Inheritance Taxation and Wealth Effects on the Labor Supply of Heirs

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Abstract

The taxation of bequests can have a positive impact on the labor supply of heirs through wealth effects. This leads to an increase in future labor income tax revenue on top of direct bequest tax revenue. We first show theoretically that a simple back-of-the-envelope calculation, based on existing estimates for the reduction in earnings after wealth transfers, fails: the marginal propensity to earn out of unearned income is not a sufficient statistic for the calculation of this effect because (i) heirs anticipate the reduction in net bequests and adjust their labor supply already prior to inheriting, and (ii) when bequest receipt is stochastic, even those who ex post end up not inheriting anything respond ex ante to a change in the distribution of net bequests. We quantitatively elaborate the size of the overall revenue effect due to labor supply changes of heirs by using a state of the art life-cycle model that we calibrate to the German economy. Besides the joint distribution of income and inheritances, quasi-experimental evidence regarding the size of wealth effects on labor supply is a key target for this calibration. We find that for each Euro of bequest tax revenue the government mechanically generates, it obtains an additional 7.6 Cents of labor income tax revenue (in net present value) through higher labor supply of (non-)heirs.

JEL Classifications: C68, D91, E21, R21

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1 Introduction

Inheritances are of growing importance for Western economies. Using data from France, Piketty (2011) shows that since the 1950s the annual flow of inheritances has been ever increasing, so that in 2010 it amounted to roughly 15 percent of national income. He also predicts that this share could become as large as 25 percent in the mid 21st century. Following his theoretical arguments, it is quite likely that a similar (and potentially even stronger) trend should be observed in other countries with low economic and population growth such as Spain, Italy and Germany (Piketty, 2011, p.1077). This development clearly highlights the increasing power of an inheritance tax in raising revenue.[1]

Despite the apparent importance of the topic, the incentive costs of inheritance taxation are not very well understood (Kopczuk, 2013). Measuring them empirically is a complicated task, because wealth transfers "are infrequent (at the extreme, occurring just at death), thereby allowing for a long period of planning, making expectations about future tax policy critical and empirical identification of the effect of incentives particularly hard" (Kopczuk, 2013, p.330). Furthermore, inheritances shape incentives along various dimensions, like wealth accumulation, labor supply and entrepreneurship.

In this paper we make progress on understanding and quantifying the revenue effects of inheritance taxation by elaborating one particular channel: labor supply of heirs. Concretely, we tackle the following policy question: For each Euro of revenue raised directly through inheritance taxes, how much additional labor income tax revenue from heirs can the government expect to obtain?

Answering this question purely empirically is problematic because it is difficult to identify the impact of inheritances on the earnings of heirs. One reason for this is that inheritances can be (imperfectly) anticipated and therefore already shape labor earnings prior to receipt. Further, settings with exogenous variation in inheritances are rare.[2] By contrast, there exists quasi-experimental evidence regarding the wealth effect of lottery gains on labor income (Imbens et al., 2001; Cesarini et al., 2017). Our methodological approach is to calibrate a version of the workhorse life-cycle model of the macroeconomics literature to be consistent with this quasi-experimental evidence and then answer our policy question through the lens of this model.

As a theoretical warm-up, we first set up a simple two-period overlapping generations framework with stochastic bequests to analyze the tax revenue effects of a change in

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1 We use the terms bequest taxes and inheritance taxes interchangeably in this paper albeit the fact that they might have different effects once an individual bequeathes to more than one heir and tax schedules are not proportional. For the experiments carried out in this paper, such a distinction, however, plays no role.

2 There exists a small empirical literature on that issue to which we relate below in the literature section.

the bequest tax rate. We formally isolate the revenue effect that is due to the labor supply of (potential) heirs. We show that the marginal propensity to earn out of unearned income is not a sufficient statistic for the change in their lifecycle labor supply (and therefore labor tax revenue) because an increase in the bequest tax is not an unanticipated reduction in wealth. Due to anticipation, two effects arise on top of the simple standard wealth effect: (i) Individuals do form expectations about the inheritances they will receive and accordingly adjust their labor supply. (ii) If inheritances are stochastic, even individuals that do not inherit but did assign a positive probability on receiving an inheritance also adjust their life-cycle labor supply.

