workers within each education level, and a finite and constant elasticity of substitution σ across the two groups. Suppose furthermore that the disutility of labor is isoelastic and that the initial tax schedule is CRP, so that the labor supply elasticities ε_r^S and ε_w^S are constant.

We obtain:

$$\hat{\mathcal{R}}(y^*) = \hat{\mathcal{R}}_{\text{ex}}(y^*) + \phi \varepsilon_r^S \left[s_C(y^*) - s_C \right] \frac{\bar{T}'_C - \bar{T}'_H}{1 - T'(y^*)} \frac{y^* f_Y(y^*)}{1 - F_Y(y^*)}, \tag{72}$$

where $\hat{\mathcal{R}}_{\text{ex}}(y^*)$ is given by (17), $\bar{T}'_i = \int T'(y)\, y f_{Y,i}(y)\, dy$ is the income-weighted average marginal tax rate in education group $i = H, C$, $s_C(y^*)$ is the share of individuals earning y^* that are college-educated, s_C is the share of aggregate income accruing to college-educated workers, and ϕ is defined as in Corollary 4.

Proof of equation (72).

Incidence of tax reforms on labor supply. The change in labor supply due to a tax reform $\hat{T}$ is given by:

$$\frac{\hat{l}_i(\omega)}{l_i(\omega)} = -\varepsilon_r^S(\omega) \frac{\hat{T}'(y(\omega))}{1 - T'(y(\omega))} + \varepsilon_w^S(\omega) \left[\gamma_{ii} \hat{\mathscr{L}}_i + \gamma_{ij} \hat{\mathscr{L}}_j \right]$$

which we can also write as a linear system of two integral equations

$$\frac{\hat{l}_i(\omega)}{l_i(\omega)} = \frac{\hat{l}_i^{\text{pe}}(\omega)}{l_i(\omega)} + \varepsilon_w^S(\omega) \left[\gamma_{ii} \int_\Omega \frac{\hat{l}_i(\omega')}{l_i(\omega')} \frac{l_i(\omega') \frac{\omega'}{w_i}}{\mathscr{L}_i} g_i\left(\frac{\omega'}{w_i}\right) d\omega' + \gamma_{ij} \int_\Omega \frac{\hat{l}_j(\omega')}{l_j(\omega')} \frac{l_j(\omega') \frac{\omega'}{w_j}}{\mathscr{L}_j} g_j\left(\frac{\omega'}{w_j}\right) d\omega' \right]$$

where we denote $\frac{\hat{l}_i^{\text{pe}}(\omega)}{l_i(\omega)} = -\varepsilon_r^S(\omega) \frac{\hat{T}'(y(\omega))}{1 - T'(y(\omega))}$. Solving this system using the usual techniques yields, after some tedious but straightforward algebra:

$$\frac{\hat{l}_i(\omega)}{l_i(\omega)} = \frac{\hat{l}_i^{\text{pe}}(\omega)}{l_i(\omega)} + \varepsilon_w^S(\omega)\frac{1}{1-\gamma_{jj}A_j-\gamma_{ii}A_i}\left[\gamma_{ii}\frac{d\mathscr{L}_i^{\text{pe}}}{\mathscr{L}_i}+\gamma_{ij}\frac{d\mathscr{L}_j^{\text{pe}}}{\mathscr{L}_j}\right],$$

where we denote

$$A_i = \int_\Omega \varepsilon_w^S(\omega')\frac{l_i(\omega')\frac{\omega'}{w_i}}{\mathscr{L}_i}g_i\left(\frac{\omega'}{w_i}\right)d\omega'$$

and

$$\hat{\mathscr{L}}_i^{\text{pe}} = \int_\Omega \frac{\hat{l}_i^{\text{pe}}(\omega')}{l_i(\omega')}l_i(\omega')\frac{\omega'}{w_i}g_i\left(\frac{\omega'}{w_i}\right)d\omega'.$$

If the tax schedule is CRP and the disutility of labor is isoelastic, we have $A_i = \varepsilon_w^S$.

Incidence of tax reforms on government revenue. The first-order effects of the tax reform $\hat{T}$ on government revenue are given by

$$\hat{\mathcal{R}} = \int_\Omega \hat{T}(y(\omega))f(\omega)d\omega + \int_\Omega T'(y(\omega))y(\omega)\left[\frac{\hat{y}_C(\omega)}{y_C(\omega)}g_C\left(\frac{\omega}{w_C}\right)+\frac{\hat{y}_H(\omega)}{y_H(\omega)}g_h\left(\frac{\omega}{w_H}\right)\right]d\omega.$$

The change in income reads

$$\begin{aligned}\frac{\hat{y}_i(\omega)}{y_i(\omega)} &= \frac{\hat{w}_i}{w_i}+\frac{\hat{l}_i(\omega)}{l_i(\omega)} = \frac{\hat{l}_i(\omega)}{l_i(\omega)}\left(1+\frac{1}{\varepsilon_w^S(\omega)}\right)+\frac{\varepsilon_r(\omega)\frac{\hat{T}'}{1-T'}}{\varepsilon_w^S(\omega)}\\ &= \frac{\hat{l}_i^{\text{pe}}(\omega)}{l_i(\omega)}+\left(1+\varepsilon_w^S(\omega)\right)\frac{\frac{\hat{\mathscr{L}}_i^{\text{pe}}}{\mathscr{L}_i}\gamma_{ii}+\frac{\hat{\mathscr{L}}_j^{\text{pe}}}{\mathscr{L}_j}\gamma_{ij}}{1-\gamma_{jj}A_j-\gamma_{ii}A_i},\end{aligned}$$

where $\varepsilon_w^S(\omega)\equiv\varepsilon_w^S$ for a CRP tax code. Now focus on the elementary tax reform at $y(\omega^*)$. We then have

$$\hat{\mathscr{L}}_i^{\text{pe}} = -\varepsilon_r^S(\omega^*)\frac{l(\omega^*)}{1-T'(y(\omega^*))}\frac{\omega^*}{w_i}g_i\left(\frac{\omega^*}{w_i}\right)\frac{1}{y'(\omega^*)}.$$

The revenue effect of an elementary tax reform at $y(\omega^*)$ thus reads

$$\begin{aligned}\hat{\mathcal{R}}(\omega^*) =& \hat{\mathcal{R}}_{\text{ex}}(\omega^*)+\left(1+\varepsilon_w^S\right)\varepsilon_r^S\frac{l(\omega^*)}{1-T'(y(\omega^*))}\int T'(y(\omega))y(\omega)\\ &\times\left\{g_C\left(\frac{\omega}{w_C}\right)\frac{\gamma_{CC}\frac{\frac{\omega^*}{w_C}g_C\left(\frac{\omega^*}{w_C}\right)\frac{1}{y'(\omega^*)}}{\mathscr{L}_C}+\gamma_{CH}\frac{\frac{\omega^*}{w_H}g_H\left(\frac{\omega^*}{w_H}\right)\frac{1}{y'(\omega^*)}}{\mathscr{L}_H}}{1-\gamma_{CC}\varepsilon_w^S-\gamma_{HH}\varepsilon_w^S}\right.\\ &\left.+g_H\left(\frac{\omega}{w_H}\right)\frac{\gamma_{HC}\frac{\frac{\omega^*}{w_C}g_C\left(\frac{\omega^*}{w_C}\right)\frac{1}{y'(\omega^*)}}{\mathscr{L}_C}+\gamma_{HH}\frac{\frac{\omega^*}{w_H}g_H\left(\frac{\omega^*}{w_H}\right)\frac{1}{y'(\omega^*)}}{\mathscr{L}_H}}{1-\gamma_{CC}\varepsilon_w^S-\gamma_{HH}\varepsilon_w^S}\right\}d\omega\end{aligned} \tag{73}$$

Now we know from (71) that

$$\frac{\frac{\omega^*}{w_C} g_C\left(\frac{\omega^*}{w_C}\right)\frac{1}{y'(\omega^*)}}{\mathscr{L}_C}\gamma_{CC} + \frac{\frac{\omega^*}{w_H} g_H\left(\frac{\omega^*}{w_H}\right)\frac{1}{y'(\omega^*)}}{\mathscr{L}_H}\gamma_{CH}$$
$$= -\frac{Y_H}{Y_C}\left(\frac{\frac{\omega^*}{w_C} g_C\left(\frac{\omega^*}{w_C}\right)\frac{1}{y'(\omega^*)}}{\mathscr{L}_C}\gamma_{HC} + \frac{\frac{\omega^*}{w_H} g_H\left(\frac{\omega^*}{w_H}\right)\frac{1}{y'(\omega^*)}}{\mathscr{L}_H}\gamma_{HH}\right).$$

and that

$$\gamma_{HC} = -\gamma_{CC}\frac{w_C\mathscr{L}_C}{w_H\mathscr{L}_H}.$$

Thus we obtain

$$\begin{aligned}\hat{\mathcal{R}}(\omega^*) =& \hat{\mathcal{R}}_{\text{ex}}(\omega^*) + \frac{\left(1+\varepsilon_w^S\right)\varepsilon_r^S}{1-\gamma_{CC}\varepsilon_w^S - \gamma_{HH}\varepsilon_w^S}\frac{1}{1-T'(y(\omega^*))}\left(\bar{T}'_C - \bar{T}'_H\right) \\ &\times\left(\gamma_{HH} g_H\left(\frac{\omega^*}{w_H}\right)\frac{1}{y'(\omega^*)} - \gamma_{CC} g_C\left(\frac{\omega^*}{w_C}\right)\frac{1}{y'(\omega^*)}\right) y(\omega^*)\end{aligned} \tag{74}$$

where $\bar{T}'_i = \frac{1}{Y_i}\int T'(y(\omega))y(\omega)g_i\left(\frac{\omega}{w_i}\right)d\omega$ is the income-weighted average marginal tax rate of sector i workers. Next, assume a CES production function, which implies $\gamma_{HH} = \frac{1}{\sigma}s_C$ and $\gamma_{CC} = \frac{1}{\sigma}s_H$. Define $s_C(\omega) = \frac{g_C\left(\frac{\omega}{w_C}\right)}{f(\omega)}$ and $s_H(\omega) = \frac{g_H\left(\frac{\omega}{w_H}\right)}{f(\omega)}$. The second term of (74) is then equal to

$$\frac{\left(1+\varepsilon_w^S\right)\varepsilon_r^S}{1+\frac{\varepsilon_w^S}{\sigma}}\frac{y(\omega^*)}{1-T'(y(\omega^*))}\left(\bar{T}'_H - \bar{T}'_C\right)\frac{1}{\sigma}f_y(y^*)\left(s_C - s_C(\omega^*)\right),$$

which proves the result in (72).

□

Comparing equation (72) to equation (20) reveals two differences arising from the alternative modeling of the production. The first is that the general-equilibrium effect now depends on the difference between the average marginal tax rates in the two education groups (or sectors), rather than on the difference between the marginal tax rate at income y^* vs. in the population as a whole. This difference becomes clear if we interpret our production function in Section 1 as one that treats each skill θ as a distinct sector. Second, the general-equilibrium contribution features an additional term that captures the difference between the share of education group C at income level y^* and the overall share of income accruing to group C. This term is positive if college educated labor is over-represented at income level y^*. This is because in this case an increase in the marginal tax rate at y^* raises wages in sector 1 and lowers them in sector 2. Note that this new term is bounded above by 1, and is equal to 1 if sector 1 is composed of all of the agents with type θ^* (and only them), as is the case in Section 1.

D.4 Roy model and sufficient statistics

In this section we derive the incidence of tax reforms in the Roy model. In Sections D.4.2 and D.4.3, we start by studying an environment where there are only two sectors and the utility function is quasilinear. We then argue in Section D.4.4 that our results do not rely on these restrictions. Our main result is that the formula derived in Section D.3 for the N-sector case with exogenous sorting remains unchanged when assignment is endogenous to taxes.

D.4.1 Overview of the results

We derive below the extension of our tax incidence formulas to the general Roy model of endogenous assignment, in which there are N tasks or sectors, and endogenous and costless sorting of agents into the various tasks. The endogeneity of assignment indeed makes the reduced-form production function endogenous to tax changes. However, we show that our main formulas of Section D.3 carry over to this environment: in response to a tax reform, the switching of some agents into different tasks is appropriately accounted for by the cross-wage elasticities that we introduced in (69). Of course, the expressions for these elasticities in terms of primitives now include additional terms reflecting the sorting, but they are still given in closed-form and, crucially, our tax incidence formulas remain identical conditional on these sufficient-statistic variables. We now briefly explain this result; the full technical details are laid out in the following sections.

- We consider the standard Roy model as analyzed by, e.g., Rothschild and Scheuer (2013). As in Section D.3, there is a continuum of skills within each of the N tasks or sectors; in particular the wage distributions in different tasks overlap. For simplicity, assume $N = 2$. The tax schedule is a function of an agent's income only, and does not depend on his task. Output is a function of the aggregate effective labor effort in each task. As in Rothschild and Scheuer (2013), we assume for simplicity that skills within each task are perfect substitutes. To model the endogenous assignment of skills into tasks, we suppose that each agent is characterized by a two-dimensional type (θ_1, θ_2) corresponding to his skill in each task. Each agent chooses to work in the task in which he earns the highest wage, equal to his corresponding skill times the marginal product of that task's labor. As a result, agents will endogenously sort into both tasks as follows. For each skill θ_1, there is an (endogenous) threshold $\theta_2^*(\theta_1)$ such that agents with skills in $\{(\theta_1, \theta_2) : \theta_2 \leq \theta_2^*(\theta_1)\}$ work in task 1, and agents with skills in $\{(\theta_1, \theta_2) : \theta_2 > \theta_2^*(\theta_1)\}$ work in task 2. Crucially, the marginal agent with types $(\theta_1, \theta_2^*(\theta_1))$, who is thus indifferent between working in taks 1 and 2 in the initial equilibrium, earns the same wage $w_1(\theta_1) = w_2(\theta_2^*(\theta_1))$ whether he works in task 1 or task 2.
- Now, an arbitrarily non-linear tax reform in this environment has several effects on individual labor supply. First, there is the standard partial-equilibrium effect, whereby a higher tax rate at income y^* lowers the labor effort of agents who initially earn that income level. Second, the wage distribution is perturbed, which leads agents to adjust their labor supply further. Recall that in the setting of Section D.3, with fixed assignment of skills to tasks, the wage of an agent with initial income y^* changed because of the adjustments in everyone's labor supply – leading to a system of integral equations. Now there is an additional effect: in response to the tax reform and the implied wage changes, some agents switch from one task/sector to another. Therefore, the change in the wage of each agent now depends on both: (i) the adjustments

of everyone's labor supplies (as before); and (ii) the adjustments of the endogenous sorting thresholds $\theta_2^*(\theta_1)$ for each θ_1. We can easily show that the adjustment of the threshold $\theta_2^*(\theta_1)$ is in turn given by the relative wage changes of the marginal agent $(\theta_1, \theta_2^*(\theta_1))$ in each task, that is,

$$\hat{\theta}_2^*(\theta_1) = \hat{w}_1(\theta_1) - \hat{w}_2(\theta_2^*(\theta_1)).$$

E.g., if the wage in task 1 of the initially marginal agent increases more than his wage in task 2, then the threshold at which agents with task-1-type θ_1 choose to work in task 2 increases, i.e., some agents move from task 2 to task 1.

- Next, note that there is a one-to-one map between agents' wage changes $\hat{w}_i(\theta_i)$ and their labor supply changes $\hat{l}_i(\theta_i)$. Indeed, it directly follows from the definition of the labor supply elasticities with respect to the wage and the marginal tax rate that:

$$\hat{w}_i(\theta_i) = \frac{1}{\varepsilon_w^S(\theta_i, i)}\hat{l}_i(\theta_i) + \frac{\varepsilon_r^S(\theta_i, i)}{\varepsilon_w^S(\theta_i, i)}\frac{\hat{T}'(y_i(\theta_i))}{1 - T'(y_i(\theta_i))}.$$

Since the marginal agent initially earns the same wage in both tasks $(w_1(\theta_1) = w_2(\theta_2^*(\theta_1)))$, and hence chooses the same labor supply and pays the same taxes, we obtain that the second term in the previous equation is the same whether it is evaluated at $(\theta_i, i) = (\theta_1, 1)$ or $(\theta_2^*(\theta_1), 2)$. Combining the previous equations, we therefore obtain that the threshold change $\hat{\theta}_2^*(\theta_1)$ can be equivalently expressed as a function of the relative labor supply changes of the marginal agent in each task, i.e.,

$$\hat{\theta}_2^*(\theta_1) = \frac{1}{\varepsilon_w^S(\theta_1, 1)}\left[\hat{l}_1(\theta_1) - \hat{l}_2(\theta_2^*(\theta_1))\right].$$

This is the key equation to understand why our results extend to the case of endogeous assignment: since the threshold change can be expressed in terms of the labor supply changes, it can therefore be incorporated as an element of the cross-wage elasticities. As a result, the labor supply changes of agents are given by the following system of integral equations:

$$\hat{l}_i(\theta_i) = -\varepsilon_r^S(\theta_i, i)\frac{\hat{T}'(y_i(\theta_i))}{1 - T'(y_i(\theta_i))} - \varepsilon_w^S(\theta_i, i)\sum_{j=1}^{2}\int_0^\infty \gamma\left((\theta_i, i), (\theta_j', j)\right)\hat{l}_j(\theta_j')d\theta_j'.$$

This system is identical to that we obtained in the baseline model and can thus be solved exactly as in Section D.3.

- Finally, and importantly, the formula describing the incidence of tax reforms on individual welfare is also identical to that in the model with exogenous assignment, since the marginal agents initially have the same wage and hence the same utility in both sectors – implying no welfare effects from the endogenous sectoral switching beyond the effects on relative wages which are incorporated in the definition of the cross-wage elasticities.

- Of course, the variables $\gamma((\theta, i), (\theta', j))$ do not have the same expression as in the model of Section D.3 where the assignment of skills to tasks is fixed. We derive these variables in closed-form in the model with endogenous assignment in the next Section. Their structural expressions can thus be used to derive properties of tax incidence in this environment. These cross-wage elasticities are constructed as the impact of an increase $\hat{l}(\theta', j)$ in the labor supply of each agent with type θ' in sector j, on the wage of type θ in sector i, keeping the labor supply of everyone else fixed, *but* accounting for the endogenous re-sorting of agents between tasks that the labor supply change $\hat{l}(\theta', j)$ induces. They are thus the natural extension to the model of endogenous sorting of the cross-wage elasticities we constructed in Section D.3. Therefore our analysis can be interpreted in a sufficient-statistic sense (Chetty (2009a)): conditional on the values of the elasticities that we uncover, the underlying structure of the model that generates them (shape of the utility function, functional form of the production function, exogenous vs. endogenous nature of the assignment of skills into tasks) is irrelevant.

D.4.2 Environment

Individuals

Agents are indexed by two types (θ_1, θ_2) representing their productivity in sectors 1 and 2, respectively. The joint distribution of types in the population is denoted by $f(\theta_1, \theta_2)$. We also denote by $f_i(\theta_i)$ the density of types in sector i, and by $f_{i|j}(\theta_i \mid \theta_j)$ the conditional density of types in sector i among agents whose type in sector j is θ_j. Assume for simplicity that the utility function is quasilinear. An agent with types (θ_1, θ_2) who works in sector i earns a wage $w_i(\theta_i)$ and chooses labor supply $l_i(\theta_i)$ that satisfies

$$v'(l_i(\theta_i)) = [1 - T'(w_i(\theta_i) l_i(\theta_i))] w_i(\theta_i).$$

The indirect utility of the agent from working in sector i is given by

$$U_i(\theta_i) = w_i(\theta_i) l_i(\theta_i) - T(w_i(\theta_i) l_i(\theta_i)) - v(l_i(\theta_i))$$

The agent of type (θ_i, θ_{-i}) works in sector i iff

$$U_i(\theta_i) \geq U_{-i}(\theta_{-i}).$$

Firms and wages

The aggregate labor inputs in efficiency units in sector i is then given by

$$\mathscr{L}_i = \int_0^\infty \int_0^\infty \theta_i l_i(\theta_i) \mathbb{I}_{\{U_i(\theta_i) \geq U_{-i}(\theta_{-i})\}} f(\theta_1, \theta_2) \, d\theta_2 d\theta_1.$$

The aggregate production function is given by $\mathscr{F}(\mathscr{L}_1, \mathscr{L}_2)$, so that the wages in sectors 1 and 2 are given by

$$w_i(\theta_i) = \mathscr{F}_i'(\mathscr{L}_1, \mathscr{L}_2) \frac{d\mathscr{L}_i}{dL_i(\theta_i)} = \mathscr{F}_i'(\mathscr{L}_1, \mathscr{L}_2)\,\theta_i.$$

In particular, we have $\frac{w_i(\theta_i')}{w_i(\theta_i)} = \frac{\theta_i'}{\theta_i}$ for any two types (θ_i, θ_i'), so that the relative wages within each sector i are given by the ratio of the corresponding exogenous skills. In particular, if the production function over the two sectors is CES, we have $\mathscr{F}(\mathscr{L}_1, \mathscr{L}_2) = \left[\mathscr{L}_1^{\frac{\sigma-1}{\sigma}} + \mathscr{L}_2^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$, and hence $w_i(\theta_i) = \left[\mathscr{L}_1^{\frac{\sigma-1}{\sigma}} + \mathscr{L}_2^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} \mathscr{L}_i^{-\frac{1}{\sigma}} \theta_i$.

Properties of the initial equilibrium

Consider an individual with types (θ_1, θ_2). His value function is

$$V(\theta_1, \theta_2) = \max\{V_1(\theta_1), V_2(\theta_1)\},$$

where for all i,

$$V_i(\theta_i) = \max_l \; w_i(\theta_i) l - T(w_i(\theta_i) l) - v(l).$$

Denote by $l_i(\theta_i)$, or $l_i(w_i(\theta_i))$ the argmax of this maximization problem.

Note first that if the agent's types θ_1, θ_2 satisfy

$$\theta_2 = \theta_1 \frac{\mathscr{F}_1'(\mathscr{L}_1, \mathscr{L}_2)}{\mathscr{F}_2'(\mathscr{L}_1, \mathscr{L}_2)} \equiv \theta_2^*(\theta_1), \tag{75}$$

then the agent is indifferent between working in sectors 1 and 2 in the initial equilibrium. Indeed, this relationship between the two types implies immediately that $w_1(\theta_1) = w_2(\theta_2^*(\theta_1))$, and hence $l_1(\theta_1) = l_2(\theta_2^*(\theta_1))$, $T(y_1(\theta_1)) = T(y_2(\theta_2^*(\theta_1)))$, and $V_1(\theta_1) = V_2(\theta_2^*(\theta_1))$.