We then study the quantitative importance of all these effects in a state of the art life-cycle labor supply model that accounts for expectations. We build such a model that features consumption, labor supply and savings decisions, heterogeneous labor productivity profiles and realistic expectations about the size and timing of bequests. We calibrate it to the German economy, most importantly to match the joint distribution on the size and timing of inheritances as well as labor earnings profiles.[3] To achieve credible magnitudes for the implied wealth effects, we target quasi-experimental evidence on wealth effects based on lottery gains (Cesarini et al., 2017). Specifically, we distribute lottery gains of different sizes among individuals of different ages in our model in the same way as they are distributed in the data set of Cesarini et al. (2017). We then measure the resulting impulse response function for labor earnings and vary preference parameters until the model predicted impulse response matches the empirical one.

The only feature of our model, for which neither quasi-experimental evidence nor the used survey data provide us with clear guidance on how to calibrate it, are expectations about the size of inheritances. Different assumptions on rational expectations can be consistent with the cross-sectional distribution of inheritances and earnings of the heirs. We therefore consider a class of expectations, that captures two special cases as polar outcomes: (i) Conditional on the age and the earnings profile, all individuals draw their inheritance from the estimated cross-sectional distribution. (ii) Conditional on the bequeather dying at a certain time, the heir knows for sure how much she inherits. Besides these two polar cases, we consider linear combinations of the two that are all consistent with the cross-sectional joint distribution of inheritances and earnings of the heirs.

Equipped with this quantitative model, we conduct the following policy experiment: We let the government levy a proportional tax of 1 percent on all bequests and calculate the resulting change in lifetime income and income tax payments for the total popula-

[3] Whereas the quasi-experimental evidence that we target was obtained for Sweden (Cesarini et al., 2017), we currently don't have access to Swedish data on earnings and bequests and therefore calibrate the model be consistent with German survey data. We will soon have access to the cross sectional features of the joint distribution of inheritances, income and age when inheriting for Sweden (based on administrative data), which will then allow us to conduct the quantitative analysis for Sweden in future versions of this paper.

tion of our model. For our benchmark calibration, we find that any Euro of bequests that is taken away from heirs increases their lifetime income by around 18.50 cent in net present value, that is discounted to the year of inheritance receipt. In terms of income tax payments this means that any Euro of revenue directly obtained through bequest taxes leads to additional tax revenues of around 7.64 cents (in net present value).

We decompose this number in two different ways. First, we show that anticipation effects constitute approximately half of it. This highlights the importance of considering a model with expectations and not only relying on a simple back-of-the envelope calculation, where one would focus on post-inheritance earnings of heirs only. More generally, our approach quantifies the bias that would occur in an estimation where only the labor supply changes of heirs after the inheritance would be taken into account and neither anticipation effects nor labor supply changes of non-heirs would be considered. Second, we consider heterogeneity in effects and answer our policy question for different earnings levels. We find that the number is increasing in earnings of the heirs, which simply reflects that a decrease in leisure is associated with a higher earnings gain for individuals with higher productivity.

Lastly, these policy implications are rather insensitive to the assumptions that we make about how informed individuals are with respect to their inheritances. Only in the polar case (or close to it) that there is no uncertainty about the size of the size of the inheritance (but not about the timing) does this number change significantly: it increases to 9.5 Cents.