Conversely, consider the population of agents with sector-1 type θ_1. Their value function in sector 1, $V_1(\theta_1)$, is independent of their sector-2 type θ_2. Their value function in sector 2, $V_1(\theta_2)$, is strictly increasing in their sector-2 type θ_2, since $\theta_2 \mapsto w_2(\theta_2)$ is strictly increasing. Therefore, there is a unique threshold type $\theta_2^*(\theta_1)$ (characterized by (75)) such that $V_1(\theta_1) = V_2(\theta_2)$.

We have proved the following result:

Lemma. *The following properties hold in the initial equilibrium:*

(i) *for each θ_1, there is an (endogenous) threshold $\theta_2^*(\theta_1)$ such that agents in $\{(\theta_1, \theta_2) : \theta_2 \leq \theta_2^*(\theta_1)\}$ work in sector 1, and agents in $\{(\theta_1, \theta_2) : \theta_2 > \theta_2^*(\theta_1)\}$ work in sector 2;*

(ii) *for each θ_2, there is an (endogenous) threshold $\theta_1^*(\theta_2)$ such that agents in $\{(\theta_1, \theta_2) : \theta_1 \geq \theta_1^*(\theta_2)\}$ work in sector 1, and agents in $\{(\theta_1, \theta_2) : \theta_1 < \theta_1^*(\theta_2)\}$ work in sector 2;*

(iii) *there is a one-to-one map* $\theta_i \mapsto \theta_{-i}^*(\theta_i)$, *with inverse* $\theta_{-i} \mapsto \theta_i^*(\theta_{-i})$, *i.e., a higher type* θ_1 *in sector* 1 *implies a higher threshold* $\theta_2^*(\theta_1)$, *and vice versa;*

(iv) *a marginal agent with types* (θ_1, θ_2) *such that* $U_1(\theta_1) = U_2(\theta_2)$, *who is thus indifferent between working in sectors 1 and 2 in the initial equilibrium (i.e.,* $\theta_2 = \theta_2^*(\theta_1)$ *or* $\theta_1 = \theta_1^*(\theta_2)$*) earns the same wage* $w_1(\theta_1) = w_2(\theta_2)$ *whether he works in sector 1 or sector 2.*

D.4.3 Incidence of tax reforms

Consider a tax reform $\hat{T}(\cdot)$. The perturbed first-order condition for labor supply of type θ_i in sector i implies that the agent's percentage change in labor supply is given by:

$$\frac{\hat{l}_i(\theta_i)}{l_i(\theta_i)} = -\varepsilon_r^S(\theta_i, i)\frac{\hat{T}'(y_i(\theta_i))}{1 - T'(y_i(\theta_i))} + \varepsilon_w^S(\theta_i, i)\frac{\hat{w}_i(\theta_i)}{w_i(\theta_i)}, \tag{76}$$

where the labor supply elasticities $\varepsilon_r^S(\theta_i, i)$ and $\varepsilon_r^S(\theta_i, i)$ are defined by

$$\varepsilon_r^S(\theta_i, i) = \frac{\frac{v'(l_i(\theta_i))}{l_i(\theta_i)v''(l_i(\theta_i))}}{1 + \frac{v'(l_i(\theta_i))}{l_i(\theta_i)v''(l_i(\theta_i))}\frac{y_i(\theta_i)T''(y_i(\theta_i))}{1-T'(y_i(\theta_i))}}$$

$$\varepsilon_w^S(\theta_i, i) = \left(1 - \frac{y_i(\theta_i)T''(y_i(\theta_i))}{1 - T'(y_i(\theta_i))}\right)\varepsilon_r^S(\theta_i, i).$$

The perturbed wage equation of type θ_i in sector i implies [note: we assume for simplicity a CES production function, but none of our results rely on this functional form]

$$\frac{\hat{w}_i(\theta_i)}{w_i(\theta_i)} = -\frac{1}{\sigma}\left[1 + \left(\frac{\mathscr{L}_i}{\mathscr{L}_{-i}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{-1}\left(\frac{\hat{\mathscr{L}}_i}{\mathscr{L}_i} - \frac{\hat{\mathscr{L}}_{-i}}{\mathscr{L}_{-i}}\right), \tag{77}$$

so that the change in the wage following the tax reform is given by the changes in aggregate labor inputs $\mathscr{L}_1, \mathscr{L}_2$. We consider two cases in turn: fixed assignment and endogenous assignment.

Fixed assignment

Suppose first that there is a positive fixed cost of switching from sector i to sector $-i$, so that the assignment of agents to sectors is unaffected by the infinitesimal tax reform. That is, for any θ_1 (resp., θ_2), the threshold $\theta_2^*(\theta_1)$ (resp., $\theta_1^*(\theta_2)$) remains unchanged. This is the assumption we made in Section D.3 above. Since

$$\mathscr{L}_1 = \int_{\theta_1=0}^{\infty}\int_{\theta_2=0}^{\theta_2^*(\theta_1)}\theta_1 l_1(\theta_1) f(\theta_1, \theta_2)\, d\theta_2 d\theta_1 = \int_0^{\infty}\theta_1 l_1(\theta_1) F_{2|1}(\theta_2^*(\theta_1) \mid \theta_1) f_1(\theta_1)\, d\theta_1$$

$$\mathscr{L}_2 = \int_{\theta_1=0}^{\theta_2^*(\theta_1)}\int_{\theta_2=0}^{\infty}\theta_2 l_2(\theta_2) f(\theta_1, \theta_2)\, d\theta_2 d\theta_1 = \int_0^{\infty}\theta_2 l_2(\theta_2) F_{1|2}(\theta_1^*(\theta_2) \mid \theta_2) f_2(\theta_2)\, d\theta_2,$$

we obtain

$$\frac{\hat{\mathscr{L}}_1}{\mathscr{L}_1} = \frac{1}{\mathscr{L}_1}\int_0^\infty \theta_1 l_1(\theta_1) F_{2|1}(\theta_2^*(\theta_1) \mid \theta_1) f_1(\theta_1) \frac{\hat{l}_1(\theta_1)}{l_1(\theta_1)} d\theta_1$$
$$\frac{\hat{\mathscr{L}}_2}{\mathscr{L}_2} = \frac{1}{\mathscr{L}_2}\int_0^\infty \theta_2 l_2(\theta_2) F_{1|2}(\theta_1^*(\theta_2) \mid \theta_2) f_2(\theta_2) \frac{\hat{l}_2(\theta_2)}{l_2(\theta_2)} d\theta_2.$$

We thus have

$$\frac{\hat{w}_i(\theta_i)}{w_i(\theta_i)} = -\int_0^\infty \gamma\left((\theta_i, i), \left(\tilde{\theta}_i, i\right)\right) \frac{\hat{l}_i\left(\tilde{\theta}_i\right)}{l_i\left(\tilde{\theta}_i\right)} d\tilde{\theta}_i + \int_0^\infty \gamma\left((\theta_i, i), \left(\tilde{\theta}_{-i}, -i\right)\right) \frac{\hat{l}_{-i}\left(\tilde{\theta}_{-i}\right)}{l_{-i}\left(\tilde{\theta}_{-i}\right)} d\tilde{\theta}_{-i} \quad (78)$$

where the cross-wage elasticities are defined by

$$\gamma\left((\theta_i, i), \left(\tilde{\theta}_j, j\right)\right) = \frac{\tilde{\theta}_j l_j\left(\tilde{\theta}_j\right) F_{-j|j}\left(\theta_{-j}^*\left(\tilde{\theta}_j\right) \mid \tilde{\theta}_j\right) f_j\left(\tilde{\theta}_j\right)}{\sigma\left[1 + \left(\mathscr{L}_i/\mathscr{L}_{-i}\right)^{\frac{\sigma-1}{\sigma}}\right] \mathscr{L}_j}.$$

We therefore obtained:

Proposition. *The changes in individual labor supplies are the solution to: for $i \in \{1, 2\}$,*

$$\frac{\hat{l}_i(\theta_i)}{l_i(\theta_i)} = -\varepsilon_r^S(\theta_i, i) \frac{\hat{T}'(y_i(\theta_i))}{1 - T'(y_i(\theta_i))} - \varepsilon_w^S(\theta_i, i) \int_0^\infty \gamma\left((\theta_i, i), \left(\tilde{\theta}_i, i\right)\right) \frac{\hat{l}_i\left(\tilde{\theta}_i\right)}{l_i\left(\tilde{\theta}_i\right)} d\tilde{\theta}_i$$
$$+\varepsilon_w^S(\theta_i, i) \int_0^\infty \gamma\left((\theta_i, i), \left(\tilde{\theta}_{-i}, -i\right)\right) \frac{\hat{l}_{-i}\left(\tilde{\theta}_{-i}\right)}{l_{-i}\left(\tilde{\theta}_{-i}\right)} d\tilde{\theta}_{-i}.$$

This is a system of linear integral equations, which can be solved as in Section D.3.

Endogenous assignment

Suppose now that there is zero cost of switching sectors in response to tax reforms. We now need to account for the endogenous responses of the thresholds $\theta_2^*(\theta_1)$ and $\theta_1^*(\theta_2)$. We now have

$$\frac{\hat{\mathscr{L}}_1}{\mathscr{L}_1} = \frac{1}{\mathscr{L}_1}\int_0^\infty \theta_1 l_1(\theta_1) F_{2|1}(\theta_2^*(\theta_1) \mid \theta_1) f_1(\theta_1) \left[\frac{\hat{l}_1(\theta_1)}{l_1(\theta_1)} + \frac{\theta_2^*(\theta_1) f_{2|1}(\theta_2^*(\theta_1) \mid \theta_1)}{F_{2|1}(\theta_2^*(\theta_1) \mid \theta_1)} \frac{\hat{\theta}_2^*(\theta_1)}{\theta_2^*(\theta_1)}\right] d\theta_1$$
$$\frac{\hat{\mathscr{L}}_2}{\mathscr{L}_2} = \frac{1}{\mathscr{L}_2}\int_0^\infty \theta_2 l_2(\theta_2) F_{1|2}(\theta_1^*(\theta_2) \mid \theta_2) f_2(\theta_2) \left[\frac{\hat{l}_2(\theta_2)}{l_2(\theta_2)} + \frac{\theta_1^*(\theta_2) f_{1|2}(\theta_1^*(\theta_2) \mid \theta_2)}{F_{1|2}(\theta_1^*(\theta_2) \mid \theta_2)} \frac{\hat{\theta}_1^*(\theta_2)}{\theta_1^*(\theta_2)}\right] d\theta_2.$$

We show the following result:

Lemma. *The change in the threshold of types $\theta_2^*(\theta_1)$ in response to a tax reform are given by:*

$$\frac{\hat{\theta}_2^*(\theta_1)}{\theta_2^*(\theta_1)} = \frac{1}{\varepsilon_w^S(\theta_1, 1)}\left[\frac{\hat{l}_1(\theta_1)}{l_1(\theta_1)} - \frac{\hat{l}_2(\theta_2^*(\theta_1))}{l_2(\theta_2^*(\theta_1))}\right]. \quad (79)$$

Proof of equation (79). By expression (75), it is easy to show that we have

$$\frac{\hat{\theta}_2^*(\theta_1)}{\theta_2^*(\theta_1)} = \frac{\hat{w}_1(\theta_1)}{w_1(\theta_1)} - \frac{\hat{w}_2(\theta_2^*(\theta_1))}{w_2(\theta_2^*(\theta_1))}. \tag{80}$$

where for any θ_2, $\hat{w}_2(\theta_2)$ denotes the change in the wage of the fixed type θ_2 in sector 2. But now recall that

$$\frac{\hat{w}_i(\theta_i)}{w_i(\theta_i)} = \frac{1}{\varepsilon_w^S(\theta_i,i)}\frac{\hat{l}_i(\theta_i)}{l_i(\theta_i)} + \frac{\varepsilon_r^S(\theta_i,i)}{\varepsilon_w^S(\theta_i,i)}\frac{\hat{T}'(y_i(\theta_i))}{1-T'(y_i(\theta_i))}$$

Using the results of the above Lemma (derived in Section D.4.2), this finally implies that

$$\frac{\hat{\theta}_2^*(\theta_1)}{\theta_2^*(\theta_1)} = \frac{1}{\varepsilon_w^S(\theta_1,1)}\left[\frac{\hat{l}_1(\theta_1)}{l_1(\theta_1)} - \frac{\hat{l}_2(\theta_2^*(\theta_1))}{l_2(\theta_2^*(\theta_1))}\right].$$

Similarly, we have

$$\frac{\hat{\theta}_1^*(\theta_2)}{\theta_1^*(\theta_2)} = \frac{1}{\varepsilon_w^S(\theta_2,2)}\left[\frac{\hat{l}_2(\theta_2)}{l_2(\theta_2)} - \frac{\hat{l}_1(\theta_1^*(\theta_2))}{l_1(\theta_1^*(\theta_2))}\right].$$

This result crucially used the fact that the initially-marginal agent earns the same wage in both sectors, and hence chooses the same labor supply, pays the same taxes and has the same labor supply elasticities, so that, in particular, $\frac{\varepsilon_r^S(\theta_i,i)}{\varepsilon_w^S(\theta_i,i)}\frac{\hat{T}'(y_i(\theta_i))}{1-T'(y_i(\theta_i))}$ is the same in both sectors for this agent.

□

We finally show that this leads to:

Proposition. *The changes in individual wages are given by:*

$$\frac{\hat{w}_i(\theta_i)}{w_i(\theta_i)} = -\int_0^\infty \gamma\left((\theta_i,i),\left(\tilde{\theta}_i,i\right)\right)\frac{\hat{l}_i\left(\tilde{\theta}_i\right)}{l_i\left(\tilde{\theta}_i\right)}d\tilde{\theta}_i + \int_0^\infty \gamma\left((\theta_i,i),\left(\tilde{\theta}_{-i},-i\right)\right)\frac{\hat{l}_{-i}\left(\tilde{\theta}_{-i}\right)}{l_{-i}\left(\tilde{\theta}_{-i}\right)}d\tilde{\theta}_{-i} \tag{81}$$

where the variables $\gamma\left((\theta,i),\left(\tilde{\theta},j\right)\right)$ *are defined below.*

Proof of equation (81). The previous Lemma implies

$$\begin{aligned}
\frac{\hat{\mathscr{L}}_1}{\mathscr{L}_1} &= \frac{1}{\mathscr{L}_1}\int_0^\infty \theta_1 l_1(\theta_1) F_{2|1}(\theta_2^*(\theta_1)\mid\theta_1) f_1(\theta_1)\,d\theta_1 \\
&\quad\times\left[\left(1+\frac{\theta_2^*(\theta_1) f_{2|1}(\theta_2^*(\theta_1)\mid\theta_1)}{\varepsilon_w^S(\theta_1,1) F_{2|1}(\theta_2^*(\theta_1)\mid\theta_1)}\right)\frac{\hat{l}_1(\theta_1)}{l_1(\theta_1)} - \left(\frac{\theta_2^*(\theta_1) f_{2|1}(\theta_2^*(\theta_1)\mid\theta_1)}{\varepsilon_w^S(\theta_1,1) F_{2|1}(\theta_2^*(\theta_1)\mid\theta_1)}\right)\frac{\hat{l}_2(\theta_2^*(\theta_1))}{l_2(\theta_2^*(\theta_1))}\right] \\
\frac{\hat{\mathscr{L}}_2}{\mathscr{L}_2} &= \frac{1}{\mathscr{L}_2}\int_0^\infty \theta_2 l_2(\theta_2) F_{1|2}(\theta_1^*(\theta_2)\mid\theta_2) f_2(\theta_2)\,d\theta_2 \\
&\quad\times\left[\left(1+\frac{\theta_1^*(\theta_2) f_{1|2}(\theta_1^*(\theta_2)\mid\theta_2)}{\varepsilon_w^S(\theta_2,2) F_{1|2}(\theta_1^*(\theta_2)\mid\theta_2)}\right)\frac{\hat{l}_2(\theta_2)}{l_2(\theta_2)} - \left(\frac{\theta_1^*(\theta_2) f_{1|2}(\theta_1^*(\theta_2)\mid\theta_2)}{\varepsilon_w^S(\theta_2,2) F_{1|2}(\theta_1^*(\theta_2)\mid\theta_2)}\right)\frac{\hat{l}_1(\theta_1^*(\theta_2))}{l_1(\theta_1^*(\theta_2))}\right].
\end{aligned}$$

A change of variables (using the Lemma derived in Section D.4.2) easily leads to (81), where the cross-wage elasticities are now defined by

$$\gamma\left((\theta_i,i),\left(\tilde{\theta}_j,j\right)\right)$$

$$= \frac{\tilde{\theta}_j l_j\left(\tilde{\theta}_j\right) F_{-j|j}\left(\theta^*_{-j}\left(\tilde{\theta}_j\right) \mid \tilde{\theta}_j\right) f_j\left(\tilde{\theta}_j\right)}{\sigma\left[1+(\mathscr{L}_i/\mathscr{L}_{-i})^{\frac{\sigma-1}{\sigma}}\right]\mathscr{L}_j}\left(1+\frac{\theta^*_{-j}\left(\tilde{\theta}_j\right) f_{-j|j}\left(\theta^*_{-j}\left(\tilde{\theta}_j\right)\mid \tilde{\theta}_j\right)}{\varepsilon^S_w\left(\tilde{\theta}_j,j\right) F_{-j|j}\left(\theta^*_{-j}\left(\tilde{\theta}_j\right)\mid\tilde{\theta}_j\right)}\right)+$$
$$\frac{\theta^*_{-j}\left(\tilde{\theta}_j\right) l_{-j}\left(\theta^*_{-j}\left(\tilde{\theta}_j\right)\right) F_{j|-j}\left(\tilde{\theta}_j\mid\theta^*_{-j}\left(\tilde{\theta}_j\right)\right) f_j\left(\tilde{\theta}_j\right)}{\sigma\left[1+(\mathscr{L}_i/\mathscr{L}_{-i})^{\frac{\sigma-1}{\sigma}}\right]\mathscr{L}_{-j}}\left(\frac{\tilde{\theta}_j f_{j|-j}\left(\tilde{\theta}_j\mid\theta^*_{-j}\left(\tilde{\theta}_j\right)\right)}{\varepsilon^S_w\left(\theta^*_{-j}\left(\tilde{\theta}_j\right),-j\right) F_{j|-j}\left(\tilde{\theta}_i\mid\theta^*_{-j}\left(\tilde{\theta}_j\right)\right)}\right).$$

This concludes the proof.

□

Equation (81) is formally identical (78). Everything else therefore goes through exactly as in the model with exogenous assignment, with these new definitions of the cross-wage elasticities.

D.4.4 General Roy model

The results of the previous section clearly do not depend on the assumptions that the utility is quasilinear or that the production function is CES. More importantly, consider the general Roy model with 3 sectors – the extension to N sectors will then be immediate. Consider an individual of type $(\theta_1,\theta_2,\theta_3)$, and define the three value functions $V_i(\theta_i)$ as in Section D.4.2. Generally an agent will work in the sector that gives him the highest wage. It is straightforward to show that for each θ_1, there are unique thresholds $\theta^*_i(\theta_i)$ for $i\in\{1,2\}$ at which an agent with sector-1 type θ_1 is indifferent between working in sector 1 and sector i, that is, $V_1(\theta_1)=V_i(\theta^*_i(\theta_1))$. Specifically, these thresholds are given by $\theta^*_i(\theta_1)=\theta_1\frac{\mathscr{F}'_1(\mathscr{L}_1,\mathscr{L}_2)}{\mathscr{F}'_2(\mathscr{L}_1,\mathscr{L}_2)}$. Similarly, for each (θ_1,θ_2) with $\theta_2>\theta^*_2(\theta_1)$, there is a threshold $\theta^*_3(\theta_2)$ such that agents work in sector 2 (resp., sector 3) if their sector-3 type θ_3 is lower (resp., higher) than $\theta^*_3(\theta_2)$. We thus obtain that the population with sector-1 type θ_1 is split in the three sectors as follows:

- agents $(\theta_1,\theta_2,\theta_3)$ with $\theta_2<\theta^*_2(\theta_1)$ and $\theta_3<\theta^*_3(\theta_1)$ work in sector 1;
- agents $(\theta_1,\theta_2,\theta_3)$ with $\theta_2>\theta^*_2(\theta_1)$ and $\theta_3<\theta^*_3(\theta_2)$ work in sector 2;
- agents $(\theta_1,\theta_2,\theta_3)$ with $\theta_2>\theta^*_2(\theta_1)$ and $\theta_3<\theta^*_3(\theta_2)$ work in sector 3;
- agents $(\theta_1,\theta_2,\theta_3)$ with $\theta_2<\theta^*_2(\theta_1)$ and $\theta_3>\theta^*_3(\theta_1)$ work in sector 3.

The marginal agents that are initially indifferent between two sectors earn the same wage in both sectors. The agents who are indifferent between all three sectors in the initial equilibrium are no problem since there is only a second-order measure of them. The rest of our analysis in the case of two sectors goes through.

E Optimal taxation

In this section, we solve the the government's optimal taxation problem to maximize a Bergson-Samuelson social welfare objective, subject to a resource constraint and the condition that wages and labor supply form an equilibrium:

$$\max_{T(\cdot)} \quad \int_{\Theta} G\left[y(\theta) - T(y(\theta)) - v(l(\theta))\right] f(\theta)\, d\theta \tag{82}$$

$$\text{s.t.} \quad \int_{\Theta} \left[y(\theta) - T(y(\theta))\right] f(\theta)\, d\theta \leq \mathscr{F}(\mathscr{L}) \tag{83}$$

$$\text{and} \quad (1),\ (2),\ \text{and}\ \ y(\theta) = w(\theta)\, l(\theta). \tag{84}$$

We propose several equivalent ways to solve this problem. First, in Section E.1 we show that the solution to this problem can be obtained as a by-product of our tax incidence analysis. In Section E.1.2, we show how the formula simplifies in the case of a CES production function, which leads to a proof of Proposition 3. In Section E.1.3 we prove the top tax formula of Corollary 5. In Section E.1.4 we further characterize the U-shape of the optimal marginal tax rates.

Next, we develop alternative derivations of the optimal tax schedule. In Section E.2, we derive the optimum formula using a novel tax reform approach. Finally, in Section E.3 we derive the optimal tax formula with the more traditional mechanism design approach.

E.1 Deriving optimal taxes from the tax incidence analysis

E.1.1 General production function

By imposing that the social welfare effects of any tax reform of the initial tax schedule T are equal to zero, our tax incidence analysis immediately delivers a characterization of the optimum tax rates. In the model with exogenous wages (Diamond, 1998), the optimum schedule $T'_{\text{pe}}(\cdot)$ is characterized by

$$\frac{T'_{\text{pe}}(y^*)}{1 - T'_{\text{pe}}(y^*)} = \frac{1}{\varepsilon_r^S(y^*)} \left(1 - \bar{g}(y^*)\right) \frac{1 - F_Y(y^*)}{y^* f_Y(y^*)}.$$

The tax rate at income y^*, $T'_{\text{pe}}(y^*)$, is decreasing in the labor supply elasticity, the average social marginal welfare weight above income y^*, and the hazard rate of the income distribution. In the general-equilibrium model, we obtain instead:

Corollary 7. *The welfare-maximizing tax schedule T satisfies: for all $y^* \in \mathbb{R}_+$,*

$$\frac{T'(y^*)}{1 - T'_{pe}(y^*)} = \frac{1}{\varepsilon_r^S(y^*)} \frac{1 - F_Y(y^*)}{y^* f_Y(y^*)} \Bigg\{ 1 - \bar{g}(y^*) + \varepsilon_r(y^*) \ldots \\ \times \int_{\mathbb{R}_+} \left[\psi(y^*) - \psi(y)\right] \frac{\Gamma(y, y^*)}{1 + \frac{\varepsilon_r^S(y^*)}{\varepsilon_r^D(y^*)}} \frac{y f_Y(y)}{1 - F_Y(y^*)} dy \Bigg\}, \tag{85}$$

where $\psi(\cdot)$ is defined by (55). Moreover, this optimal tax formula (85) can be expressed as an integral equation in $T'(\cdot)$, which can then be solved using analogous techniques as in Section 2.1.

Note that in equation (85), the variables $\varepsilon_r^S(y^*)$, $\bar{g}(y^*)$, and $\frac{1-F_Y(y^*)}{y^* f_Y(y^*)}$ that appear in $T'_{\text{pe}}(y^*)$ are evaluated in an economy where the general-equilibrium optimum tax schedule T (and not the exogenous-wage optimum T'_{pe}) is implemented.

Proof of Corollary 7.

Formula (85). The impact of the elementary tax reforms on social welfare is given by (51) with

$$\frac{\hat{w}(y)}{w(y)} = (1-F_Y(y^*))^{-1}\frac{1}{\varepsilon_w^S(y)}[(\varepsilon_r^S(y)-\varepsilon_r(y))\frac{\delta(y-y^*)}{1-T'(y)} - \frac{\varepsilon_w(y)\Gamma(y,y^*)\varepsilon_r(y^*)}{1-T'(y^*)}].$$

This implies

$$\begin{aligned}\hat{\mathcal{W}}(y^*) = &\int_{y^*}^{\infty}(1-g(y))\frac{f_Y(y)}{1-F_Y(y^*)}dy - \frac{T'(y^*)}{1-T'(y^*)}\varepsilon_r^S(y^*)\frac{y^* f_Y(y^*)}{1-F_Y(y^*)}\\ &+\frac{\varepsilon_r(y^*)}{1-T'(y^*)}\frac{y^* f_Y(y^*)}{1-F_Y(y^*)}\left[\left(1+\varepsilon_w^S(y^*)\right)T'(y^*)+(1-T'(y^*))g(y^*)\right]\frac{1}{\varepsilon_w^D(y)}\\ &-\frac{\varepsilon_r(y^*)}{1-T'(y^*)}\frac{y^* f_Y(y^*)}{1-F_Y(y^*)}\int\left[\left(1+\varepsilon_w^S(y)\right)T'(y)+(1-T'(y))g(y)\right]\frac{\Gamma(y,y^*)}{1+\frac{\varepsilon_w^S(y)}{\varepsilon_w^D(y)}}\frac{ydF_Y(y)}{y^* f_Y(y^*)}.\end{aligned}$$

Using Euler's theorem (25) to substitute for $\frac{1}{\varepsilon_w^D(y)}$ in the second line, imposing $\hat{\mathcal{W}}(y^*)=0$ for all y^* and rearranging the terms leads to

$$\begin{aligned}\frac{T'(y^*)}{1-T'(y^*)} =&\frac{1}{\varepsilon_r^S(y^*)}(1-\bar{g}(y))\frac{1-F_Y(y^*)}{y^* f_Y(y^*)}\\ &+\frac{\varepsilon_r(y^*)}{\varepsilon_r^S(y^*)}\frac{1}{1-T'(y^*)}\int_{\mathbb{R}_+}[\psi(y^*)-\psi(y)]\frac{\Gamma(y,y^*)}{1+\frac{\varepsilon_w^S(y)}{\varepsilon_w^D(y)}}\frac{yf_Y(y)}{y^* f_Y(y^*)}dy.,\end{aligned}$$

where $\psi(y) = \left(1+\varepsilon_w^S(y)\right)T'(y)+g(y)(1-T'(y))$. Multiplying this equation by $1-T'(y^*)$ and solving for $T'(y^*)$ easily leads to equation (85).

Integral equation formulation. We can rewrite the optimal tax formula (85) as

$$\begin{aligned}T'(y^*) =&\frac{1}{A(y^*)}\frac{T'_{\text{pe}}(y^*)}{1-T'_{\text{pe}}(y^*)}+\frac{1}{A(y^*)}\frac{\varepsilon_r(y^*)}{\varepsilon_r^S(y^*)}\int(g(y^*)-g(y))\frac{\Gamma(y,y^*)}{1+\frac{\varepsilon_w^S(y)}{\varepsilon_w^D(y)}}\frac{yf_Y(y)}{y^* f_Y(y^*)}dy\\ &-\frac{1}{A(y^*)}\frac{\varepsilon_r(y^*)}{\varepsilon_r^S(y^*)}\int\left(1-g(y)+\varepsilon_w^S(y)\right)\frac{\Gamma(y,y^*)}{1+\frac{\varepsilon_w^S(y)}{\varepsilon_w^D(y)}}\frac{yf_Y(y)}{y^* f_Y(y^*)}T'(y)\,dy\end{aligned}$$

where

$$A(y^*) \equiv \frac{1}{1-T'_{\text{ex}}(y^*)} - \frac{\varepsilon_r(y^*)}{\varepsilon_r^S(y^*)}\left(1-g(y^*)+\varepsilon_w^S(y^*)\right)\int\frac{\Gamma(y,y^*)}{1+\frac{\varepsilon_w^S(y)}{\varepsilon_w^D(y)}}\frac{yf_Y(y)}{y^* f_Y(y^*)}dy.$$

This is a linear Fredholm integral equation in $T'(\cdot)$ that can be solved using the same techniques as in Section 2.1.

□

E.1.2 CES production function

Proof of Proposition 3. If the production function is CES, we have $\varepsilon_w^D(y) = \sigma$ and $\Gamma(y, y^*) = \frac{y^* f_Y(y^*)}{\sigma \mathbb{E}[(1+\frac{1}{\sigma}\varepsilon_w^S(x))^{-1} x]}$. Using these expressions, formula (85) can then be rewritten as

$$[1 + \frac{1}{\sigma}(g(y^*) - 1)]T'(y^*) = \mathscr{T}(y^*)(1 - T'(y^*)) + \frac{1}{\sigma} g(y^*) - \frac{A}{\sigma},$$

where

$$\mathscr{T}(y^*) \equiv \frac{1}{\varepsilon_r(y^*)}(1 - \bar{g}(y^*)) \frac{1 - F_Y(y^*)}{y^* f_Y(y^*)}$$

denote the optimal tax formula for $\frac{T'(y^*)}{1-T'(y^*)}$ in the partial-equilibrium model, and

$$A \equiv \frac{1}{\mathbb{E}[\frac{y}{1+\frac{1}{\sigma}\varepsilon_w^S(y)}]} \int \frac{g(y) + \left[(1 - g(y)) + \varepsilon_w^S(y)\right] T'(y)}{1 + \frac{1}{\sigma}\varepsilon_w^S(y)} y f_Y(y)\, dy. \tag{86}$$

Note that A is a constant, as it does not depend on y^*. The previous equation can then be rewritten as

$$T'(y^*) = \frac{\mathscr{T}(y^*) + \frac{1}{\sigma}(g(y^*) - A)}{1 + \mathscr{T}(y^*) + \frac{1}{\sigma}(g(y^*) - 1)} \tag{87}$$

We now show that $A = 1$, so that we get expression (22), i.e., $\frac{T'(y^*)}{1-T'(y^*)} = \mathscr{T}(y^*) + \frac{1}{\sigma}(g(y^*) - 1)$. (Note that equation (86)-(87) is an integral equation in $T'(y^*)$. Moreover, its kernel is separable in (y^*, y), since A does not depend on y^*. We can thus easily solve it following the same steps as in the proof of equation (13).) Consider the following tax reform:

$$\hat{T}_2(y) = -\frac{\varepsilon_r(y^*)}{1 - T'(y^*)} \gamma(y, y^*)(1 - T'(y))\, y,$$
$$\hat{T}_2'(y) = -\frac{\varepsilon_r(y^*)}{1 - T'(y^*)} \gamma(y, y^*)(1 - T'(y) - yT''(y)),$$

where $\gamma(y, y^*) = \frac{1}{\sigma}\frac{y^* f_Y(y^*)}{\int x f_Y(x) dx}$ is independent of y since the production function is CES. As we show below (Section E.2), this tax reform is the one that cancels out the general equilibrium effects on individual labor supply of the elementary reform at y^*. Tedious but straightforward algebra shows that the incidence of this counteracting tax reform $\hat{T}_2$ on social welfare is given by

$$\begin{aligned}
\hat{\mathcal{W}}(\hat{T}_2) = & \int \hat{\mathcal{W}}(y^*)\, \hat{T}_2'(y^*)(1 - F_Y(y^*))\, dy^* \\
= & -\frac{1}{\sigma}\frac{\varepsilon_r(y^*)}{1 - T'(y^*)} \frac{y^* f_Y(y^*)}{\int x f_Y(x)\, dx} \Bigg\{ \int (1 - g(y))(1 - T'(y))\, y f_Y(y)\, dy \ldots \\
& - \int \varepsilon_w(y) \left(\left[1 + \frac{1}{\sigma}(g(y) - 1)\right] T'(y) - \frac{1}{\sigma} g(y) \right) y f_Y(y)\, dy \\
& - \frac{1}{\sigma} \frac{\int \varepsilon_w(y)\, y dF_Y(y)}{\mathbb{E}[\frac{x}{1+\frac{1}{\sigma}\varepsilon_w^S(x)}]} \int \frac{1}{1 + \frac{1}{\sigma}\varepsilon_w^S(x)} \left[\left(1 - g(x) + \varepsilon_w^S(x)\right) T'(x) + g(x)\right] x dF_Y(x) \Bigg\}.
\end{aligned}$$

Using expression (86) for the constant A implies

$$\hat{\mathcal{W}}(\hat{T}_2) = -\frac{1}{\sigma}\frac{\varepsilon_r(y^*)}{1-T'(y^*)}\frac{y^* f_Y(y^*)}{\int x f_Y(x)\,dx}\left\{\int \frac{(1-g(y)) + \frac{1-A}{\sigma}\varepsilon_w^S(y)}{1+\frac{1}{\sigma}\varepsilon_w^S(y)} y f_Y(y)\,dy \right. \\ \left. - \int \frac{(1-g(y)) + \varepsilon_w^S(y)}{1+\frac{1}{\sigma}\varepsilon_w^S(y)} T'(y)\, y f_Y(y)\,dy\right\}.$$

This expression must be equal to zero for the tax schedule to be optimal, so

$$\int \frac{(1-g(y)) + \varepsilon_w^S(y)}{1+\frac{1}{\sigma}\varepsilon_w^S(y)} T'(y)\, y f_Y(y)\,dy = \int \frac{(1-g(y)) + \frac{1-A}{\sigma}\varepsilon_w^S(y)}{1+\frac{1}{\sigma}\varepsilon_w^S(y)} y f_Y(y)\,dy. \tag{88}$$

Using the solution (87) for the optimal tax schedule and solving for A yields

$$\frac{A}{\sigma} = \frac{\int \frac{(1-g(y))+\frac{1}{\sigma}\varepsilon_w^S(y)}{1+\frac{1}{\sigma}\varepsilon_w^S(y)} y f_Y(y)\,dy - \int \frac{(1-g(y))+\varepsilon_w^S(y)}{1+\frac{1}{\sigma}\varepsilon_w^S(y)} \frac{\mathscr{T}(y)+\frac{1}{\sigma}g(y)}{1+\mathscr{T}(y)+\frac{1}{\sigma}(g(y)-1)} y f_Y(y)\,dy}{\int \frac{\varepsilon_w^S(y)}{1+\frac{1}{\sigma}\varepsilon_w^S(y)} y f_Y(y)\,dy - \int \frac{(1-g(y))+\varepsilon_w^S(y)}{1+\frac{1}{\sigma}\varepsilon_w^S(y)} \frac{1}{1+\mathscr{T}(y)+\frac{1}{\sigma}(g(y)-1)} y f_Y(y)\,dy}.$$

Now compare expressions (86) and (88). These two equations imply

$$\int \frac{\left[(1-g(y)) + \varepsilon_w^S(y)\right] T'(y)}{1+\frac{1}{\sigma}\varepsilon_w^S(y)} y f_Y(y)\,dy = A\,\mathbb{E}\left[\frac{1-g(y)}{1+\frac{1}{\sigma}\varepsilon_w^S(y)} y\right] \\ = \int \frac{(1-g(y)) + \frac{1-A}{\sigma}\varepsilon_w^S(y)}{1+\frac{1}{\sigma}\varepsilon_w^S(y)} y f_Y(y)\,dy.$$

Solving for A easily implies $A = 1$. This concludes the proof.

□

E.1.3 Optimal top tax rate

Proof of Corollary 5. Suppose that the disutility of labor is isoelastic with parameter e, and that the aggregate production function is CES. Assume that in the data (i.e., given the current tax schedule with a constant top tax rate), the income distribution has a Pareto tail, so that the (observed) hazard rate $\frac{1-F_{\hat{Y}}(y^*)}{y f_{\hat{Y}}(y^*)}$ converges to a constant. We show that under these assumptions, the income distribution at the optimum tax schedule is also Pareto distributed at the tail with the same Pareto coefficient. That is, the hazard rate of the income distribution at the top is independent of the level of the top tax rate. At the optimum, we have

$$\frac{1-F_Y(y(\theta))}{y(\theta) f_Y(y(\theta))} = \frac{1-F(\theta)}{\frac{y(\theta)}{y'(\theta)} f(\theta)} = \frac{1-F(\theta)}{\theta f(\theta)}\frac{\theta y'(\theta)}{y(\theta)} = \frac{1-F(\theta)}{\theta f(\theta)}\varepsilon_{y,\theta}, \tag{89}$$

where we define the income elasticity $\varepsilon_{y,\theta} \equiv d\ln y(\theta)/d\ln\theta$. To compute this elasticity, use the individual first order condition (1) with isoelastic disutility of labor to get $l(\theta) = r(\theta)^e w(\theta)^e$, where

$r(\theta)$ is agent θ's retention rate. Thus we have

$$\varepsilon_{l,\theta} \equiv \frac{d\ln l\left(\theta\right)}{d\ln\theta} = e\frac{d\ln r(\theta)}{d\ln\theta} + e\frac{d\ln w(\theta)}{d\ln\theta}.$$

But since the production function is CES, we have shown above that

$$\begin{aligned}\frac{d\ln w(\theta)}{d\ln\theta} =& \frac{d\ln a\left(\theta\right)}{d\ln\theta} - \frac{1}{\sigma}\frac{d\ln L\left(\theta\right)}{d\ln\theta} \\ =& \frac{d\ln a\left(\theta\right)}{d\ln\theta} - \frac{1}{\sigma}\frac{d\ln l\left(\theta\right)}{d\ln\theta} - \frac{1}{\sigma}\frac{d\ln f\left(\theta\right)}{d\ln\theta} = \frac{\theta a'\left(\theta\right)}{a\left(\theta\right)} - \frac{1}{\sigma}\varepsilon_{l,\theta} - \frac{1}{\sigma}\frac{\theta f'\left(\theta\right)}{f\left(\theta\right)}.\end{aligned}$$

Thus, substituting this expression for $\frac{\theta w'(\theta)}{w(\theta)}$ in the previous equation, we obtain

$$\varepsilon_{l,\theta} = e\left[\frac{\theta a'\left(\theta\right)}{a\left(\theta\right)} - \frac{1}{\sigma}\varepsilon_{l,\theta} - \frac{1}{\sigma}\frac{\theta f'\left(\theta\right)}{f\left(\theta\right)} + \frac{\theta r'(\theta)}{r(\theta)}\right].$$

Moreover, since we assume that the second derivative of the optimal marginal tax rate, $T''\left(y\right)$, converges to zero for high incomes, we have $\lim_{\theta\to\infty} r'(\theta) = 1$. Therefore, the previous equation yields

$$\lim_{\theta\to\infty}\varepsilon_{l,\theta} = \frac{e}{1+\frac{e}{\sigma}}\left[\lim_{\theta\to\infty}\frac{\theta a'\left(\theta\right)}{a\left(\theta\right)} - \frac{1}{\sigma}\lim_{\theta\to\infty}\frac{\theta f'\left(\theta\right)}{f\left(\theta\right)}\right].$$

Note that the variables $\frac{\theta a'(\theta)}{a(\theta)}$ and $\frac{\theta f'(\theta)}{f(\theta)}$ are primitive parameters that do not depend on the tax rate. Assuming that they converge to constants as $\theta\to\infty$, we obtain that $\lim_{\theta\to\infty}\varepsilon_{l,\theta}$ is a constant independent of the tax rates, and hence

$$\varepsilon_{y,\theta} = \varepsilon_{l,\theta} + \varepsilon_{w,\theta} = \left(1+\frac{1}{e}\right)\varepsilon_{l,\theta}$$

converges to a constant independent of the tax rate as $\theta\to\infty$. Therefore, the hazard rate of the income distribution at the optimum tax schedule, given by (89), converges to the same constant $1/\rho$ as the hazard rate of incomes observed in the data.

Now let $y^*\to\infty$ in equation (22), to obtain an expression for the optimal top tax rate $\tau^* = \lim_{y^*\to\infty} T'\left(y^*\right)$. Since the production function is CES, the disutility of labor is isoelastic, and the top tax rate is constant, we have seen that

$$\lim_{y^*\to\infty}\varepsilon_r\left(y^*\right) = \frac{e}{1+e/\sigma}.$$

Furthermore assume that $\lim_{y^*\to\infty} g\left(y^*\right) = \bar{g}$, so that $\lim_{y^*\to\infty}\bar{g}\left(y^*\right) = \bar{g}$. Therefore (22) implies

$$\frac{\tau^*}{1-\tau^*} = \frac{1+e/\sigma}{e}\left(1-\bar{g}\right)\frac{1}{\Pi} + \frac{\bar{g}-1}{\sigma} = \frac{1-\bar{g}}{\Pi e} + \frac{1-\bar{g}}{\Pi\sigma} + \frac{\bar{g}-1}{\sigma},$$

where π is the Pareto parameter of the tail of the income distribution. Solving for τ^* leads to equation (23) and concludes the proof.

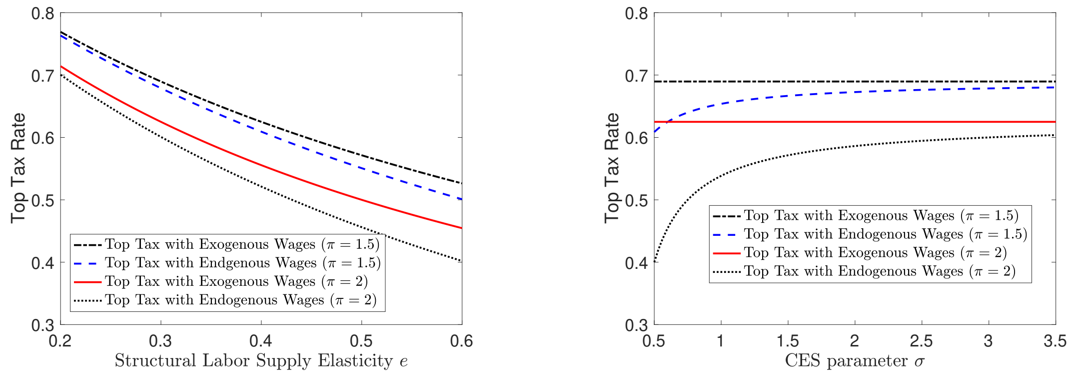


Figure 4: Optimal top tax rate as a function of the labor supply elasticity e (left panel, $\sigma = 3.1$ fixed) and the elasticity of substitution σ (right panel, $e = 0.33$ fixed) and for varying Pareto parameters Π.

□

Figure 4 shows the comparative statics implied by formula (23). (See also Green and Phillips (2015) who study quantitatively the size of the optimal top tax rate in a two-sector model.)

E.1.4 Additional results: U-shape of the optimum tax schedule

In this section we analyze the impact of general equilibrium on the shape of optimal tax rates. Suppose for simplicity that the social planner is Rawlsian, i.e., it maximizes the lump-sum component of the tax schedule, so that $g(y) = 0$ for all $y > 0$. Thus, if the income of the lowest type is positive, we assume that there are some additional agents in the economy who are unable to work and whose consumption equals the demogrant. The partial-equilibrium equivalent of formula (22) for optimal taxes (for which the second term on the right hand side is equal to zero) generally implies a U-shaped pattern of marginal tax rates (Diamond, 1998; Saez, 2001) because the hazard rate $\frac{1-F_Y(y)}{yf_Y(y)}$ is a U-shaped function of income y.

If this is the case, then the additional term $-1/\sigma < 0$ in (22) leads to a general equilibrium correction for $T'(\cdot)$ that is also U-shaped, because the optimal marginal tax rate $T'(y^*)$ is increasing and concave in the right hand side of (22). That is, if the function $\frac{h(y)}{1+h(y)}$ with $h(y) = \frac{1}{\varepsilon_r(y)} \frac{1-F_Y(y)}{yf_Y(y)}$ is U-shaped, it is easy to check that the general-equilibrium correction to marginal tax rates $y \mapsto \frac{h(y)-\sigma^{-1}}{1+h(y)-\sigma^{-1}} - \frac{h(y)}{1+h(y)}$ is then also U-shaped. This suggests that the general equilibrium forces tend to reinforce the U-shape of the optimum tax schedule.

To formalize this intuition using our tax incidence analysis of Section 3, we start by defining a benchmark optimal tax schedule with exogenous wages, to which we can compare our general-equilibrium formula.

Defining a benchmark with exogenous wages. First, we define the marginal tax rates that a partial-equilibrium planner would set from Diamond (1998) using the same data to calibrate the model, and making the same assumptions about the utility function, but wrongly assuming that the wage distribution is exogenous:

$$\frac{T'_{\text{ex}}\left(y\left(\theta\right)\right)}{1-T'_{\text{ex}}\left(y\left(\theta\right)\right)}=\left(1+\frac{1}{\varepsilon_r^S(\hat{w}(\theta))}\right)\frac{1-F_{\hat{W}}(\hat{w}(\theta))}{f_{\hat{W}}(\hat{w}(\theta))\hat{w}(\theta)}, \tag{90}$$

where $\hat{w}(\theta)$ are the wages inferred from the data, i.e., obtained from the incomes observed empirically and the first-order conditions (1), and $F_{\hat{W}}$ is the corresponding wage distribution. Formula (90) is the benchmark to which we compare our optimal policy results numerically, thus directly highlighting how our policy implications differ from those of Diamond (1998).