We conclude that the additional labor tax revenue of heirs is likely to be of sizable magnitude and should be taken into account in fiscal planning (dynamic scoring).[4]

Related Literature. The paper is related to and motivated by a small but growing quasi-experimental literature of wealth effects on labor supply. Imbens et al. (2001) is the first paper to use lottery data to estimate the impact of wealth on labor supply. They document that on average a one dollar wealth increase triggers a decrease in earnings of 11 Cents. Cesarini et al. (2017) use a similar setting in Sweden and obtain surprisingly similar numbers. Picchio et al. (2015) study lottery winners in the Netherlands. While they find no effects along the extensive margin, the impact along the intensive margin is a bit smaller than in Imbens et al. (2001) and Cesarini et al. (2017). Gelber et al. (2017) analyze the wealth effect for individuals who receive disability insurance. The individuals they consider receive around $1,700 of DI benefits per month. The sample is particular in the sense that monthly income among the studied subjects is very low, on average around $200 per month. The authors have a very clean identification strategy (regression-kink design) and find an income effect from one dollar of

[4] To put this number into perspective, note that Saez et al. (2012) report the marginal excess burden per dollar of federal income tax raised to be below 20 cents.

additional unearned income of about 20 Cents.[5]

Further, our paper is related to the literature that estimates the impact of inheritances on labor supply of the heirs. Papers along these lines include Holtz-Eakin et al. (1993), who document the effect of bequests on labor force participation, and Brown et al. (2010), who investigate retirement choices. In a recent study Doorley and Pestel (2016) use the German Socio-Economic Panel (SOEP) to analyze the effect of inheritances on (actual and desired) hours worked, self-employment and hiring of entrepreneurs. The authors find that women who receive an inheritance reduce their labor supply by about 1.5 hours a week, while men's labor supply is by and large unaffected.

More relatedly, Elinder et al. (2012) study the impact of inheritances on earnings of the heirs and use variation in the size of inheritances for identification. The sample they consider is very small, however. They do find effects on earnings that are significantly larger than the one implied by our model. More recently, Bø et al. (2018) study this impact with Norwegian administrative data using a propensity score matching approach. The authors also find significantly larger effects. Whereas our model is consistent with wealth effects that are measured in the perhaps cleanest experimental studies one can think of (lotteries), their approach, while relying on less clean identification, has the advantage to rely directly on inheritance data. In that sense, we consider the two approaches of quantifying earnings changes as complementary. More importantly, our contribution is not to per se measure these labor supply effects, but to elaborate the implications for public finances in a transparent way. Changing our calibration such that the model is consistent with this alternative empirical evidence would significantly strengthen our policy implications that inheritance taxes have significant positive implications for labor tax revenue from heirs. In that sense, our numbers can be interpreted as lower bounds.

A recent related public economics paper is Koeniger and Prat (2018), who analyze the policy implications of wealth effects. In a dynastic Mirrleesian environment, they find that such wealth effects create a force for less educational investment of children from wealthy families.

The remainder of this paper is organized as follows. In section 2 we illustrate the main mechanisms within a tractable two-period OLG model. In section 3 we present the full life-cycle model. We present our parameterization of expectations in Section 4. The calibration is explained in section 5. In section 6 we summarize the results and perform several robustness checks. Section 7 concludes.

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5 Another recent related study is Bick et al. (2018), who document differences in hours worked across countries at different development stages. They find that both labor force participation (extensive margin) and hours worked conditional on employment (intensive margin) are lower in high income countries. This pattern is very much in line with wealth effects on labor supply.

2 A Two-Period OLG Framework

In this section, we illustrate our general ideas using a simple two-period overlapping generations framework. At each point in time $t \in \{0, 1, \ldots, \infty\}$, there are two generations alive, the sizes of which we normalize to one without loss of generality. From one period to the next, the older of the two generations dies, the younger generation turns old and a new generation is born. We denote by $j = 1, 2$ the age of a generation. Members of each generation have to decide about how much to consume c and how much effort l to put into working. When old, they might receive an inheritance b with a certain probability π from their parent generation.[6] In addition, they can choose themselves how much of a bequest to leave to their descendants. For the sake of simplicity and in line with the recent literature (see e.g. Piketty and Saez (2013)), we focus on the case where net bequests of descendants directly enter the utility function instead of considering a dynastic Barro-Becker model.