A government that would implement this tax formula, however, would then observe that the implied distribution of wages does change and is not consistent with the optimality of the tax schedule (90). To overcome this inconsistency, we consider a self-confirming policy equilibrium (SCPE) $T'_{\text{scpe}}\left(y\left(\theta\right)\right)$, as originally proposed by Rothschild and Scheuer (2013, 2016), which is such that implementing the tax schedule (90) generates a wage distribution given which these tax rates are optimal – in other words, this construction solves for the fixed point between the wage distribution and the tax schedule. We use this concept as our exogenous-wage benchmark for our theoretical analysis below.

Comparing optimal taxes to those obtained with exogenous wages. We can apply our tax incidence result of Proposition 2 using the SCPE tax schedule as our initial tax schedule. This exercise gives the (first-order) welfare gains of reforming this tax schedule at any income level, and hence the shape of the general-equilibrium correction to the optimal policy obtained assuming exogenous wages.

Corollary 8. *Suppose that the production function is Cobb-Douglas ($\sigma=1$), that the initial tax schedule $T=T_{scpe}$ is the SCPE, and that the disutility of labor is isoelastic with parameter e. The incidence of the elementary tax reform at income y^* on government revenue (or Rawlsian welfare) is given by*

$$\hat{\mathcal{R}}_{scpe}\left(y^*\right)=1-\zeta\,\frac{1}{T'\left(y^*\right)}\left[p(y^*)+\frac{1}{e}\right], \tag{91}$$

where $\zeta^{-1}\equiv\frac{1}{\bar{T}'}[\bar{p}+\frac{1}{e}]>0$ is a constant that depends on the income-weighted averages of the marginal tax rate $\bar{T}'=\mathbb{E}[\frac{y}{\mathbb{E}y}T'\left(y\right)]$ and of the local rate of progressivity $\bar{p}=\mathbb{E}[\frac{y}{\mathbb{E}y}p\left(y\right)]$ in the initial economy.

Proof of Corollary 8. The revenue effects of elementary tax reforms are given by Proposition 2. Now assume that our starting tax schedule is a self-confirming policy equilibrium that implies an income distribution $f_Y(y)$. Then we know that this initial tax schedule satisfies

$$\frac{T'_{\text{scpe}}\left(y\left(\theta\right)\right)}{1-T'_{\text{scpe}}\left(y\left(\theta\right)\right)}=\frac{1}{\varepsilon_r^S(y(\theta))}\frac{1-F_Y(y(\theta))}{f_Y(y(\theta))y(\theta)}. \tag{92}$$

This implies that the incidence on government revenue in the model with exogenous wages is equal to $\hat{\mathcal{R}}_{\text{ex}}\left(y^*\right)=0$ if the initial tax schedule is the SCPE. Moreover, equations (13) and (41) imply that $\int_{\mathbb{R}_+} \frac{\Gamma(y,y^*)}{1+\frac{1}{\sigma}\varepsilon_w^S(y)} \frac{yf_Y(y)}{y^* f_Y(y^*)} dy = \frac{1}{\sigma}$. Thus, with endogenous wages, we obtain

$$\hat{\mathcal{R}}(y^*) = \varepsilon_r(y^*) \frac{T'_{\text{scpe}}(y^*)}{1-T'_{\text{scpe}}(y^*)} \frac{y^* f_Y(y^*)}{1-F_Y(y^*)} \left(1+\varepsilon_w^S(y^*)\right) \frac{1}{\sigma}$$
$$- \varepsilon_r(y^*) \frac{1}{1-T'_{\text{scpe}}(y^*)} \frac{y^* f_Y(y^*)}{1-F_Y(y^*)} \int_{\mathbb{R}_+} \left[T'_{\text{scpe}}(y)\left(1+\varepsilon_w^S(y)\right)\right] \frac{\Gamma(y,y^*)}{1+\frac{1}{\sigma}\varepsilon_w^S(y)} \frac{yf_Y(y)}{y^* f_Y(y^*)} dy.$$

Now we can use the property (92) of T'_{scpe} to obtain

$$\hat{\mathcal{R}}(y^*) = \frac{1+\varepsilon_w^S(y^*)}{\sigma+\varepsilon_w^S(y^*)} \left[1 - \frac{1}{\sigma+\varepsilon_w^S(y^*)} \frac{1}{1-\frac{1}{\sigma}\mathbb{E}[\frac{y}{\mathbb{E}y}\varepsilon_w(y)]} \frac{\int_{\mathbb{R}_+} T'_{\text{scpe}}(y) \frac{1+\varepsilon_w^S(y)}{\sigma+\varepsilon_w^S(y)} y f_Y(y)\, dy}{T'_{\text{scpe}}(y^*) \frac{1+\varepsilon_w^S(y^*)}{\sigma+\varepsilon_w^S(y^*)} \mathbb{E}y}\right].$$

Assume further that $\sigma = 1$, i.e. that the production function is Cobb-Douglas. We get

$$\hat{\mathcal{R}}(y^*) = 1 - \frac{1}{1+\varepsilon_w^S(y^*)} \frac{1}{1-\mathbb{E}[\frac{y}{\mathbb{E}y}\varepsilon_w(y)]} \int_{\mathbb{R}_+} \frac{T'_{\text{scpe}}(y)}{T'_{\text{scpe}}(y^*)} \frac{y}{\mathbb{E}y} f_Y(y)\, dy.$$

Letting $\bar{T}'_{\text{scpe}} = \int_{\mathbb{R}_+} \frac{y}{\mathbb{E}y} T'_{\text{scpe}}(y) f_Y(y)\, dy$ and $\bar{\varepsilon}_w = \mathbb{E}[\frac{y}{\mathbb{E}y}\varepsilon_w(y)]$ denote the income-weighted average marginal tax rate and labor supply elasticity, and recalling that $\varepsilon_w(y) = \frac{(1-p(y))e}{1+p(y)e}$, we can rewrite the previous expression as

$$\hat{\mathcal{R}}(y^*) = 1 - \frac{p(y^*)+\frac{1}{e}}{1+\frac{1}{e}} \frac{1}{1-\bar{\varepsilon}_w} \frac{\bar{T}'_{\text{scpe}}}{T'_{\text{scpe}}(y^*)} \equiv 1 - \frac{p(y^*)+\frac{1}{e}}{T'_{\text{scpe}}(y^*)}\zeta,$$

where $\zeta = \frac{\bar{T}'_{\text{scpe}}}{(1+\frac{1}{e})(1-\bar{\varepsilon}_w)}$. Now note that the elasticity $\varepsilon_r(y) = \frac{\varepsilon_r^S(y)}{1+\frac{1}{\sigma}(1-p(y))\varepsilon_r^S(y)} = \frac{e}{1+e}$ is constant. Therefore we can rewrite ζ as $\zeta = \frac{\bar{T}'_{\text{scpe}}}{\frac{1}{e}+\mathbb{E}[\frac{y}{\mathbb{E}y}p(y)]}$.

□

The map $y^* \mapsto \hat{\mathcal{R}}_{\text{scpe}}(y^*)$ in (91) gives the shape of the general-equilibrium correction to the optimal tax schedule obtained assuming exogenous wages. Importantly, just as the result of Corollary 4, formula (91) shows that the general-equilibrium effects of the tax reform have a shape that is inherited from that of the initial tax schedule. In particular, if the marginal tax rates $T'(y^*)$ of the SCPE are U-shaped as a function of income, the term $-1/T'(y^*)$ in equation (91) leads to a general-equilibrium correction that is itself U-shaped. Note, however, that the additional term in general equilibrium depends also on the rate of progressivity $p(y^*)$ of the initial (SCPE) tax schedule. Nevertheless, since $|p(y^*)| < 1 \ll \frac{1}{e} \approx 3$ (Chetty (2012)), the shape of $\hat{\mathcal{R}}(y^*)$ as a function of y^* is mostly driven by the term $-1/T'(y^*)$. Our numerical simulations below confirm this intuition.

E.2 Alternative derivation I: counteracting perturbation

E.2.1 Heuristic derivation

Consider as in Section 3 an elementary tax reform at income y^*, i.e., $\hat{T}_1(y) = \mathbb{I}_{\{y \geq y^*\}}$ and $\hat{T}'_1(y) =$

$\delta(y-y^*)$, that consists of an increase in the marginal tax rate at income level y^* and hence an increase in the tax payment of incomes $y \geq y^*$ by a uniform lump-sum amount. Denote by θ^* the type that earns $y(\theta^*) = y^*$ in the initial equilibrium.

Mechanical effect. First, as in Saez (2001), this tax reform implies a mechanical effect $M_1(y^*)$ which captures the direct welfare consequences of the higher tax bill faced by individuals with incomes above y^*, absent any behavioral responses. There is a fraction $1 - F_Y(y^*))$ of such agents, who all pay an additional unit of income in taxes. The corresponding increase in revenue from individual with income y is only valued $(1 - g(y))$ by the government, by definition of the marginal social welfare weight $g(y)$. We therefore obtain

$$M_1(y^*) = (1 - \bar{g}(y^*))\,(1 - F_Y(y^*)))\,, \tag{93}$$

where $\bar{g}(y^*) = \int_{y^*}^{\infty} g(y) \frac{f_Y(y)}{1-F_Y(y^*))} dy$ denotes the average welfare weight above income y^*.

Behavioral effect. Second, individuals with income y^* reduce their labor supply as a response to their higher marginal tax rate, by an amount proportional to the elasticity $\varepsilon_r(y^*)$. Since a share $T'(y^*)$ of the corresponding income change accrues to the government, and there are $f_Y(y^*)$ such agents, this has an impact on government revenue given by

$$B(y^*) = -\frac{T'(y^*)}{1 - T'(y^*)} \varepsilon_r(y^*)\, y^* f_Y(y^*). \tag{94}$$

This term is the same as the behavioral effect in Saez (2001), except that the relevant labor supply elasticity is $\varepsilon_r(y^*)$ rather than $\varepsilon_r^S(y^*)$ – it now accounts for the decreasing marginal product labor of agent y^*, i.e., the endogeneity of $w(\theta^*)$ due to the own-wage effect. Note that the change in labor supply of agents of type θ^* has no first-order impact on their utility, because of the envelope theorem.

Own-wage general-equilibrium effect. Third, the increase in the wage of individuals θ^* due to their own labor supply reduction has a first-order positive impact on both government revenue and individual utility (see equations (16) and (15)). This own-wage general-equilibrium effect is given by

$$GE_1(y^*) = [T'(y^*) + g(y^*)(1 - T'(y^*))]\, y^* \frac{1}{\varepsilon_w^D(y^*)} \frac{\varepsilon_r(y^*)}{1 - T'(y^*)}. \tag{95}$$

The change in labor supply implies a percentage increase of the wage by $\frac{1}{\varepsilon_w^D(y^*)} \frac{\varepsilon_r(y^*)}{1-T'(y^*)}$. A share $T'(y^*)$ of the implied income change accrues to the government, while the share $1 - T'(y^*)$ translates into consumption, which is valued by the planner by the amount $g(y^*)$, by definition of the social marginal welfare weight.

Accounting for the cross-wage effects. Fourth, the tax reform impacts wages (and hence government revenue and individual utilities) through cross-wage effects. Fully accounting for the cross-wage effects would imply, as we analyzed in Section , solving an integral equation. Here we follow a different route and design a second tax reform τ_2 such that the labor supply of all individuals $\theta \neq \theta^*$ remains constant in response to the combination of the two perturbations $\hat{T}_1 + \hat{T}_2$, therefore avoiding the need to carry around the integral term in equation (9) (and hence (10)).

The tax reform $\hat{T}_1$ induces a direct adjustment in the wage of type $\theta \neq \theta^*$ equal to

$$\frac{\hat{w}(\theta)}{w(\theta)} = -\gamma(y(\theta), y^*) \frac{\varepsilon_r(y^*)}{1 - T'(y^*)} \tag{96}$$

To keep the labor supply of agent θ fixed, the net-of-tax wage rate $(1 - T'(y(\theta)))\, w(\theta)$ must remain unchanged, i.e., the tax change at income $y(\theta)$ exactly compensates the wage change of agent θ:

$$\mathrm{d} \ln (1 - T'(y(\theta))) = -\frac{\hat{w}(\theta)}{w(\theta)} \tag{97}$$

Note that the right-hand side of this expression (given by formula (96)) only accounts for the first-round of cross-wage effects, since it features the structural elasticity $\gamma(y(\theta), y^*)$ rather than the aggregate elasticity $\Gamma(y(\theta), y^*)$. This is because, by construction, the combination of tax reforms $\hat{T}_1 + \hat{T}_2$ leaves labor supplies fixed, so that there will be no further rounds of cross-wage effects – therefore the right-hand side of (97) effectively captures the full adjustment in $w(\theta)$.

We now derive the tax reform $\hat{T}_2$ that ensures that equation (97) is satisfied. If the initial tax schedule T were linear, the tax reform $\hat{T}_2$ would impact the retention rate at income $y(\theta)$ simply by $-\hat{T}_2'(y(\theta))$, so that (97) would require a counteracting change in the marginal tax rate of the same magnitude (in percentage terms) as the wage adjustment, i.e. $\frac{\hat{T}_2'(y(\theta))}{1-T'(y(\theta))} = \frac{\hat{w}(\theta)}{w(\theta)}$. Instead, for a nonlinear tax schedule, the change in income induced by the perturbation $\hat{T}_2$ also triggers an indirect endogenous marginal tax rate adjustment. The relation between $\mathrm{d}\ln(1 - T')$ and $\hat{T}_2'$ is thus given by

$$\mathrm{d} \ln (1 - T'(y(\theta))) = -\frac{\hat{T}_2'(y(\theta)) + T''(y(\theta))\, l(\theta) \frac{\hat{w}(\theta)}{w(\theta)}}{1 - T'(y(\theta))}. \tag{98}$$

Equations (96), (97) and (98) then imply that the counteracting perturbation is given by:

$$\hat{T}_2'(y) = -\frac{\varepsilon_r(y^*)}{1 - T'(y^*)} \left(1 - T'(y) - yT''(y)\right) \gamma(y, y^*). \tag{99}$$

(Note that this counteracting tax perturbation is able to undo all of the general equilibrium effects on labor supply thanks to the assumption that agents who earn the same income y have the same wage and identical cross-wage elasticities $\gamma(y, y^*)$. In a model with multidimensional heterogeneity as in Rothschild and Scheuer (2014), the perturbation $\hat{T}_2$ would not be a flexible enough tool to exactly cancel out everyone's labor supply response to $\hat{T}_1$, as different agents earning the same income would have different behavioral responses yet face the same tax change.)

We can now derive the revenue and welfare implications of (i) the wage adjustments (96), and (ii) the (counteracting) tax reform (99).

Effect of the wage adjustments. The welfare impact of the cross-wage effects (96) is similar to that of the own wage effect (95). Individuals of type θ lose a share $1 - T'(y(\theta))$ of their income change $y(\theta)\frac{\hat{w}(\theta)}{w(\theta)}$, while the share $T'(y(\theta))$ accrues to the government. The total impact on social welfare is thus equal to

$$GE_2(y^*) = -\frac{\varepsilon_r(y^*)}{1 - T'(y^*)} \int_{\mathbb{R}_+} \left[T'(y) + g(y)(1 - T'(y))\right] \gamma(y, y^*) \, y f_Y(y) \, dy. \tag{100}$$

Effect of the counteracting tax reform. The counteracting tax changes (99) induce a mechanical change in social welfare given by

$$M_2(y^*) = \int_{\mathbb{R}_+} (1 - \bar{g}(y)) \hat{T}_2'(y) (1 - F_Y(y))) \, dy. \tag{101}$$

This is because the compensating marginal tax rate decrease (say) at income $y(\theta)$, $\hat{T}_2'(y(\theta))$, uniformly decreases the tax bill of individuals with income above $y(\theta)$, which therefore lowers government revenue by $\hat{T}_2'(y(\theta))(1 - F_Y(y(\theta)))$. This revenue gain is valued $(1 - \bar{g}(y(\theta))) \times \hat{T}_2'(y(\theta)) \times (1 - F_Y(y(\theta)))$ by the planner. Summing over incomes $y(\theta)$ gives the change in social welfare from the counteracting perturbation. Note that we do not have to consider the behavioral effects of this counteracting tax reform since by construction it mutes the labor supply responses.

Taking stock. To sum up the reasoning: in response to a (say, higher) marginal tax rate, θ^* decreases her labor supply, which affects (say, lowers) the wage of type θ. This wage effect directly lowers government revenue. An equivalent reduction in the marginal tax rate at income $y(\theta)$ is then necessary to cancel out the induced change in the labor supply of θ. However, due the limited nature of the government's policy instrument, there is a key difference between the initial wage adjustment and the counteracting tax change: the former impacts only the income of type θ, while the latter impacts that of *all* individuals with skill higher than θ. Summing the effects of these two perturbations on government revenue and social welfare immediately yields the general-equilibrium correction $GE_2 + M_2$ to the optimal tax rates.

Optimal tax schedule. The optimal tax schedule is then described by

$$M_1(y^*) + B(y^*) + GE_1(y^*) + GE_2(y^*) + M_2(y^*) = 0$$

for all $y^* \in \mathbb{R}_+$. Straightforward algebra (see below) leads to the following formula for optimal taxes:

$$\begin{aligned} \frac{T'(y^*)}{1 - T'(y^*)} = & \frac{1}{\varepsilon_r(y^*)} (1 - \bar{g}(y^*)) \left(\frac{1 - F_Y(y^*)}{y^* f_Y(y^*)}\right) + \frac{1}{\varepsilon_w^D(y^*)} (g(y^*) - 1) \\ & - \int_{\mathbb{R}_+} \left[(1 - \bar{g}(y)) \left(\frac{1 - F_Y(y)}{y^* f_Y(y^*)}\right) \left(\frac{1 - T'(y)}{1 - T'(y^*)}\right) y\right]' \gamma(y, y^*) \, dy. \end{aligned} \tag{102}$$

E.2.2 Rigorous derivation

Formal proof of equation (102).

Proof of equation (99). Consider the perturbation $\hat{T}_1$ defined by $\hat{T}_1(y) = \mathbb{I}_{\{y \geq y^*\}}$ and $\hat{T}_1'(y) = \delta(y - y^*)$. As usual, denote by θ^* the type such that $y(\theta^*) = y^*$. We impose that $\hat{T}_1 + \hat{T}_2$ has the same effects on labor supply as those that $\hat{T}_1$ induces in the partial equilibrium framework. The general equilibrium response to $\hat{T}_1 + \hat{T}_2$ is given by the solution to the following integral equation: for all $\theta \in \Theta$,

$$\begin{aligned}\frac{\hat{l}(\theta|\hat{T}_1+\hat{T}_2)}{l(\theta)} =& -\frac{\varepsilon_r^S(\theta)}{1+\varepsilon_w^S(\theta)/\varepsilon_w^D(\theta)}\left[\frac{\hat{T}_1'(y(\theta))+\hat{T}_2'(y(\theta))}{1-T'(y(\theta))}\right] \\ &+\frac{\varepsilon_w^S(\theta)}{1+\varepsilon_w^S(\theta)/\varepsilon_w^D(\theta)}\int_\Theta \gamma(\theta,\theta')\frac{\hat{l}(\theta'|\hat{T}_1+\hat{T}_2)}{l(\theta')}d\theta'.\end{aligned} \tag{103}$$

The effect of $\hat{T}_1$ in the model with exogenous wages, on the other hand, is given by: for all $\theta \in \Theta$,

$$\hat{l}_{\text{pe}}(\theta|\hat{T}_1) = -\varepsilon_r^S(\theta)\frac{\hat{T}_1'(y(\theta))}{1-T'(y(\theta))} = -\varepsilon_r^S(\theta)\frac{\delta(y(\theta)-y^*)}{1-T'(y(\theta^*))}. \tag{104}$$

In particular, note that with exogenous wages, we have $\hat{l}_{\text{ex}}(\theta|\hat{T}_1) = 0$ for all $\theta \neq \theta^*$, i.e., the only individuals who respond to a change in the marginal tax rate at income y^* are those whose type is θ^* (and hence whose income is y^*). Substituting for (104) in the left hand side and under the integral sign of (103) yields, after a change of variables in the integral (recall that $\gamma(y,y') = (\frac{dy}{d\theta}(\theta'))^{-1}\gamma(\theta,\theta')$),

$$\frac{\hat{T}_2'(y(\theta))}{1-T'(y(\theta))} = -\frac{1}{1-T'(y(\theta^*))}\frac{\varepsilon_w^S(\theta)\,\varepsilon_r^S(\theta^*)}{\varepsilon_r^S(\theta)}\left[-\frac{1}{\varepsilon_w^D(\theta^*)}\delta(y(\theta)-y^*)+\gamma(y,y^*)\right],$$

which easily leads to equation (99). Note that $\hat{T}_2'(y(\theta))$ is a smooth function, except for a jump (formally, a Dirac term) at $\theta = \theta^*$, which adds to the jump in marginal tax rates defined by the tax reform $\hat{T}_1$ at θ^* so that the total response of labor supply of individuals with income y^* is equal to their response to $\hat{T}_1$ in the partial equilibrium environment. Now, integrate this expression from 0 to y (letting $\hat{T}_2(0) = 0$) to get $\hat{T}_2(y)$:

$$\begin{aligned}\hat{T}_2(y(\theta)) = -\frac{\varepsilon_r^S(\theta^*)}{1-T'(y(\theta^*))}\Big[&-\frac{1}{\varepsilon_w^D(\theta^*)}(1-T'(y^*)-y^*T''(y^*))\,\mathbb{I}_{\{y\geq y^*\}} \\ &+\int_0^y (1-T'(y')-y'T''(y'))\,\gamma(y',y^*)\,dy'\Big].\end{aligned}$$

Proof of equation (102). We now derive the effects of the combination of perturbations $\hat{T}_1 + \hat{T}_2$ on social welfare. Tedious but straightforward algebra shows that this has the following first order effects on individual tax paments:

$$dT\left(y\left(\theta\right)\right)=\mathbb{I}_{\left\{y\left(\theta\right)\geq y^{*}\right\}}-\frac{T'\left(y\left(\theta\right)\right)}{1-T'\left(y^{*}\right)}\varepsilon_{r}^{S}\left(\theta^{*}\right)y\left(\theta\right)\delta\left(y\left(\theta\right)-y^{*}\right)\ldots$$
$$-\frac{\varepsilon_{r}^{S}\left(\theta^{*}\right)}{1-T'\left(y\left(\theta^{*}\right)\right)}\left\{\left[-\frac{1}{\varepsilon_{w}^{D}\left(\theta\right)}\delta\left(y\left(\theta\right)-y^{*}\right)+\gamma\left(y\left(\theta\right),y^{*}\right)\right]y\left(\theta\right)T'\left(y\left(\theta\right)\right)\ldots\right.$$
$$\left.-\left(1-T'\left(y^{*}\right)-y^{*}T''\left(y^{*}\right)\right)\frac{1}{\varepsilon_{w}^{D}\left(y^{*}\right)}\mathbb{I}_{\left\{y\left(\theta\right)\geq y^{*}\right\}}+\int_{0}^{y}\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\gamma\left(y',y^{*}\right)dy'\right\},$$

and on aggregate tax payments:

$$\begin{aligned}\hat{\mathcal{R}}=&1-F_{Y}\left(y^{*}\right)-\frac{T'\left(y^{*}\right)}{1-T'\left(y^{*}\right)}\varepsilon_{r}^{S}\left(y^{*}\right)y^{*}f_{Y}\left(y^{*}\right)\\&+\frac{\varepsilon_{r}^{S}\left(y^{*}\right)}{1-T'\left(y^{*}\right)}\left[T'\left(y^{*}\right)y^{*}f_{Y}\left(y^{*}\right)+\left(1-T'\left(y^{*}\right)-y^{*}T''\left(y^{*}\right)\right)\left(1-F_{Y}\left(y^{*}\right)\right)\right]\frac{1}{\varepsilon_{w}^{D}\left(y^{*}\right)}\\&-\frac{\varepsilon_{r}^{S}\left(y^{*}\right)}{1-T'\left(y^{*}\right)}\int_{\mathbb{R}_{+}}\left[T'\left(y\right)yf_{Y}\left(y\right)+\left(1-T'\left(y\right)-yT''\left(y\right)\right)\left(1-F_{Y}\left(y\right)\right)\right]\gamma\left(y,y^{*}\right)dy.\end{aligned}$$

where we used the fact that

$$\begin{aligned}&\int_{y=0}^{\infty}\int_{y'=0}^{y}\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\gamma\left(y',y^{*}\right)dy'f_{Y}\left(y\right)dy\\=&\int_{y'=0}^{\infty}\left(1-T'\left(y'\right)-y'T''\left(y'\right)\right)\left(1-F_{Y}\left(y'\right)\right)\gamma\left(y',y^{*}\right)dy'.\end{aligned}$$

Now, using Euler's homogeneous function theorem (24), we can rewrite this expression as

$$\begin{aligned}\hat{\mathcal{R}}=&1-F_{Y}\left(y^{*}\right)-\frac{T'\left(y^{*}\right)}{1-T'\left(y^{*}\right)}\varepsilon_{r}^{S}\left(y^{*}\right)y^{*}f_{Y}\left(y^{*}\right)\\&+\frac{\varepsilon_{r}^{S}\left(y^{*}\right)}{1-T'\left(y^{*}\right)}\left[A'\left(y^{*}\right)\frac{1}{\varepsilon_{w}^{D}\left(y^{*}\right)}-\int_{\mathbb{R}_{+}}A'\left(y\right)\gamma\left(y,y^{*}\right)dy\right],\end{aligned}$$

where

$$\begin{aligned}A'\left(y\right)\equiv&\left[\left(1-T'\left(y\right)\right)y\left(1-F_{Y}\left(y\right)\right)\right]'\\=&-\left(1-T'\left(y\right)\right)yf_{Y}\left(y\right)+\left(1-T'\left(y\right)-yT''\left(y\right)\right)\left(1-F_{Y}\left(y\right)\right).\end{aligned}$$

Alternatively, we could also write

$$\begin{aligned}\hat{\mathcal{R}}=&1-F_{Y}\left(y^{*}\right)-\frac{T'\left(y^{*}\right)}{1-T'\left(y^{*}\right)}\varepsilon_{r}^{S}\left(y^{*}\right)y^{*}f_{Y}\left(y^{*}\right)\\&+\frac{\varepsilon_{r}^{S}\left(y^{*}\right)}{1-T'\left(y^{*}\right)}y^{*}f_{Y}\left(y^{*}\right)\int_{\mathbb{R}_{+}}\left[B\left(y^{*}\right)-B\left(y\right)\right]\gamma\left(y,y^{*}\right)\frac{yf_{Y}\left(y\right)}{y^{*}f_{Y}\left(y^{*}\right)}dy,\end{aligned}$$

where

$$B\left(y\right)=T'\left(y\right)+\left(1-T'\left(y\right)\right)\left(1-p\left(y\right)\right)\frac{1-F_{Y}\left(y\right)}{yf_{Y}\left(y\right)}.$$

Next, we derive the effect of the perturbation $\hat{T} = \hat{T}_1 + \hat{T}_2$ on individual welfare:

$$\hat{U}(\theta) = -\mathbb{I}_{\{y(\theta)\geq y^*\}} - \frac{\varepsilon_r^S(\theta^*)}{1-T'(y^*)}\Bigg\{(1-T'(y(\theta)))\,y(\theta)\left[-\frac{1}{\varepsilon_w^D(y^*)}\delta(y(\theta)-y^*) + \gamma(y(\theta),y^*)\right]$$
$$+(1-T'(y^*)-y^*T''(y^*))\frac{1}{\varepsilon_w^D(y^*)}\mathbb{I}_{\{y(\theta)\geq y^*\}} - \int_0^y (1-T'(y')-y'T''(y'))\,\gamma(y',y^*)\,dy'\Bigg\},$$

and on social welfare:

$$\hat{\mathcal{G}} = -\int_{y^*}^{\infty} g(y)\,f_Y(y)\,dy + \frac{\varepsilon_r^S(y^*)}{1-T'(y^*)}\times\ldots$$
$$\Bigg\{\left[(1-T'(y^*))\,g(y^*)\,y^* f_Y(y^*) - (1-T'(y^*)-y^*T''(y^*))\int_{y^*}^{\infty} g(y)\,f_Y(y)\,dy\right]\frac{1}{\varepsilon_w^D(y^*)}$$
$$-\int_0^{\infty}\left[(1-T'(y))\,g(y)\,yf_Y(y) - (1-T'(y)-yT''(y))\int_y^{\infty} g(y')\,f_Y(y')\,dy'\right]\gamma(y,y^*)\,dy\Bigg\}.$$

But note that

$$(1-T'(y))\,g(y)\,yf_Y(y) - (1-T'(y)-yT''(y))\left(\int_y^{\infty} g(y')\,f_Y(y')\,dy'\right)$$
$$= -\left[(1-T'(y))\,y\left(\int_y^{\infty} g(y')\,f_Y(y')\,dy'\right)\right]'.$$

Substituting in the expression for $\hat{\mathcal{G}}$, we obtain that the normalized effect of the perturbation on social welfare is given by:

$$\hat{\mathcal{W}} = \hat{\mathcal{G}} + \hat{\mathcal{R}} = 1 - F_Y(y^*) - \int_{y^*}^{\infty} g(y)\,f_Y(y)\,dy - \frac{T'(y^*)}{1-T'(y^*)}\varepsilon_r^S(y^*)\,y^* f_Y(y^*) + \frac{\varepsilon_r^S(\theta^*)}{1-T'(y(\theta^*))}$$
$$\times\Bigg\{\left[(1-T'(y))\,y\,(1-F_Y(y)) - (1-T'(y))\,y\left(\int_y^{\infty} g(x)\,f_Y(x)\,dx\right)\right]'_{y=y^*}\frac{1}{\varepsilon_w^D(y^*)}$$
$$+\int_{\mathbb{R}_+}\left[-(1-T'(y))\,y\,(1-F_Y(y)) + (1-T'(y))\,y\left(\int_y^{\infty} g(x)\,f_Y(x)\,dx\right)\right]'\gamma(y,y^*)\,dy\Bigg\}.$$

Now we have

$$\left[(1-T'(y))\,y\,(1-F_Y(y)) - (1-T'(y))\,y\left(\int_y^{\infty} g(x)\,f_Y(x)\,dx\right)\right]'$$
$$= \left[(1-T'(y))\,y\,(1-F_Y(y))\left(1-\int_y^{\infty} g(x)\,\frac{f_Y(x)}{1-F_Y(y)}dx\right)\right]',$$

so that

$$\hat{\mathcal{W}} = (1-F_Y(y^*))\,(1-\bar{g}(y^*)) - \frac{T'(y^*)}{1-T'(y^*)}\varepsilon_r^S(y^*)\,y^* f_Y(y^*)$$
$$+\frac{\varepsilon_r^S(y^*)}{1-T'(y^*)}\left[A'(y^*)\,\frac{1}{\varepsilon_w^D(y^*)} - \int_{\mathbb{R}_+} A'(y)\,\gamma(y,y^*)\,dy\right]$$

where we denote

$$A(y) \equiv (1-T'(y))\, y\,(1-F_Y(y))\,(1-\bar{g}(y)),$$
$$A'(y^*) = (1-T'(y^*))\,(g(y^*)-1)\, y^* f_Y(y^*) + (1-T'(y^*)-y^*T''(y^*))\,(1-\bar{g}(y^*))\,(1-F_Y(y^*)).$$

Now, at the optimum we must have $\hat{\mathcal{W}}/(1-F_Y(y^*))=0$. This implies:

$$\begin{aligned} 0 =& 1-\bar{g}(y^*) - \frac{T'(y^*)}{1-T'(y^*)}\varepsilon_r^S(y^*)\frac{y^* f_Y(y^*)}{1-F_Y(y^*)} + \varepsilon_r^S(y^*)\left(1-\frac{y^*T''(y^*)}{1-T'(y^*)}\right)(1-\bar{g}(y^*))\frac{1}{\varepsilon_w^D(y^*)} \\ &+ \varepsilon_r^S(y^*)\,(g(y^*)-1)\,\frac{y^* f_Y(y^*)}{1-F_Y(y^*)}\frac{1}{\varepsilon_w^D(y^*)} - \frac{\varepsilon_r^S(y^*)}{1-T'(y^*)}\frac{1}{1-F_Y(y^*)}\int_{\mathbb{R}_+} A'(y)\,\gamma(y,y^*)\,dy. \end{aligned}$$

Rearranging terms and using $\frac{1}{\varepsilon_r^S(y^*)} + (1-p(y^*))\frac{1}{\varepsilon_w^D(y^*)} = \frac{1}{\varepsilon_r(y^*)}$ leads to the optimal tax formula (102).

Proof of Proposition 3. We now derive equation (22) from the general formula (102). Integrating by parts the term

$$\int_{\mathbb{R}_+} \frac{A'(y)}{(1-T'(y^*))\, y^* f_Y(y^*)}\gamma(y,y^*)\,dy$$

in the optimal tax formula using $A(0)=0$ and $A(\bar{y})=0$ yields

$$\begin{aligned} \frac{T'(y^*)}{1-T'(y^*)} =& \frac{1}{\varepsilon_r(y^*)}(1-\bar{g}(y^*))\frac{1-F_Y(y^*)}{y^* f_Y(y^*)} + (g(y^*)-1)\frac{1}{\varepsilon_w^D(y^*)} \\ &+ \int_{\mathbb{R}_+}(1-\bar{g}(y))\left(\frac{1-F_Y(y)}{y^* f_Y(y^*)}\right)\left(\frac{1-T'(y)}{1-T'(y^*)}\right)\frac{d\gamma(y,y^*)}{dy}dy. \end{aligned}$$

Equation (22) follows immediately from this expression since $\frac{d\gamma(y,y^*)}{dy}=0$ when the production function is CES.

Integral equation for the optimal tax schedule. We finally show how to rewrite formula (102) as an integral equation. Denote by $\bar{T}'$ the optimal tax schedule in the case of a CES production function (formula (22)). Multiplying both sides of the previous equation by $1-T'(y(\theta^*))$, we obtain the following formula for the optimal retention rate $r(\theta^*) \equiv 1-T'(y(\theta^*))$, letting $\bar{r}(\theta^*)$ denote the retention rate in the CES case:

$$\begin{aligned} r(\theta^*) =& \frac{1}{1+\frac{\bar{T}'(y(\theta^*))}{1-\bar{T}'(y(\theta^*))}}\left[1-\int_{\Theta} r(\theta)\frac{y(\theta)\,(1-\bar{g}(\theta))\,(1-F(\theta))\,\gamma'(\theta,\theta^*)}{y(\theta^*)\, f(\theta^*)}d\theta\right] \\ =& \bar{r}(\theta^*)\left[1-\int_{\Theta}(1-\bar{g}(\theta))\,(1-F(\theta))\, y(\theta)\frac{\gamma'(\theta,\theta^*)}{y(\theta^*)\, f(\theta^*)}r(\theta)\,d\theta\right]. \end{aligned}$$

This is now a well-defined integral equation in $r(\theta)$, so that we can use the mathematical tools introduced in Section 2 to characterize further the optimal tax schedule.

□

E.2.3 Generalization: preferences with income effects

In this section, we allow for general (non-quasilinear) preferences and show how to construct the counteracting perturbation in this case.

Construction of the counteracting perturbation. Consider the tax reform $\hat{T}_1(y) = \mathbb{I}_{\{y \geq y^*\}}$, so that $\hat{T}_1'(y) = \delta_{y^*}(y)$ is the Dirac delta function at y^*. Denote by θ^* the type that earns $y(\theta^*) = y^*$ in the baseline equilibrium. We construct a counteracting perturbation $\hat{T}_2$ that cancels out the general equilibrium effects on labor supply. In particular, the combination of perturbations $\hat{T}_1 + \hat{T}_2$ leaves the labor supply of every individual $\theta \neq \theta^*$ unchanged, i.e., $\hat{l}(\theta) = 0$. We have, in response to this combination of perturbations,

$$\begin{aligned}\frac{\hat{l}(\theta|\hat{T}_1+\hat{T}_2)}{l(\theta)} =& -\varepsilon_r(\theta)\frac{\hat{T}_1'(y(\theta))+\hat{T}_2'(y(\theta))}{1-T'(y(\theta))} + \varepsilon_R(\theta)\frac{\hat{T}_1(y(\theta))+\hat{T}_2(y(\theta))}{(1-T'(y(\theta)))\,y(\theta)} \\ &+ \varepsilon_w(\theta)\int_\Theta \gamma(\theta,\theta')\frac{\hat{l}(\theta'|\hat{T}_1+\hat{T}_2)}{l(\theta')}d\theta'.\end{aligned}$$

By contrast, the impact of the perturbation $\hat{T}_1$ in the model with exogenous wages is given by

$$\hat{l}_{\text{pe}}(\theta|\hat{T}_1) = -\varepsilon_r^S(\theta)\frac{\hat{T}_1'(y(\theta))}{1-T'(y(\theta))} + \varepsilon_R^S(\theta)\frac{\hat{T}_1(y(\theta))}{(1-T'(y(\theta)))\,y(\theta)}.$$

Imposing equality of the two expressions for all $\theta \in \Theta$ leads to

$$\begin{aligned}&\varepsilon_r(\theta)\frac{\hat{T}_2'(y(\theta))}{1-T'(y(\theta))} - \varepsilon_R(\theta)\frac{\hat{T}_2(y(\theta))}{(1-T'(y(\theta)))\,y(\theta)} \\ =&\varepsilon_w(\theta)\frac{1}{\varepsilon_w^D(\theta)}\left[\varepsilon_r^S(\theta)\frac{\hat{T}_1'(y(\theta))}{1-T'(y(\theta))} - \varepsilon_R^S(\theta)\frac{\hat{T}_1(y(\theta))}{(1-T'(y(\theta)))\,y(\theta)}\right] \\ &+ \varepsilon_w(\theta)\int_\Theta \gamma(\theta,\theta')\left[-\varepsilon_r^S(\theta')\frac{\hat{T}_1'(y(\theta'))}{1-T'(y(\theta'))} + \varepsilon_R^S(\theta')\frac{\hat{T}_1(y(\theta'))}{(1-T'(y(\theta')))\,y(\theta')}\right]d\theta'.\end{aligned}$$

This is an ordinary differential equation for $\hat{T}_2$ that can easily be solved (note that the right hand side is a known function). Changing variables from types to incomes in this equation can be rewritten as

$$\varepsilon_r(y)\frac{\hat{T}_2'(y)}{1-T'(y)} - \varepsilon_R(y)\frac{\hat{T}_2(y)}{(1-T'(y))\,y} = H_1(y)$$

where

$$\begin{aligned}H_1(y) =&\varepsilon_w(y)\frac{1}{\varepsilon_w^D(\theta)}\left[\varepsilon_r^S(y)\frac{\hat{T}_1'(y)}{1-T'(y)} - \varepsilon_R^S(y)\frac{\hat{T}_1(y)}{(1-T'(y))\,y}\right] \\ &+ \varepsilon_w(y)\int \gamma(y,y')\left[-\varepsilon_r^S(y')\frac{\hat{T}_1'(y')}{1-T'(y')} + \varepsilon_R^S(y')\frac{\hat{T}_1(y')}{(1-T'(y'))\,y'}\right]dy'.\end{aligned}$$

The solution to this differential equation reads

$$\hat{T}_2(y) = -\int_0^y \frac{1-T'(y')}{\varepsilon_r(y')} \exp\left(-\int_{y'}^y \frac{\varepsilon_R(y'')}{\varepsilon_r(y'')}\frac{dy''}{y''}\right) H_1(y')\,dy'.$$

Thus the construction of a counteracting tax reform designed to cancel out the general-equilibrium effects on labor supply can be easily extended to a general utility function $u(c,l)$. The derivation of the optimal tax formula then follows the same steps as in the case of a quasilinear utility function.

□

E.3 Alternative derivation II: mechanism design

In this section we study the government problem (82)-(84) using mechanism-design arguments to derive the optimal informationally-constrained efficient consumption and labor supply allocations $\{c(\theta), l(\theta)\}_{\theta\in\Theta}$, subject to feasibility and incentive compatibility of these allocations.

We start by introducing some useful notations. From definition (2), the wage $w(\theta)$ can be represented as a functional $\omega : \Theta \times \mathbb{R}_+ \times \mathcal{M} \to \mathbb{R}_+$ that has three arguments: the individual's type $\theta \in \Theta$, her labor supply $L(\theta) \in \mathbb{R}_+$, and the measure $\mathscr{L} \in \mathcal{M}$ that describes all agents' labor effort

$$w(\theta) = \omega(\theta, L(\theta), \mathscr{L}). \tag{105}$$

We define the cross-wage elasticity as

$$\gamma(\theta,\theta') = L(\theta') \times \lim_{\mu\to 0} \frac{1}{\mu}\{\ln\omega(\theta, L(\theta), \mathscr{L} + \mu\delta_{\theta'}) - \ln\omega(\theta, L(\theta), \mathscr{L})\},$$

where $\delta_{\theta'} \equiv \delta(\theta - \theta')$ is the Dirac delta function at θ', and the own-wage elasticity as

$$-\alpha(\theta) \equiv -\frac{1}{\varepsilon_w^D(\theta)} = \frac{\partial \ln\omega(\theta, L(\theta), \mathscr{L})}{\partial \ln L(\theta)}.$$

We finally define the total wage elasticity $\mathring{\gamma}(\theta,\theta')$, for any $(\theta,\theta') \in \Theta^2$, by

$$\mathring{\gamma}(\theta,\theta') = \gamma(\theta,\theta') - \alpha(\theta')\,\delta_{\theta'}(\theta). \tag{106}$$

Government's problem

Rather than solving for the allocation $\{c(\theta), l(\theta)\}_{\theta\in\Theta}$, it is useful to change variables and optimize over $\{V(\theta), l(\theta)\}_{\theta\in\Theta}$, where $V(\theta) \equiv c(\theta) - v(l(\theta))$. The mechanism design problem then reads

$$\max_{V(\cdot),l(\cdot)} \quad \int_\Theta G(V(\theta)) f(\theta) d\theta \tag{107}$$

$$\text{s.t.} \quad \int_\Theta [V(\theta) + v(l(\theta))] f(\theta) d\theta \le \mathscr{F}(\mathscr{L}) \tag{108}$$

$$\text{and} \quad V(\theta) \ge V(\theta') + v(l(\theta')) - v\left(l(\theta')\frac{w(\theta')}{w(\theta)}\right), \ \forall(\theta,\theta') \in \Theta^2. \tag{109}$$

The incentive compatibility constraint (109) of type θ can be expressed as a standard envelope condition $V'(\theta) = v'(l(\theta))\, l(\theta) \frac{w'(\theta)}{w(\theta)}$, along with the monotonicity constraints $w'(\theta) > 0$ and $y'(\theta) \geq 0$. (As is standard in the literature, we assume that these monotonicity conditions are satisfied and verify them ex-post in our numerical simulations.) An issue with this envelope condition is that $w'(\theta)$ is not only a function of the control variable $l(\theta)$, but also of its derivative $l'(\theta)$. Indeed, from (105), we have $w'(\theta) = \omega_1(\theta, L(\theta), \mathscr{L}) + \omega_2(\theta, L(\theta), \mathscr{L}) L'(\theta)$, where $L'(\theta) = l'(\theta) f(\theta) + l(\theta) f'(\theta)$. We thus define $b(\theta) = l'(\theta)$ and maximize (107) subject to (108), the envelope condition

$$V'(\theta) = v'(l(\theta))\, l(\theta) \frac{\omega_1 [\theta, l(\theta) f(\theta), \mathscr{L}] + [l(\theta) f'(\theta) + b(\theta) f(\theta)]\, \omega_2 [\theta, l(\theta) f(\theta), \mathscr{L}]}{w(\theta)}, \tag{110}$$

and

$$l'(\theta) = b(\theta). \tag{111}$$

This is now a well-defined optimal control problem with two state variables, $V(\theta)$ and $l(\theta)$, and one control variable, $b(\theta)$ (see Seierstad and Sydsaeter (1986)).

Optimal tax schedule

We now characterize the solution to the government problem (107), (108), (110), (111). Below we prove the following proposition:

Proposition 4. *For any $\theta \in \Theta$, the optimal marginal tax rate $\tau(\theta) \equiv T'(y(\theta))$ of type θ satisfies*

$$\frac{\tau(\theta)}{1-\tau(\theta)} = \left(1 + \frac{1}{e(\theta)}\right) \frac{\mu(\theta)}{\lambda w(\theta) f_W(w(\theta))} - \frac{\int_\Theta [\mu(x) v'(l(x)) l(x)]' \, \mathring{\gamma}(x, \theta)\, dx}{\lambda (1-\tau(\theta)) y(\theta) f(\theta)}, \tag{112}$$

where $\mu(\theta) = \lambda \int_\theta^{\bar{\theta}} (1 - g(x)) f(x) dx$ is the Lagrange multiplier on the envelope condition (110) of type θ.

Before proceeding to the formal proof, we start by describing the economic meaning of formula (112).

Interpretation of formula (112)

The first term on the right hand side of (112) is the formula for optimal taxes we would obtain in partial equilibrium (see Diamond (1998)). The second term captures the effect of a variation in type-θ labor supply on each incentive constraint.