Life-time utility of a household is given by

$$\begin{aligned} U_t = u(c_{1t}, l_{1t}) + \beta \Big[\pi \cdot v\left(c^I_{2t+1}, l^I_{2t+1}, (1-\tau_b) b^I_{t+2}\right) \\ + (1-\pi) \cdot v\left(c^N_{2t+1}, l^N_{2t+1}, (1-\tau_b) b^N_{t+2}\right) \Big], \quad (1) \end{aligned}$$

where I denotes the case in which the agent receives an inheritance and N the case in which she does not. The instantaneous utility functions u and v are assumed to be strictly increasing and concave in c and $(1 - \tau_b)b$ as well as strictly decreasing and convex in l.

The agent maximizes her life-time utility given the budget constraint

$$c_{1t} + a_{t+1} \leq (1 - \tau_l) w_1 l_{1t} + T_1 \quad (2)$$

in the first period and the state-dependent constraints

$$\begin{aligned} c^K_{2t+1} + b^K_{t+2} \leq (1-\tau_l) w_2 l^K_{2t+1} + (1+r) a_{t+1} \\ + \mathbb{1}_{K=I} (1-\tau_b) b_{t+1} + T_2 \quad \text{for} \quad K = I, N. \quad (3) \end{aligned}$$

In the first period, households use their labor earnings net of proportional labor taxes τ_l as well as (potential) lump-sum transfers from the government to either consume or save into the next period. When they are old, they split their net labor earnings, gross savings, potential net bequest and the lump-sum transfer received between own consumption and bequests to their descendants. Note that we assume prices to be constant over time, but allow wages to be age dependent, reflecting potential wage growth over the life cycle. For the sake of simplicity, we assume that all bequests a generation leaves to their descendants are pooled and then distributed evenly across

[6] In the following, we use the words bequest and inheritance synonymously.

the group of heirs of the subsequent cohort.[7] In order to guarantee that all bequests are transferred to the descendant generation we require

$$\pi b_{t+2} = \pi b^I_{t+2} + (1-\pi) b^N_{t+2}, \tag{4}$$

which directly follows from the fact that only a share π of the population receives an inheritance.

Let us finally define the expected lifetime tax payments of the generation that is born at time t in present value terms as

$$R_t = \tau_l \cdot \left[y_{1t} + \frac{\pi y^I_{2t+1} + (1-\pi) y^N_{2t+1}}{1+r} \right] + \frac{\pi \tau_b b_{t+1}}{1+r} - T_1 - \frac{T_2}{1+r}. \tag{5}$$

Before thinking about how tax revenues change when bequest tax rates vary, let us first define what an equilibrium and a steady state of the above model are.

Definition 1 *Given an initial level of bequests b_1, an equilibrium allocation is a set of household decision rules $\{c_{1t}, a_{t+1}, c^I_{2t+1}, c^N_{2t+1}, b^I_{t+2}, b^N_{t+2}\}_{t=0}^{\infty}$ that maximize the household's utility function* (1) *subject to the budget constraints* (2) *and* (3)*, a set of bequest levels $\{b_t\}_{t=2}^{\infty}$ that is consistent with* (4) *and a set of lifetime tax revenues $\{R_t\}_{t=0}^{\infty}$ derived from* (5).

A steady state is an equilibrium allocation in which all variables are constant over time. We denote a steady state allocation as $\{c_1, a, c^I_2, c^N_2, b^I, b^N, b, R\}$.

2.1 The Effect of Changes in Bequests on Household Labor Earnings

In our modeling framework, we now want to work towards clarifying what the effect of a change in the proportional bequest tax τ_b on the life-time tax revenue R_t of a generation born in t is. Before, we however need to define how labor earnings of a household change with respect to exogenous variations in unearned income.