To gain intuition, consider first a model with two types, as in Stiglitz (1982). In this case, a decrease in the tax on the high type increases her labor supply, which in turn decreases her wage rate; conversely a higher tax on the low type raises her wage. This compression of the pre-tax wage distribution in general equilibrium is beneficial as it relaxes the downward incentive constraint (109) of the high type. Therefore, optimal taxes are more regressive in general equilibrium: the optimal marginal tax rate on the high type is negative (rather than zero) and it is higher for the low type than in partial equilibrium.

Now suppose that there is a discrete set of types, $\Theta = \{\theta_i\}_{i=1,..,N}$. An increase in the wage $w(\theta_i)$ reduces the gap between $w(\theta_{i+1})$ and $w(\theta_i)$, and therefore relaxes the downward incentive constraint (109) of type θ_{i+1}. Denoting by $\mu(\theta_{i+1})$ the Lagrange multiplier on this constraint, this perturbation has a welfare impact equal to

$$\mu(\theta_{i+1})v'\left(l(\theta_i)\frac{w(\theta_i)}{w(\theta_{i+1})}\right)\frac{l(\theta_i)}{w(\theta_{i+1})}dw(\theta_i) > 0. \tag{113}$$

On the other hand, the perturbation increases the gap between $w(\theta_i)$ and $w(\theta_{i-1})$, and therefore tightens the downward incentive constraint of type θ_i, which has a welfare impact equal to

$$-\mu(\theta_i)v'\left(l(\theta_{i-1})\frac{w(\theta_{i-1})}{w(\theta_i)}\right)l(\theta_{i-1})\frac{w(\theta_{i-1})}{w(\theta_i)^2}dw(\theta_i) < 0. \tag{114}$$

Whether the perturbation increases or decreases welfare depends on whether (113) or (114) is larger in magnitude. First, this depends on the relative size of the Lagrange multipliers, $\mu(\theta_{i+1})-\mu(\theta_i)$, that is, on which incentive constraint binds more strongly. In the continuous-type limit $\Theta = [\underline{\theta}, \overline{\theta}]$, this yields the term $\mu'(x)$ in (112). Second, it depends on the relative change in the values of deviating, captured by the difference between $v'\left(l(\theta_{i-1})\frac{w(\theta_{i-1})}{w(\theta_i)}\right)l(\theta_{i-1})\frac{w(\theta_{i-1})}{w(\theta_i)^2}$ and $v'\left(l(\theta_i)\frac{w(\theta_i)}{w(\theta_{i+1})}\right)\frac{l(\theta_i)}{w(\theta_{i+1})}$. In the continuous-type limit, this yields the term $[v'(l(x))l(x)]'$ in (112).

Proofs

We now prove formula (112), i.e., we solve the optimal control problem (107, 108, 110, 111).

Proof of Proposition 4. The Lagrangian writes:

$$\begin{aligned}\mathcal{L} = &\int_\Theta G\left(V(\theta)\right)f(\theta)d\theta + \lambda\left\{\mathscr{F}\left(\mathscr{L}\right) - \int_\Theta \left[V(\theta) + v\left(l(\theta)\right)\right]f(\theta)d\theta\right\} \\ &- \int_\Theta \mu(\theta)v'\left(l(\theta)\right)l(\theta)\frac{\omega_1\left[\theta, l(\theta)f(\theta), \mathscr{L}\right] + \left[l(\theta)f'(\theta) + b(\theta)f(\theta)\right]\omega_2\left[\theta, l(\theta)f(\theta), \mathscr{L}\right]}{\omega\left[\theta, l(\theta)f(\theta), \mathscr{L}\right]}d\theta \\ &- \int_\Theta \mu'(\theta)V(\theta)d\theta - \int_\Theta \eta(\theta)b(\theta)d\theta - \int_\Theta \eta'\left(\theta\right)l(\theta)d\theta.\end{aligned}$$

We denote by $\tilde{w}(\theta) \equiv \hat{w}\left(\theta\right)/w\left(\theta\right)$ the percentage change in the wage as skills increase:

$$\begin{aligned}\tilde{w}(\theta) \equiv &\tilde{\omega}\left[\theta, l(\theta)f(\theta), \mathscr{L}\right] \\ \equiv &\frac{\omega_1\left[\theta, l(\theta)f(\theta), \mathscr{L}\right] + \left[l(\theta)f'(\theta) + b(\theta)f(\theta)\right]\omega_2\left[\theta, l(\theta)f(\theta), \mathscr{L}\right]}{\omega\left[\theta, l(\theta)f(\theta), \mathscr{L}\right]}.\end{aligned} \tag{115}$$

The first-order condition for $V(\theta)$ reads:

$$G'\left(V(\theta)\right)f(\theta) - \lambda f(\theta) - \mu'(\theta) = 0. \tag{116}$$

The first-order conditions for $b(\theta)$ reads:

$$-\mu(\theta)v'(l(\theta))\,l(\theta)\frac{\partial\tilde{w}(\theta)}{\partial b(\theta)}-\eta(\theta)=0. \tag{117}$$

The first-order conditions for $l(\theta)$ is obtained by perturbing $\mathscr{L}$ in the Dirac direction δ_θ and evaluating the Gateaux derivative of $\mathcal{L}$ (i.e., heuristically, $\partial\mathcal{L}/\partial l(\theta)$"):

$$\begin{aligned} d\mathcal{L}(\mathscr{L},\delta_\theta)=&\lambda w(\theta)f(\theta)-\lambda v'(l(\theta))f(\theta)-\mu(\theta)v''(l(\theta))\,l(\theta)\tilde{w}(\theta)-\mu(\theta)v'(l(\theta))\tilde{w}(\theta)\\ &-\int_\Theta \mu(\theta')v'(l(\theta'))l(\theta')d\tilde{\omega}(\theta',\delta_\theta)\,d\theta'-\eta'(\theta)=0, \end{aligned} \tag{118}$$

where $d\tilde{\omega}(\theta',\delta_\theta)$ (or, heuristically, $\partial\tilde{w}(\theta')/\partial l(\theta)$) is defined by:

$$d\tilde{\omega}(\theta',\delta_\theta)\equiv\lim_{\mu\to 0}\frac{1}{\mu}\left\{\tilde{\omega}\left[\theta',\left(l(\theta')+\mu\delta_\theta(\theta')\right)f(\theta'),\mathscr{L}+\mu\delta_\theta\right]-\tilde{\omega}\left[\theta',l(\theta')f(\theta'),\mathscr{L}\right]\right\}. \tag{119}$$

Now, note that

$$\frac{\partial\tilde{w}(\theta)}{\partial b(\theta)}=\frac{\omega_2\left[\theta,l(\theta)f(\theta),\mathscr{L}\right]f(\theta)}{\omega(\theta,l(\theta)f(\theta),\mathscr{L})}=-\frac{\alpha(\theta)}{l(\theta)}.$$

(Intuitively, the second equality comes from the fact that, by definition of the own-wage elasticity, $\alpha=-\frac{l}{w}\frac{\partial w}{\partial l}$, keeping $\mathscr{L}$ constant). Substituting for $\frac{\partial\tilde{w}(\theta)}{\partial b(\theta)}$ in (117) and differentiating with respect to θ leads to:

$$\eta'(\theta)=\mu'(\theta)v'(l(\theta))\,\alpha(\theta)+\mu(\theta)v''(l(\theta))\,l'(\theta)\alpha(\theta)+\mu(\theta)v'(l(\theta))\,\alpha'(\theta).$$

We can use this expression to substitute for $\eta'(\theta)$ into (118). We now analyze the integral term in this equation. From (119), we have

$$\begin{aligned} d\tilde{\omega}(\theta',\delta_\theta)=&\tilde{\omega}_2\left(\theta',l(\theta')f_\theta(\theta'),\mathscr{L}\right)f(\theta')\,\delta_\theta(\theta')\\ &+\lim_{\mu\to 0}\frac{1}{\mu}\left\{\tilde{\omega}\left[\theta',l(\theta')f(\theta'),\mathscr{L}+\mu\delta_\theta\right]-\tilde{\omega}\left[\theta',l(\theta')f(\theta'),\mathscr{L}\right]\right\}\\ \equiv&\tilde{\omega}_2\left(\theta',l(\theta')f(\theta'),\mathscr{L}\right)f(\theta')\,\delta_\theta(\theta')+\tilde{\omega}_{3,\theta}\left[\theta',l(\theta')f(\theta'),\mathscr{L}\right], \end{aligned}$$

where we introduce the short-hand notation $\tilde{\omega}_{3,\theta}$ in the last line for simplicity. Denote by $\tilde{\omega}_{13,\theta}$ and $\tilde{\omega}_{23,\theta}$ the derivatives of $\tilde{\omega}_{3,\theta}$ with respect to its first and second variables, respectively. Now recall the notation (115) and note that

$$\begin{aligned} \tilde{\omega}_2(\theta,l(\theta)f(\theta),\mathscr{L})f(\theta)=&\frac{\left[f(\theta)\,\omega_{12}+f'(\theta)\omega_2+\left(l(\theta)f'(\theta)+b(\theta)f(\theta)\right)f(\theta)\,\omega_{22}\right]w(\theta)}{w^2(\theta)}\\ &-\frac{\left[\omega_1+\left(l(\theta)f'(\theta)+b(\theta)f(\theta)\right)\omega_2\right]f(\theta)\,\omega_2}{w^2(\theta)},\\ =&\frac{\omega_{12}+\frac{f'(\theta)}{f(\theta)}\omega_2+\left[l(\theta)f'(\theta)+b(\theta)f(\theta)\right]\omega_{22}}{w(\theta)}f(\theta)+\frac{\tilde{w}(\theta)}{l(\theta)}\alpha(\theta). \end{aligned}$$

Now, we have

$$
\begin{aligned}
-\left(\frac{\alpha(\theta)}{l(\theta)}\right)' =& \frac{\{[\omega_{21}+(l(\theta)f'(\theta)+b(\theta)f(\theta))\,\omega_{22}]\,f(\theta)+\omega_2 f'(\theta)\}\,w(\theta)}{w^2(\theta)} \\
&-\frac{\omega_2 f(\theta)\,[\omega_1+(l(\theta)f'(\theta)+b(\theta)f(\theta))\,\omega_2]}{w^2(\theta)} \\
=& \frac{\omega_{12}+\frac{f'(\theta)}{f(\theta)}\omega_2+[l(\theta)f'(\theta)+b(\theta)f(\theta)]\,\omega_{22}}{w(\theta)} f(\theta)+\frac{\tilde{w}(\theta)}{l(\theta)}\alpha(\theta).
\end{aligned}
$$

Therefore, the previous two equalities imply

$$
\tilde{\omega}_2[\theta, l(\theta)f(\theta), \mathscr{L}]\,f(\theta) = -\left(\frac{\alpha(\theta)}{l(\theta)}\right)' = \frac{-\alpha'(\theta)\,l(\theta)+\alpha(\theta)\,b(\theta)}{l^2(\theta)}. \tag{120}
$$

Next, from definition (115) we have (omitting the arguments $(\theta', L(\theta'), \mathscr{L})$ on the right hand side)

$$
\begin{aligned}
&\tilde{\omega}_{3,\theta}[\theta', l(\theta')f(\theta'), \mathscr{L}] \\
=& \frac{\omega_{13,\theta}+(l(\theta')f'(\theta')+b(\theta')f(\theta'))\,\omega_{23,\theta}}{w(\theta')} - \frac{[\omega_1+(l(\theta')f'(\theta')+b(\theta')f(\theta'))\,\omega_2]\,\omega_{3,\theta}}{w^2(\theta')} \\
=& \frac{\omega_{13,\theta}+(l(\theta')f'(\theta')+b(\theta')f(\theta'))\,\omega_{23,\theta}}{w(\theta')} - \frac{\tilde{w}(\theta')}{l(\theta)}\gamma(\theta',\theta),
\end{aligned} \tag{121}
$$

where the second equality follows from the definition of $\tilde{w}(\theta')$ and of the cross-wage elasticities

$$
\begin{aligned}
\gamma(\theta',\theta) =& \frac{l(\theta)}{w(\theta')} \times \lim_{\mu\to 0}\frac{1}{\mu}\{\omega[\theta', l(\theta')f(\theta'), \mathscr{L}+\mu\delta_\theta]-\omega[\theta', l(\theta')f(\theta'), \mathscr{L}]\} \\
=& \frac{l(\theta)}{w(\theta')}\omega_{3,\theta}(\theta', l(\theta')f(\theta'), \mathscr{L}).
\end{aligned}
$$

Note moreover that this equality implies

$$
\begin{aligned}
\frac{\partial\gamma(\theta',\theta)}{\partial\theta'} =& l(\theta)\,\frac{\omega_{13,\theta}(\theta', l(\theta')f(\theta'), \mathscr{L})+(l(\theta')f'(\theta')+b(\theta')f(\theta'))\,\omega_{23,\theta}(\theta', l(\theta')f(\theta'), \mathscr{L})}{w(\theta')} \\
&- l(\theta)\,\frac{\omega_{3,\theta}(\theta', l(\theta')f(\theta'), \mathscr{L})}{w^2(\theta')} \times \ldots \\
&\quad [\omega_1(\theta', l(\theta')f(\theta'), \mathscr{L})+(l(\theta')f'(\theta')+b(\theta')f(\theta'))\,\omega_2(\theta', l(\theta')f(\theta'), \mathscr{L})] \\
=& l(\theta)\,\frac{\omega_{13,\theta}+(l(\theta')f'(\theta')+b(\theta')f(\theta'))\,\omega_{23,\theta}}{w(\theta')} - \tilde{w}(\theta')\,\gamma(\theta',\theta),
\end{aligned}
$$

and thus, from (121)

$$
\tilde{\omega}_{3,\theta}[\theta', l(\theta')f(\theta'), \mathscr{L}] = \frac{1}{l(\theta)}\frac{\partial\gamma(\theta',\theta)}{\partial\theta'}. \tag{122}
$$

Substitute equations (120) and (122) in (118) to get:

$$
\begin{aligned}
0 =& \lambda w(\theta) f(\theta) - \lambda v'(l(\theta)) f(\theta) - \mu(\theta) v''\left(l(\theta)\right) l(\theta)\tilde{w}(\theta) - \mu(\theta) v'(l(\theta))\tilde{w}(\theta) \\
& - \mu'(\theta) v'\left(l(\theta)\right) \alpha\left(\theta\right) - \mu(\theta) v''\left(l(\theta)\right) b\left(\theta\right) \alpha\left(\theta\right) - \mu(\theta) v'\left(l(\theta)\right) \frac{b\left(\theta\right)}{l\left(\theta\right)} \alpha\left(\theta\right) \\
& - \int_{\Theta} \mu(\theta') v'(l(\theta')) \frac{l(\theta')}{l\left(\theta\right)} \frac{\partial \gamma\left(\theta', \theta\right)}{\partial \theta'} d\theta'.
\end{aligned} \tag{123}
$$

Using the definition of the labor supply elasticity $e\left(\theta\right)$ and defining the wedge $(1 - \tau(\theta))w(\theta) = v'(l(\theta))$, we obtain:

$$
\begin{aligned}
0 =& \lambda \tau(\theta) w(\theta) f(\theta) + \mu(\theta)(1 - \tau(\theta)) w(\theta) \left(1 + \frac{1}{e(\theta)}\right) \left\{ - \frac{l'\left(\theta\right)}{l\left(\theta\right)} \alpha\left(\theta\right) - \tilde{w}(\theta) \right\} \\
& - \mu'(\theta)(1 - \tau(\theta)) w(\theta) \alpha\left(\theta\right) - \frac{1}{l\left(\theta\right)} \int_{\Theta} \mu(\theta')(1 - \tau(\theta')) y(\theta') \frac{\partial \gamma\left(\theta', \theta\right)}{\partial \theta'} d\theta'.
\end{aligned}
$$

Using the fact that $\tilde{w}(\theta) = \frac{w'(\theta)}{w(\theta)}$, dividing through by $\lambda(1-\tau(\theta))w(\theta)f(\theta)$, and using the relationship between the densities of productivities and wages yields:

$$
\begin{aligned}
\frac{\tau(\theta)}{1 - \tau(\theta)} =& \left(1 + \frac{1}{e(\theta)}\right) \frac{\mu(\theta)}{\lambda w(\theta) f_W(w(\theta))} \left(1 + \alpha\left(\theta\right) \frac{\frac{l'(\theta)}{l(\theta)}}{\frac{w'(\theta)}{w(\theta)}}\right) + \frac{\mu'(\theta)}{\lambda f(\theta)} \alpha\left(\theta\right) \\
& + \frac{1}{\lambda(1 - \tau(\theta)) y(\theta) f(\theta)} \int_{\Theta} \mu(\theta')(1 - \tau(\theta')) y(\theta') \frac{\partial \gamma\left(\theta', \theta\right)}{\partial \theta'} d\theta'.
\end{aligned} \tag{124}
$$

Note that for a CES production function, we have $\frac{\partial \gamma(\theta', \theta)}{\partial \theta'} = 0$.

Finally, an alternative optimal tax formula is given by integrating the previous equation by parts (with the appropriate boundary conditions on $\mu\left(\theta'\right)$):

$$
\int_{\Theta} \mu(\theta')(1 - \tau(\theta')) y(\theta') \frac{\partial \gamma\left(\theta', \theta\right)}{\partial \theta'} d\theta' = - \int_{\Theta} \gamma\left(\theta', \theta\right) \frac{d}{d\theta'} \left[\mu(\theta')(1 - \tau(\theta')) y(\theta')\right] d\theta'.
$$

We therefore obtain

$$
\begin{aligned}
\frac{\tau(\theta)}{1 - \tau(\theta)} =& \left(1 + \frac{1}{e(\theta)}\right) \frac{\mu(\theta)}{\lambda w(\theta) f_W(w(\theta))} \left(1 + \alpha\left(\theta\right) \frac{\frac{l'(\theta)}{l(\theta)}}{\frac{w'(\theta)}{w(\theta)}}\right) \\
& + \frac{\mu'(\theta)}{\lambda f(\theta)} \alpha\left(\theta\right) - \frac{\int_{\Theta} \left[\mu(x) v'(l(x)) l(x)\right]' \gamma\left(x, \theta\right) dx}{\lambda(1 - \tau(\theta)) y(\theta) f(\theta)} \\
=& \left(1 + \frac{1}{e(\theta)}\right) \frac{\mu(\theta)}{\lambda w(\theta) f_W(w(\theta))} - \frac{\int_{\Theta} \left[\mu(x) v'(l(x)) l(x)\right]' \mathring{\gamma}\left(x, \theta\right) dx}{\lambda(1 - \tau(\theta)) y(\theta) f(\theta)} \\
& + \alpha\left(\theta\right) \left\{ \left(1 + \frac{v''(l(\theta)) l(\theta)}{v'(l(\theta))}\right) \frac{\mu(\theta)}{\lambda f(\theta)} \frac{l'\left(\theta\right)}{l\left(\theta\right)} + \frac{\mu'(\theta)}{\lambda f(\theta)} - \ldots \right. \\
& \left. \frac{\mu'(\theta) v'(l(\theta)) l(\theta) + \mu(\theta) v''(l(\theta)) l'(\theta) l(\theta) + \mu(\theta) v'(l(\theta)) l'(\theta)}{\lambda v'(l(\theta)) l(\theta) f(\theta)} \right\}.
\end{aligned}
$$

where the second equality uses the definition (106) of $\mathring{\gamma}(x,\theta)$ and rearranges terms. The terms in the curly brackets in the second and third lines cancel each other out. Finally, the first-order condition (116) implies $\mu'(\theta) = G'(V(\theta)) f(\theta) - \lambda f(\theta)$, so that, using $\mu(\bar{\theta}) = 0$,

$$\begin{aligned}\mu(\theta) =& -\int_{\theta}^{\bar{\theta}} \left[G'(V(x)) f(x) - \lambda f(x)\right] dx \\ =& \lambda \int_{\theta}^{\bar{\theta}} \left[1 - \frac{G'(V(x)) f(x)}{\lambda f(x)}\right] f(x) dx = \lambda \int_{\theta}^{\bar{\theta}} (1 - g(x)) f(x) dx.\end{aligned}$$

This concludes the proof.

□

We now prove that the optimal tax formula obtained by the variational approach coincides with the formula obtained by solving the mechanism design problem.

E.3.1 Equivalence of the mechanism-design and perturbation approaches

Proof of the equivalence of (112) and (102). Substitute for $\mu(\theta) = \lambda \int_{\theta}^{\bar{\theta}} (1-g(x)) dF(x)$ in the optimal tax formula (112) evaluated at θ^* to get:

$$\begin{aligned}&\frac{T'(y(\theta^*))}{1 - T'(y(\theta^*))} \equiv \frac{\tau(\theta^*)}{1-\tau(\theta^*)} \\ =& \left(1 + \frac{1}{e(\theta^*)}\right) \frac{\int_{\theta^*}^{\bar{\theta}} (1-g(x)) f(x) dx}{f_W(w(\theta^*)) w(\theta^*)} - \frac{\int_{\Theta} \left[v'(l(x)) l(x) \left(\int_x^{\bar{\theta}} (1-g(x')) f(x') dx'\right)\right]' \mathring{\gamma}(x,\theta^*) dx}{(1-\tau(\theta^*)) y(\theta^*) f(\theta^*)} \\ =& \left(1 + \frac{1}{e(\theta^*)}\right) \frac{1 - F(\theta^*)}{f_W(w(\theta^*)) w(\theta^*)} \left(\int_{\theta^*}^{\bar{\theta}} (1-g(x)) \frac{f(x)}{1-F(\theta^*)} dx\right) \\ &- \frac{\int_{\Theta} \left[(1 - T'(y(x))) y(x) (1-F(x)) \left(\int_x^{\bar{\theta}} (1-g(x')) \frac{f(x')}{1-F(x)} dx'\right)\right]' \mathring{\gamma}(x,\theta^*) dx}{(1 - T'(y(\theta^*))) y(\theta^*) f(\theta^*)},\end{aligned}$$

where the last equality uses individual x's first order condition (1). Using the definition of the average marginal welfare weight $\bar{g}(\theta) = \int_{\theta}^{\bar{\theta}} g(x)) \frac{f(x)}{1-F(\theta)} dx$, and multiplying and dividing the first term on the right hand side by $w'(\theta^*)/w(\theta^*)$, we can rewrite this expression as

$$\begin{aligned}\frac{T'(y(\theta^*))}{1 - T'(y(\theta^*))} =& \left(1 + \frac{1}{e(\theta^*)}\right) \frac{w'(\theta^*)}{w(\theta^*)} (1 - \bar{g}(\theta^*)) \frac{1 - F(\theta^*)}{f_W(w(\theta^*)) w'(\theta^*)} \\ &- \frac{\int_{\Theta} \left[(1 - T'(y(\theta))) y(\theta) (1 - F(\theta)) (1 - \bar{g}(\theta))\right]' \mathring{\gamma}(\theta,\theta^*) d\theta}{(1 - T'(y(\theta^*))) y(\theta^*) f(\theta^*)}.\end{aligned}$$

We now change variables from types and wages to incomes in each of the terms of this equation. First, recall that $F(\theta^*) = F_W(w(\theta^*)) = F_Y(y(\theta^*))$, and $f(\theta^*) = f_Y(y(\theta^*)) \times \left.\frac{dy(\theta)}{d\theta}\right|_{\theta=\theta^*}$. Second, we can rewrite the integral as

$$\begin{aligned}
&\int_{\Theta}\frac{d}{d\theta}\left[\left(1-T'\left(y\left(\theta\right)\right)\right)y(\theta)\left(1-F(\theta)\right)\left(1-\bar{g}\left(\theta\right)\right)\right]\times\mathring{\gamma}\left(\theta,\theta^{*}\right)d\theta\\
=&\int_{\Theta}\Bigg[\left(1-T'\left(y\left(\theta\right)\right)-y\left(\theta\right)T''\left(y\left(\theta\right)\right)\right)\left(1-F(\theta)\right)\left(1-\bar{g}\left(\theta\right)\right)\frac{dy\left(\theta\right)}{d\theta}\\
&\quad-\left(1-T'\left(y\left(\theta\right)\right)\right)y\left(\theta\right)\left(1-g\left(\theta\right)\right)f\left(\theta\right)\Bigg]\mathring{\gamma}\left(\theta,\theta^{*}\right)d\theta\\
=&\left(\frac{dy\left(\theta^{*}\right)}{d\theta}\right)\int_{\mathbb{R}_{+}}\Bigg[\left(1-T'\left(y\right)-yT''\left(y\right)\right)\left(1-F_{Y}(y)\right)\left(1-\bar{g}\left(y\right)\right)\\
&\quad-\left(1-T'\left(y\right)\right)y\left(1-g\left(y\right)\right)f_{Y}\left(y\right)\Bigg]\mathring{\gamma}\left(y,y^{*}\right)dy\\
=&\left(\frac{dy\left(\theta^{*}\right)}{d\theta}\right)\int_{\mathbb{R}_{+}}\frac{d}{dy}\left[\left(1-T'\left(y\right)\right)y\left(1-F_{Y}(y)\right)\left(1-\bar{g}\left(y\right)\right)\right]\times\mathring{\gamma}\left(y,y^{*}\right)dy,
\end{aligned}$$

where the second equality follows from a change variables from types to incomes. Third, we have

$$\begin{aligned}
\frac{w'(\theta^{*})}{w(\theta^{*})}\frac{1-F(\theta^{*})}{f_{W}(w(\theta^{*}))w'(\theta^{*})}&=\frac{w'(\theta^{*})}{w(\theta^{*})}\frac{1-F(\theta^{*})}{f(\theta^{*})}=\frac{\frac{w'(\theta^{*})}{w(\theta^{*})}}{\frac{y'(\theta^{*})}{y(\theta^{*})}}\frac{1-F_{Y}\left(y(\theta^{*})\right)}{y(\theta^{*})f_{Y}\left(y(\theta^{*})\right)}.\\
&=\frac{1}{1+\varepsilon_{w}^{S}(\theta^{*})}\frac{1-F_{Y}\left(y(\theta^{*})\right)}{y(\theta^{*})f_{Y}\left(y(\theta^{*})\right)}.
\end{aligned}$$

Collecting all the terms and using $\frac{1}{1+\varepsilon_{w}^{S}(\theta)}=\frac{e(\theta)}{1+e(\theta)}\frac{1}{\varepsilon_{r}^{S}(\theta)}$, we obtain

$$\begin{aligned}
\frac{T'\left(y\left(\theta^{*}\right)\right)}{1-T'\left(y\left(\theta^{*}\right)\right)}=&\frac{1}{\varepsilon_{r}^{S}\left(\theta^{*}\right)}\left(1-\bar{g}\left(\theta^{*}\right)\right)\frac{1-F_{Y}\left(y(\theta^{*})\right)}{y(\theta^{*})f_{Y}\left(y(\theta^{*})\right)}\\
&-\frac{\int_{\mathbb{R}_{+}}\frac{d}{dy}\left[\left(1-T'\left(y\right)\right)y\left(1-F_{Y}(y)\right)\left(1-\bar{g}\left(y\right)\right)\right]\mathring{\gamma}\left(y,y^{*}\right)dy}{\left(1-T'\left(y\left(\theta^{*}\right)\right)\right)y\left(\theta^{*}\right)f_{Y}\left(y\left(\theta^{*}\right)\right)},
\end{aligned}$$

which is exactly formula (102).

□

F Numerical simulations: details and robustness

F.1 Calibration of the model

F.1.1 Wage calibration as in Saez (2001)

Income distribution

We assume that incomes are log-normally distributed apart from the top, where we append a Pareto distribution for incomes above $150,000. To obtain a smooth hazard ratio $\frac{1-F_{y}(y)}{yf_{y}(y)}$, we decrease the thinness parameter of the Pareto distribution linearly between \$150,000 and \$350,000 and let it be constant at 1.5 afterwards (Diamond and Saez, 2011). In the last step we use a standard kernel smoother to ensure differentiability of the hazard ratios at \$150,000 and \$350,000. We set the mean and variance of the lognormal distribution at 10 and 0.95, respectively. The mean parameter is chosen such that the resulting income distribution has a mean of \$64,000, i.e., approximately the average US yearly earnings. The variance parameter was chosen such that the hazard ratio at level \$150,000 is equal to that reported by Diamond and Saez (2011, Fig.2). The resulting hazard ratio is illustrated in Figure 5.

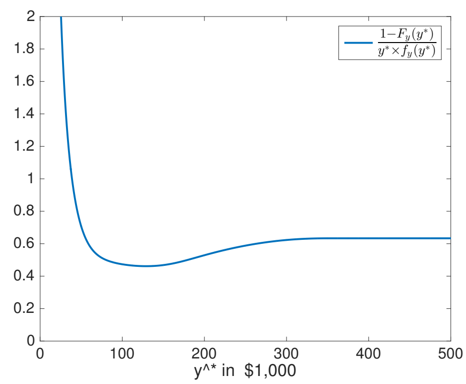


Figure 5: Calibrated hazard ratio $\frac{1-F_y(y)}{yf_y(y)}$ of the U.S. income distribution

Model Primitives

Denote by θ_y the type of an agent who earns income y given the current tax system. Our first step is then the same as in Saez (2001): we use the individual's first order condition $1-T'(y) = v'\left(\frac{y}{w}\right)\frac{1}{w}$ and her observed income and marginal tax rate in the data, to back out her wage. As in Saez (2001), this gives us both the wage $w(\theta_y)$ as well as the labor supply $l(\theta_y) = \frac{y}{w(\theta_y)}$ that correspond to that income level y, given the current tax schedule.

CES. Assume that the production function is CES with a given parameter σ. Once we know the wage $w(\theta_y)$, the labor supply $l(\theta_y)$, and the density of incomes $f_Y(y)$, we can back out the primitive parameters $a(\theta_y)$ of the CES production function (3) using the following formula

$$w(\theta_y) = a(\theta_y)\left(\frac{l(\theta_y)f_Y(y)y'(\theta_y)}{\mathscr{F}(\mathscr{L})}\right)^{\frac{1}{\sigma}},$$

where we know everything but $a(\theta_y)$ and $y'(\theta_y) \equiv \left.\frac{dy(\theta)}{d\theta}\right|_{\theta_y}$. We can without loss of generality assume that θ is uniformly distributed in the unit interval. This pins down $y'(\theta_y)$, since we observe the income percentiles in the data. We can therefore infer the parameter $a(\theta_y)$ for each y.

Translog. For a Translog production the steps are very similar. We only have to make some additional assumptions about how the elasticity of substitution varies with the distance between types. Recall that the wage of type θ_y is given by

$$w(\theta_y) = \frac{\mathscr{F}(\mathscr{L})}{l(\theta_y)\,y'(\theta_y)} \left\{ a_{\theta_y} + \beta_{\theta_y} \ln\left(l(\theta_y)\,y'(\theta_y)\right) + \int_\Theta \chi_{\theta_y,\theta''} \ln\left(l(\theta'')\,y'(\theta'')\right) d\theta'' \right\},$$

where we assume w.l.o.g. that θ is uniformly distributed in the unit interval, and where we parameterize the functions $\beta(\cdot)$ and $\chi(\cdot,\cdot)$ as explained in Section F.1.4 below. This only leaves $a(\theta_y)$ as unknown parameters, which can thus be inferred directly from this wage equation.

F.1.2 The elasticity of taxable income in general equilibrium

Here, we briefly describe the connection between our model and the empirical literature that estimates the elasticity of taxable income. This can be best understood for an elementary tax reform where the marginal tax rate would be raised by for individuals with income $y(\theta^*)$ and would be unchanged for all income levels outside of this interval. We know from Proposition 1 that the relative labor supply change of individuals of type θ^* is given by:

$$\frac{\hat{l}(\theta^*)}{l(\theta^*)} = -\frac{\varepsilon_r(\theta^*)}{1 - T'(y(\theta^*))}\delta_{y^*}(y^*) - \varepsilon_w(\theta^*)\Gamma(y(\theta^*), y(\theta^*))\frac{\varepsilon_r(\theta^*)}{1 - T'(y(\theta^*))}.$$

The second term in this expression is dominated by the first (since the Dirac is infinite at y^*), so that the estimate for the taxable income elasticity would be given by $\varepsilon_r(\theta^*) < \varepsilon_r^S(\theta^*)$. In the limiting case $\alpha(\theta^*) = \infty$ this exactly identifies the structural labor supply elasticity $\varepsilon_r^S(\theta^*)$ (along the nonlinear budget constraint). For $\alpha(\theta^*) < \infty$, however, the estimate would underestimate the labor supply elasticity $\varepsilon_r^S(\theta^*)$. Note that, if we have an estimate of the taxable income elasticity $\varepsilon_r(\theta^*)$ and of the structural parameter $\varepsilon_r^S(\theta^*)$, we can easily back out the implied the elasticity of substitution. Indeed, recall that

$$\varepsilon_r(\theta^*) = \frac{\varepsilon_r^S(\theta^*)}{1 + \alpha(\theta^*)\varepsilon_r^S(\theta^*)}.$$

For instance, if the structural elasticity is $\varepsilon_r^S(\theta^*) = 0.33$ (Chetty (2012), assuming a locally linear tax schedule) and the taxable income elasticity is $\varepsilon_r(\theta^*) = 0.25$ (Saez, Slemrod, and Giertz (2012)), we obtain

$$\alpha(\theta^*) = \frac{1}{\varepsilon_r(\theta^*)} - \frac{1}{\varepsilon_r^S(\theta^*)} \approx 0.97.$$

If the production function is CES, this implies an approximately Cobb-Douglas technology.

F.1.3 Further results concerning the calibration of CES production functions

Consider an economy where the distribution of incomes is observed. Denote by $\theta \in [0,1]$ the income percentile. I.e. $y(\theta)$ is the income of an individual at percentile θ. We now consider the calibration of two different CES production functions for this economy: a CES production function with a continuum of inputs, and a CES production function with 4 types of labor inputs (a "quartile CES production function"). We then show that if we impose the same elasticity of substitution for both production functions, they imply the same relative change in the average earnings of a given quartile. Therefore, the structural wage effects that are estimated for coarser groups (i.e. quartiles) are informative for the calibration of the production function with a continuum of types. That is, if the true underlying production function is a CES with a continuum of types, then a researcher that would estimate the structural wage changes through the lens of a quartile CES production function would infer the correct elasticity of substitution.

Continuum CES production function

Consider the production function

$$Y = \left(\int_0^1 a(\theta) l(\theta)^{\frac{\sigma-1}{\sigma}} d\theta \right)^{\frac{\sigma}{\sigma-1}}. \tag{125}$$

As described above in Appendix , we can apply the calibration method of Saez (2001) to infer $l(\theta)$ and $w(\theta)$ for each worker. And hence, given a choice for σ, we can also infer $a(\theta)$.

Now consider an increase in the labor supply of workers in a given income percentile θ' by one percent, where θ' is located within the lowest quartile, that is, $\theta' < .25$. By equation (40), the wage at percentile $\theta \neq \theta'$ adjusts (in percentage terms) by the direct (structural) effect

$$\frac{\hat{w}(\theta)}{w(\theta)} = \frac{1}{\sigma} \frac{y(\theta')}{Y} \; \forall \; \theta \neq \theta'$$

and the wage at percentile θ' adjusts by

$$\frac{\hat{w}(\theta')}{w(\theta')} = -\frac{1}{\sigma} \left(1 - \frac{y(\theta')}{Y} \right).$$

We now derive the structural change in the average earnings in each quartile. For the top 3 quartiles, the average earnings in group $j \in \{2, 3, 4\}$,

$$w_j \equiv \int_{(j-1)\times 0.25}^{j\times 0.25} w(\theta) l(\theta) d\theta,$$

increases, in response to the one percent increase in $l(\theta')$, by

$$\frac{\hat{w}_j}{w_j} = \frac{1}{\sigma} \frac{y(\theta')}{Y} \; \forall \; j \in \{2, 3, 4\} \tag{126}$$

in percentage terms. Indeed, since the percentage change is the same for each individual (which is a consequence of the CES assumption), the percentage change of the quartile's average is also the same. For the lowest quartile, the percentage change in average earnings, in response to the one percent increase in $l(\theta')$, is given by

$$\begin{aligned}\frac{\hat{w}_1}{w_1} =& \frac{-\frac{1}{\sigma}w(\theta')l(\theta') + \int_0^{0.25}\frac{1}{\sigma}\frac{w(\theta')l(\theta')}{Y}w(\theta)l(\theta)d\theta}{\int_0^{0.25}w(\theta)l(\theta)d\theta} \\ =& -\frac{1}{\sigma}\frac{w(\theta')l(\theta')}{\int_0^{0.25}w(\theta)l(\theta)d\theta}\left(1-\frac{\int_0^{0.25}w(\theta)l(\theta)d\theta}{Y}\right). \end{aligned} \tag{127}$$

Quartile CES production function

We want to show that the relative changes in the average earnings of all four quartiles are consistent with the relative changes that would occur in case of a quartile production function that is defined as follows

$$Y = \left(\sum_{j=1}^{4}\mathscr{L}_j^{\frac{s-1}{s}}\right)^{\frac{s}{s-1}} \tag{128}$$

where

$$\mathscr{L}_j = \int_{(j-1)\times 0.25}^{j\times 0.25}A(\theta)l(\theta)d\theta.$$

Denote the wage for labor of type $j \in \{1,2,3,4\}$ by

$$\omega_j = \frac{\partial Y}{\partial \mathscr{L}_j} = \left(\frac{\mathscr{L}_j}{Y}\right)^{-\frac{1}{s}}.$$

Thus, the wage per unit of effort (or hour) of type θ in percentile $j \in \{1,2,3,4\}$ is given by

$$\frac{\partial Y}{\partial l(\theta)} = \omega_j A(\theta).$$

The relationship $\omega_j A(\theta) = w(\theta)$, where $w(\theta)$ is the wage obtained for the CES production function with a continuum of inputs defined above, has to hold between the calibrated parameters of the two different models. Now, based on the income levels $y(\theta)$ and the calibration as in Saez (2001), we can thus infer $\omega_j A(\theta)$ for each θ in quartile $j \in \{1,2,3,4\}$. Note that this does not fully pin down the schedule of $A(\theta)$ and ω_j for all $j \in \{1,2,3,4\}$.

As above, consider an increase in labor supply of a given type $\theta' \in [0, 0.25)$ by one percent, for the quartile production function. The percentage change in average earnings in quartile $j \in \{1,2,3,4\}$, $\int_{(j-1)\times 0.25}^{j\times 0.25}\omega_j A(\theta)\, l(\theta)\, d\theta$, due to this percentage increase in labor supply of type θ', is given by

$$\frac{\hat{\omega}_1}{\omega_1} = -\frac{1}{s}\left(1-\frac{\omega_1\int_0^{0.25}A(\theta)l(\theta)d\theta}{Y}\right)\frac{A(\theta')l(\theta')}{\int_0^{0.25}A(\theta)l(\theta)d\theta}$$

which is the same relative change of the average earnings of workers in the first quartile as (127) since $\omega_j A(\theta) = w(\theta)$. The relative change in average earnings for $j = 2, 3, 4$ is given by

$$\frac{\hat{\omega}_j}{\omega_j} = \frac{1}{s} \frac{\omega_1 \int_0^{0.25} A(\theta) l(\theta) d\theta}{Y} \frac{A(\theta') l(\theta')}{\int_0^{0.25} A(\theta) l(\theta) d\theta} \; \forall \, j \in \{2, 3, 4\}$$

which is the same as in (126).

Hence, the two models (continuum and quartile technologies (125) and (128), respectively) are observationally equivalent if $s = \sigma$. That is, both production functions predict the same structural change for the average earnings in each quartile in response to a labor supply change. That is, a researcher who would group workers into quartiles (having in mind (128) as the production function) and would infer the elasticity of substitution s by observing the structural own- and cross-wage elasticities $\hat{\omega}_j$, would obtain the correct value σ even if the true underlying production function is (125).

F.1.4 Calibration of Translog production function with distance-dependent elasticities of substitution

A criticism of the CES production function with a continuum of types is that high-skill workers (say) are equally substitutable with middle-skill workers as they are with low-skill workers. We therefore propose a more flexible parametrization of the production function that allows us to obtain distance-dependent elasticities of substitution, i.e., such that closer skill types are stronger substitutes. (Teulings (2005) obtains this distance-dependent property in an assignment model.)

Specifically, in this paragraph we explore quantitatively the implications of the transcendental-logarithmic (Translog) production function. This specification can be used as a second-order local approximation to any production function (Christensen, Jorgenson, and Lau, 1973). With a continuum of labor inputs, its functional form is given by

$$\begin{aligned} \ln \mathscr{F}\left(\{L(\theta)\}_{\theta\in\Theta}\right) &= a_0 + \int_\Theta a(\theta) \ln L(\theta)\, d\theta + \dots \\ \frac{1}{2}\int_\Theta \beta(\theta) (\ln L(\theta))^2 d\theta &+ \frac{1}{2}\int_{\Theta\times\Theta} \chi(\theta,\theta') (\ln L(\theta)) (\ln L(\theta'))\, d\theta d\theta', \end{aligned} \tag{129}$$

where for all θ, θ', $\int_\Theta a(\theta')\, d\theta' = 1$, $\chi(\theta,\theta') = \chi(\theta',\theta)$, and $\beta(\theta) = -\int_\Theta \chi(\theta,\theta')\, d\theta'$. These restrictions ensure that the technology has constant returns to scale. When $\chi(\theta,\theta') = 0$ for all θ, θ', the production function is Cobb-Douglas.

The elasticity of substitution between the labor of types θ and θ' is given by

$$\sigma(\theta,\theta') = \left[1 + \left(\frac{1}{\rho(\theta)} + \frac{1}{\rho(\theta')}\right)\chi(\theta,\theta')\right]^{-1},$$

where $\rho(\theta) = \frac{w(\theta)L(\theta)}{\mathscr{F}(\mathscr{L})} = \frac{y(\theta)f(\theta)}{\mathbb{E}(y)}$ denotes the type-θ labor share of output. (We derived the results about the Translog production function in Appendix A.4.2 above.) To obtain distance-dependent elasticities, we propose the following specification of the exogenous parameters:

$$\chi(\theta,\theta') = \left(\frac{1}{\rho^c(\theta)} + \frac{1}{\rho^c(\theta')}\right)^{-1} \left[c_1 - c_2 \exp\left(-\frac{1}{2s^2}\left(y^c(\theta) - y^c(\theta')\right)^2\right)\right],$$

where c_1, c_2 are constants, and where $\rho^c(\theta)$ and $y^c(\theta)$ are the current (i.e., empirically measured given the actual tax system) income share and income of type θ. The local (i.e., such that $\rho(\theta) = \rho^c(\theta)$ and $y(\theta) = y^c(\theta)$) elasticity of substitution between workers in percentiles θ and θ' is then given by

$$\sigma(\theta,\theta') = \left\{1 + c_1\left[c_2 - \exp\left(-\frac{1}{2s^2}\left(y^c(\theta) - y^c(\theta')\right)^2\right)\right]\right\}^{-1}. \tag{130}$$

The parameters c_1 and c_2 determine the values of the elasticity of substitution between types (θ,θ') with $|y(\theta) - y(\theta')| \to \infty$ and $\theta \approx \theta'$, respectively. The parameter s specifies the rate at which $\sigma(\theta,\theta')$ falls as θ' moves away from θ.

The left panel of Figure 6 shows the elasticity of substitution $\sigma(\theta,\theta')$ as a function of θ for such a specification, where $\sigma(\theta,\theta')$ varies between 0.5 and 10. We let $\theta \in \Theta = [0,1]$ be the agent's percentile in the income distribution. We choose two values for θ': the type that earns the median income (\$33,500) and the type at the 95th percentile of the income distribution (\$126,500), i.e., $\theta' = 0.5$ (red bold line) and $\theta' = 0.95$ (black dashed line). This illustrates how substitutable is the labor supply of a given skill type, measured by its income level $y(\theta)$ on the x-axis, with the skills at the median and the 95th percentile. By construction, the elasticity of substitution equals 10 as $\theta \to \theta'$, then decreases with the distance $|\theta - \theta'|$, and converges to a value of 0.5 as $\theta \to 1$. As a comparison, we also plot the elasticity of substitution for a Cobb-Douglas production function, which is equal to 1 for any pair of types (θ,θ'). In Appendix F.2 we illustrate the cross-wage elasticities $\gamma(\theta,\theta')$ and also explore alternative Translog specifications.

The right panel of Figure 6 plots the incidence on government revenue of the elementary tax reforms at each income $y(\theta)$ (equation 18) for the Translog specification (130) (black dashed curve) and compares them to the Cobb-Douglas technology (blue dashed-dotted curve). The general-equilibrium contribution with distance-dependence is also positive for high incomes and of slightly larger magnitude.

F.2 Tax incidence analysis: additional simulations

F.2.1 Alternative specification of the tax-and-transfer system

Our numerical simulations of Section 4 assume that the initial tax schedule (in the calibrated U.S. economy) has a constant rate of progressivity (CRP). We now consider an alternative calibration that differs in two ways: (i) we use a Gouveia-Strauss approximation for the income tax, taken from Guner, Kaygusuz, and Ventura (2014); (ii) we also account for the phasing-out of means-tested transfers programs that increase effective marginal tax rates, in particular for low incomes. Concretely, for Gouveia-Strauss, we use the specification including state taxes and consider the average overall households, i.e. we take the third to last column in Table 12 of Guner, Kaygusuz, and

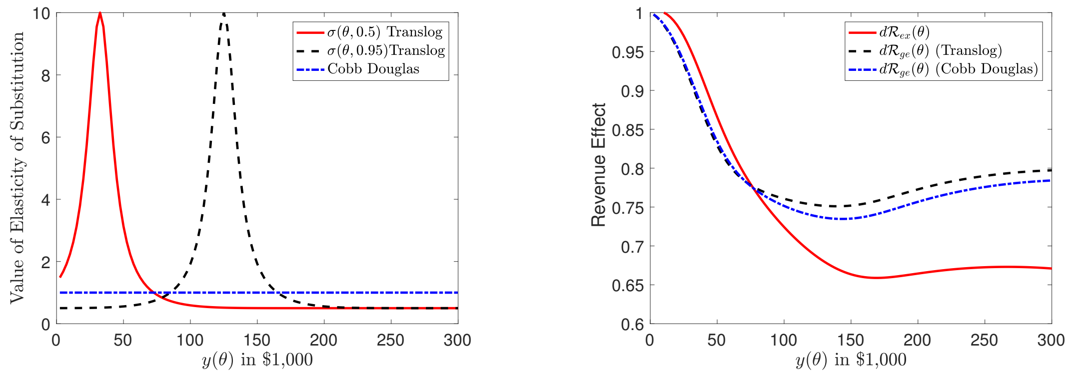


Figure 6: Left panel: Red bold (resp., black dashed) line: elasticity of substitution between types with income $y(\theta)$ and the 50th (resp., 95th) percentile, for the Translog specification (130). Blue dashed-dotted lines: Cobb-Douglas specification. Right panel: Black dashed line (resp., red bold line, blue dashed-dotted line): Revenue gains of elementary tax reforms at income $y(\theta)$ for the Translog specification (130) (resp., for exogenous wages, Cobb-Douglas production).

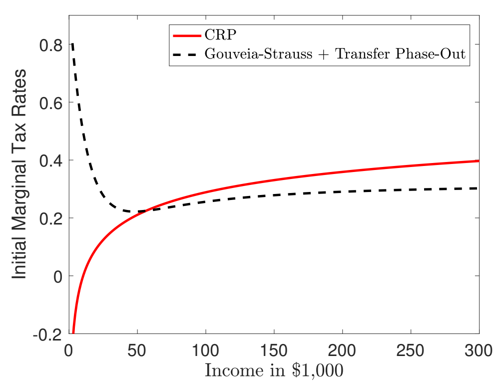


Figure 7: Alternative specification of the baseline tax schedule. Red bold line: CRP tax schedule. Black dashed line: Gouveia-Strauss approximation with additional distortions due to means-tested transfers.

Ventura (2014). For the phasing-out of transfers, we use parametric estimates from Guner, Rauh, and Ventura (2017), who consider a Ricker model $T(I) = \exp(\alpha)\exp(\beta_0 I)I^{\beta_1}$. We take their estimates for all households and all transfer programs, i.e. $\alpha = -1.816$, $\beta_0 = -4.290$ and $\beta_1 = -0.006$, where I is expressed in multiples of average income. Since Guner, Rauh, and Ventura (2017) express everything in year 2015 dollars and Guner, Kaygusuz, and Ventura (2014) in year 2000 dollars, we use a CPI deflator and express everything in terms of year 2000 dollars. This alternative baseline schedule of marginal tax rates is illustrated in Figure 7 (black dashed line), which also shows for comparison our specification of a CRP tax schedule (red bold line) that we use in the main body.

Figure F.2.1 then illustrates the normalized revenue gains of elementary tax reforms for a CES parameter $\sigma = 0.6$ (left panel) and $\sigma = 3.1$ (right panel). The additional general-equilibrium revenue

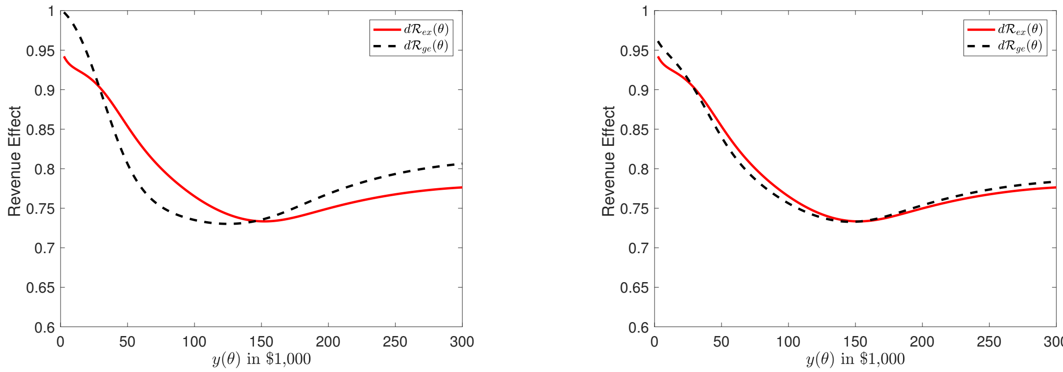


Figure 8: Revenue gains of elementary tax reforms at $y(\theta)$. The initial tax schedule is the black dashed line of Figure 7. Red bold line: model with exogenous wages. Black dashed lines: CES production function with $\sigma = 0.6$ (left panel) and $\sigma = 3.1$ (right panel).

effects due to the endogeneity of wages are a bit smaller in magnitude than for a CRP initial tax schedule, the general insight (Corollary and Figure 2) does not change.

F.2.2 Translog specification

Figure 9 shows the cross-wage elasticities $\gamma(\theta, \theta') = \rho(\theta') + \frac{\chi(\theta,\theta')}{\rho(\theta)}$ for our main Translog specification (130). They are also distance-dependent: an increase in the labor supply of type θ' tends to have a larger impact on the wage of types that are more distant, and therefore less substitutable, to θ'. Note that the cross-wage elasticities are decreasing in $y(\theta)$ for large θ, because of the term $\rho(\theta)$.

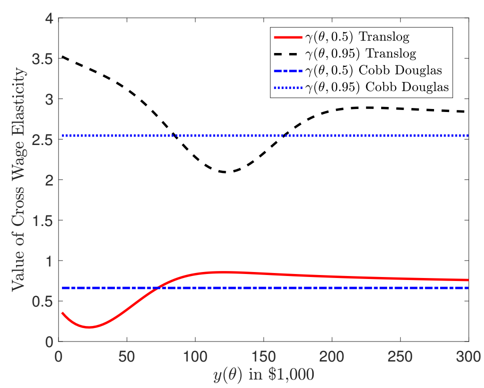


Figure 9: Red bold (resp., black dashed) line: cross-wage elasticities between types with income $y(\theta)$ and the 50th (resp., 95th) percentile, for the Translog specification (130). Blue dashed-dotted lines: Cobb-Douglas specification.

The left panel of Figure 10 shows the elasticities of substitution between individuals with income $y(\theta)$ and individuals at the 95th percentile for four cases. Case 1 is the specification of the main body. Case 2 implies the same minimal and maximal values for the elasticities of substitution but they decrease more slowly with distance (we chose $s = 100,000$ instead of $s = 50,000$). Cases 3 and 4 are analogouss to Cases 1 and 2 with the difference that the maximal value for the elasticity of substitution is equal to 20 instead of 10. The right panel of Figure 10 shows the revenue gains of elementary tax reforms at income $y(\theta)$ for all four cases. The case with exogenous wages is shown as a comparison. The normalized revenue gains are similar for all four cases. They are almost indistinguishable if we compare Cases 1 and 3, and Cases 2 and 4 respectively. The reason is that in these cases, the respective values of the own-wage elasticities are very similar. This is illustrated in Figure 11.

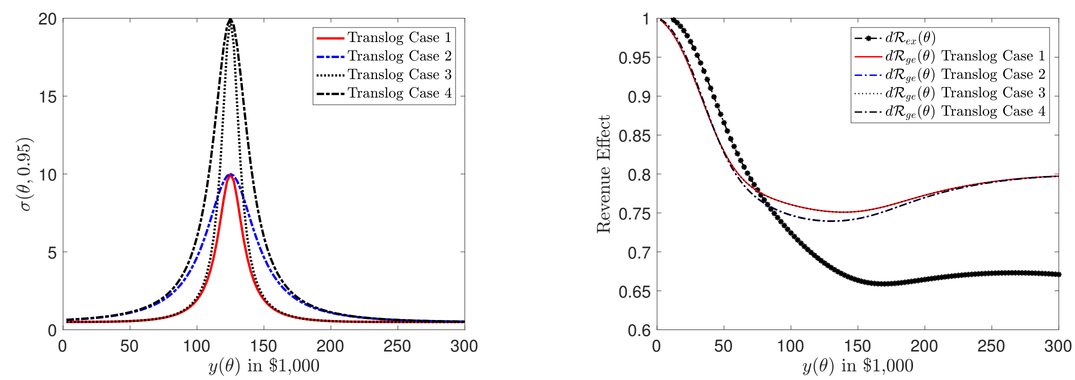


Figure 10: Left panel: elasticity of substitution between income $y(\theta)$ and the 95th income percentile in four cases. Right panel: normalized revenue gain of elementary tax reforms at $y(\theta)$ in these four cases and in the exogenous-wage case.

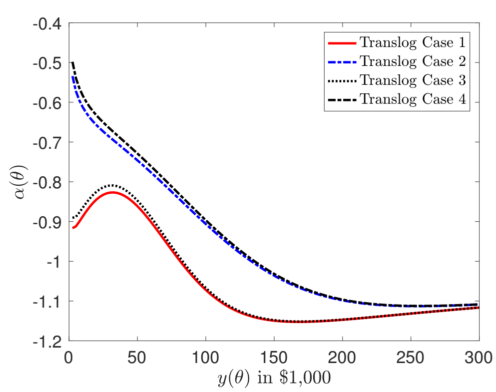
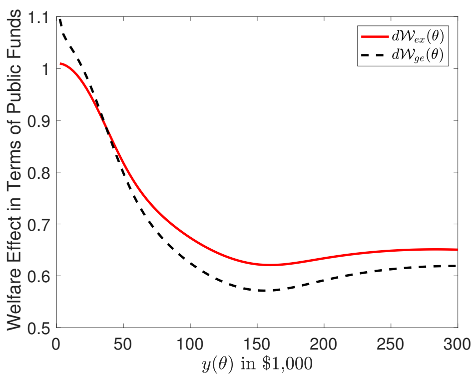


Figure 11: Own-wage elasticities $\alpha(\theta)$ as a function of $y(\theta)$ for the four Translog specifications.

F.2.3 Incidence for non-Rawlsian welfare functions

Here we ask how our tax incidence results differ if, instead of focusing on revenue effects (i.e., Rawlsian welfare), we consider alternative concave social welfare functions $G(u) = \frac{u^{1-\kappa}}{1-\kappa}$. Welfare gains are expressed in terms of public funds. For a low taste for redistribution ($\kappa = 1$, see the left panel of Figure 12), the welfare gains of raising tax rates on high incomes are muted due to general equilibrium. For a stronger taste for redistribution ($\kappa = 3$, see the right panel of Figure 12), general equilibrium effects make raising top tax rates more desirable. The reason is as follows. General equilibrium effects make raising top tax rates more desirable because the tax revenue increase is higher. At the same time the implied wage decreases for the working poor make them worse-off. In case of very strong redistributive tastes (i.e., when the social marginal welfare weights decrease sufficiently fast with income, the extreme case being the Rawlsian welfare criterion), the tax revenue gain gets a higher weight (since these gains are used for lump-sum redistribution at the margin). In the case where relatively richer workers (for whom the lump-transfer is less important relative to the very poor) still have significant welfare weights, the wage effects dominates. The CES parameter in the figures is $\sigma = 3.1$.



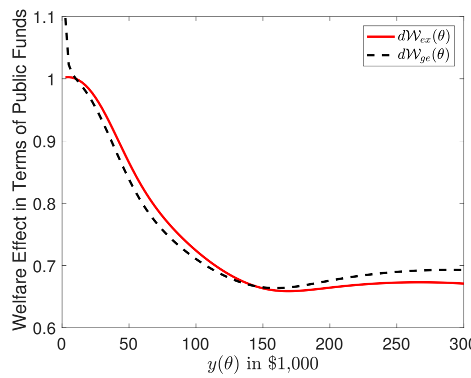


Figure 12: Welfare effect of elementary tax reforms at $y(\theta)$ for the welfare function $G(u) = \frac{u^{1-\kappa}}{1-\kappa}$. Left panel: $\kappa = 1$. Right panel: $\kappa = 3$.

F.3 Optimal taxation

F.3.1 Main simulations

We first consider a Rawlsian social objective. In Appendix F.3.3 we simulate optimal taxes for concave social welfare functions G; our results are similar.

The role of the elasticity of substitution. The left panel of Figure 13 plots the optimal marginal tax rates as a function of types for two different values of the elasticity of substitution, and for the exogenous-wage planner defined in (90). (The scale on the horizontal axis on the left panel is measured in income; e.g., the value of the optimal marginal tax rate at the notch \$100,000 is that of a type θ who earns an income $y(\theta) = \$100,000$ in the calibration to the U.S data. The income that this type earns in the optimal allocation is generally different (see the right panel).) The latter schedule has a familiar U-shape (Diamond, 1998; Saez, 2001). In line with our theoretical results of Section 6, the top tax rate is lower in general equilibrium and decreasing with σ. Moreover, the optimal marginal tax rates are reduced by an even larger amount at income levels close to the bottom of the U (around \$100,000), and are higher at low income levels (below \$40,000). Note that, since the exogenous-wage tax rates are already very high at those low income levels, the general equilibrium effects are quantitatively very small (at most 1.8 percentage points). This confirms our findings of Corollary 8 and implies that the U-shape obtained for exogenous-wages is reinforced by the general equilibrium considerations.

Translog production function. In the right panel of Figure 13, we illustrate the optimal marginal tax rates in case of the Translog production function with distance-dependent elasticities of substitution, as calibrated above, and compare it to the optimal tax schedule in the case of a Cobb-Douglas production function; the graph shows that the policy implications are hardly altered, which justifies our focus on the case of a CES production for the theoretical analysis of this section. In Section F.3.4 below, we consider alternative Translog specifications and obtain similar conclusions.

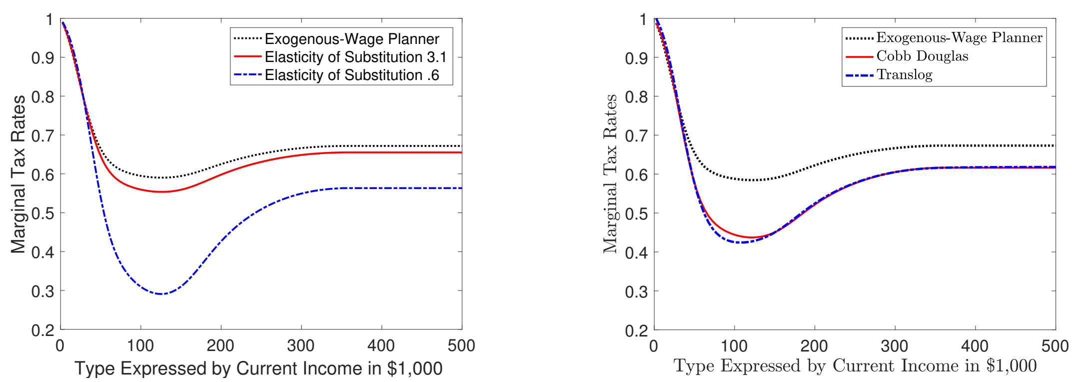


Figure 13: Optimal marginal tax rates as a function of types. Black dashed lines: exogenous wages. Left panel: CES technology with $\sigma = 3.1$ (red bold line) and $\sigma = 0.6$ (blue dashed-dotted line). Right panel: Translog technology (130) (blue dashed-dotted line) and Cobb-Douglas ($\sigma = 1$) (red bold line).

U-shape of the general-equilibrium effect. Next, we plot in Figure 14 the shape of the general-equilibrium correction to the optimal taxes obtained in the model with exogenous wages. We do so by applying our incidence formula (18) using (90) (i.e., the black-dotted curve in Figure 13) as our initial tax schedule. Recall that Corollary 8 addresses the same question analytically using the SCPE as the exogenous-wage benchmark. The red bold line plots the effects of the tax reform according to the exogenous-wage planner (90). These effects are uniformly equal to zero by construction. The black dashed line shows that when the low-income marginal tax rates are high (as in the exogenous-wage optimum) rather than low (as in the CRP tax code assumed in Corollary), the general equilibrium forces call for lower tax rates for intermediate and high incomes, and higher marginal tax rates for low incomes. This graph implies that starting from the exogenous-wage optimum, the gains from

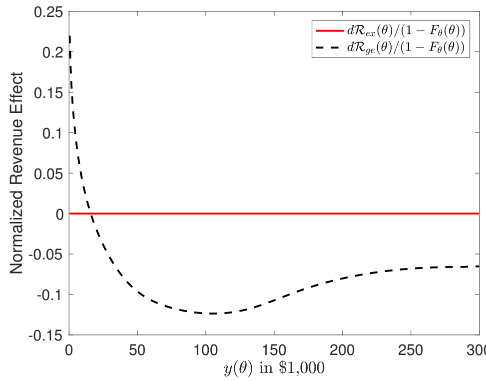


Figure 14: Tax incidence around the exogenous-wage optimum (90). Red bold line: model with exogenous wages. Black dashed line: CES production function ($\sigma = 3.1$).

perturbing the marginal tax rates are themselves U-shaped and negative, except at the very bottom of the income distribution, thus confirming our theoretical result of Corollary 8.

F.3.2 Alternative graphical representation

Figure 15 is the equivalent of the left panel of Figure 13 and illustrates optimal marginal tax rates as a function of income in the optimal allocation (instead of income in the current allocation). Marginal tax rates in this graph reflect the policy recommendations of the optimal tax exercise which is to set marginal tax rates at each income (rather than unobservable productivity) level. A general pattern is that the marginal tax rate schedule is shifted to the left because individuals work less for optimal taxes than with current taxes. This is visible most clearly for the top bracket and the bottom of the U that start earlier.

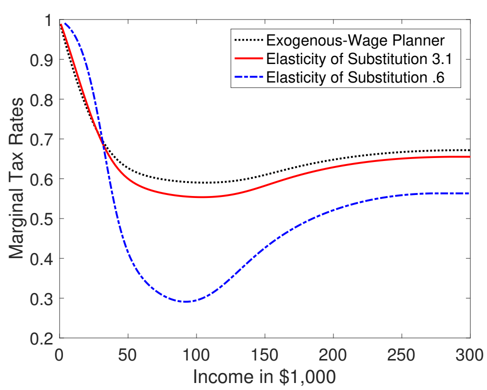


Figure 15: Optimal Rawlsian marginal tax rates plotted as a function of income in the optimal allocation.

F.3.3 Non-Rawlsian welfare function

Here we consider a social welfare function $G(u) = \frac{1}{1-\kappa} u^{1-\kappa}$. Figure 16 shows the optimal marginal tax rates for two values of κ (1 and 3). As in the Rawlsian case, the optimal U-shape of marginal tax rates is reinforced. Given that low income levels now also have positive welfare weights, there is a force for higher marginal tax rates at low income levels. Thus, the result that marginal tax rates should be higher at the bottom is stronger than in the Rawlsian case, because (i) the magnitude of this effect is larger, and (ii) it holds for a broader range (up to \$50,000). The CES parameter in the figures is $\sigma = 3.1$.

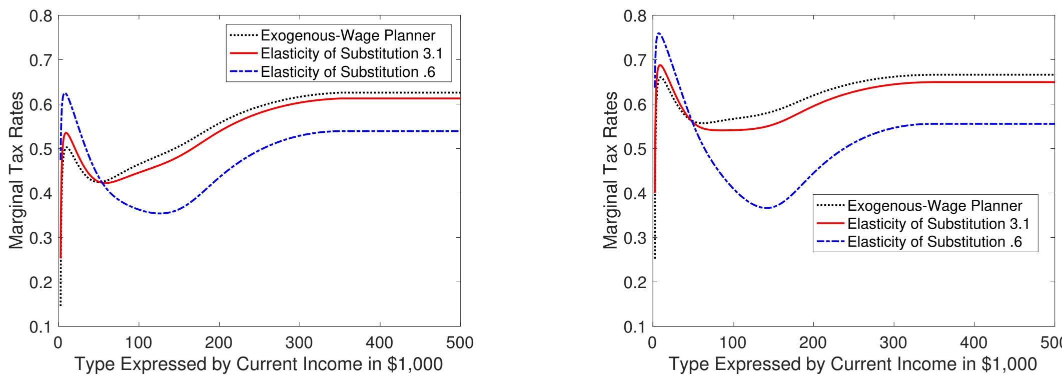


Figure 16: Optimal marginal tax rates for the welfare function $G(u) = \frac{u^{1-\kappa}}{1-\kappa}$. Left panel: $\kappa = 1$. Right panel: $\kappa = 3$.

F.3.4 Alternative specifications of the Translog production function

We now consider optimal Rawlsian marginal tax rates for all four Translog cases described in Section F.2. The results are illustrated in Figure 17. The shape of marginal tax rates is very similar for all four cases. Thus the quantitative implications for optimal taxes obtained in the main body for our baseline specification of the Translog production function are robust to different specifications.

F.3.5 Welfare gains

The right panel of Figure 18 plots the welfare gains of moving from the optimal taxes assuming exogenous wages, to the optimal taxes in general equilibrium (assuming a CES production function), as a function of the elasticity of substitution σ. These gains are expressed in consumption equivalent, which (given our welfare criterion) corresponds to a uniform increase in the lump-sum transfer. Naturally these gains are decreasing in σ and converge to zero as $\sigma \to \infty$. For low values of σ, they can be as high as 3.5 percent, and they remain nontrivial for the whole range of plausible values of the elasticity.

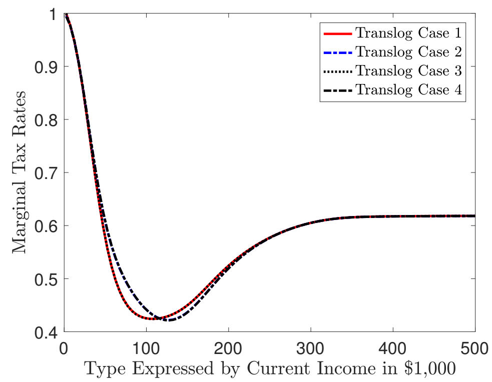


Figure 17: Optimal Rawlsian marginal tax rates for four Translog specifications.

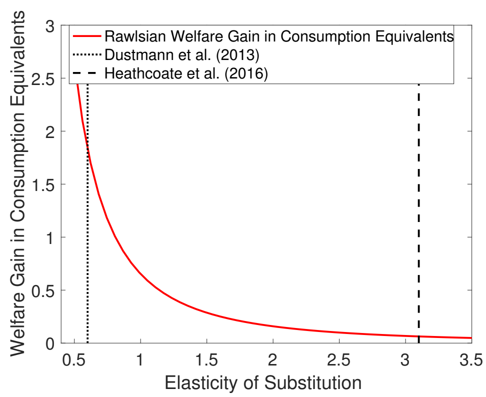


Figure 18: Welfare gains