Definition 2 *Let us define*

$$\eta_1 = -\left.\frac{dy_1}{dT_1}\right|_{da=0} \quad , \quad \eta^K_2 = -\left.\frac{dy^K_2}{dT_2}\right|_{da=0} \quad \textit{and} \quad \alpha = -(1+r) \cdot \frac{da}{d\left[(1-\tau_b)b\right]}. \tag{6}$$

η_1 and η^K_2 denote the instantaneous wealth effects on labor earnings, meaning the decline in labor earnings as a result of an exogenous increase in lump-sum transfers under the assumption that savings are kept constant. α is the reaction in savings to an exogenous increase in the amount of bequests heirs receive at old age.

[7] An alternative would be to create a dynastic model in which there is a direct link between parents and children. Yet, in such a model, we would need another mechanism that can account for the fact that some children receive an inheritance and others don't. Since in a long-run equilibrium, this might result in a continuous distribution of bequests over households, it is beyond the scope of our theoretical analysis.

The following proposition summarizes the impact of a change in the net-of-tax-rate $1-\tau_b$ on household labor earnings in different periods of life, evaluated and linearized around a steady state with a constant tax rate τ_b.

Proposition 1 *A change in the net-of-tax rate on bequests $1-\tau_b$ leads to a total labor earnings reaction of*

$$\frac{dy_1}{d(1-\tau_b)\cdot b} = -\eta_1 \cdot (1+\varepsilon) \cdot \frac{\alpha}{1+r} \quad \text{and} \tag{7}$$

$$\frac{dy_2^K}{d(1-\tau_b)\cdot b} = \eta_2^K \cdot (1+\varepsilon)\left[-\mathbb{1}_{K=I} + \alpha\right] + \eta_2^K \cdot \xi_\tau^K, \tag{8}$$

where

$$\varepsilon = \frac{db}{d(1-\tau_b)} \cdot \frac{1-\tau_b}{b}$$

is the elasticity of bequests the household receives with respect to the net-of-tax rate $1-\tau_b$. ξ_τ^K measures the effect of a change in the net-of-tax-rate $1-\tau_b$ on the willingness of a household of type $K = I, N$ to bequeath to her own descendants.

Proof: see Appendix A. □

Proposition 1 tells us that upon an exogenous change in the net-of-tax-rate, the household labor earnings reaction has three components. First, there is a direct wealth effect on the earnings y_2^I of those who inherit some bequests. Second, in anticipation of a change in future bequest levels, the household can adjust her savings behavior in period one, which influences labor supply in period 1 as well as labor supply of both household types in period 2. Note that the intensity of the wealth effect on labor supply is itself due to two components: On the one hand, a net-of-tax rate increase leads to a mechanical wealth effect, on the other hand, the change in the net-of-tax rate might induce some behavioral reactions on the parent's bequeathing behavior. The sum of the two effects is captured in the term $1+\varepsilon$, where ε measures the elasticity of gross bequests a household receives from her parents with respect to the net-of-tax rate. Finally, when the tax rate on bequests declines, leaving bequests to her own descendants becomes more attractive to the household. Note that owing to our specification of utility, this argument holds for net bequests. A change in $1-\tau_b$, however, already mechanically leads to a rise in net bequests. The extent to which this influences the gross bequest level b^K is measured by the parameter ξ_τ^K, the sign of which is ambiguous. In any case, whether gross bequests increase ($\xi_\tau^K > 0$) or decrease ($\xi_\tau^K < 0$), labor supply will have to adjust accordingly, which is captured by $\eta_2^K \cdot \xi_\tau^K$. As we explain further below, however, our focus is on wealth effects and not on the price effect $\eta_2^K \cdot \xi_\tau^K$.

The following corollary shows we can put a lot of structure on these wealth effects, if we impose the assumption that all goods are normal goods.

Corollary 1 *If consumption and leisure in both periods as well as bequests are normal goods, we have*

$$\eta_1 \geq 0 \quad , \quad \eta_2^K \geq 0 \quad and \quad \alpha \geq 0.$$

Hence, if the assumptions in the preceding corollary hold, we can expect that upon an increase in expected net bequests in the second period:

(i) The household generates less labor earnings in the case she receives an inheritance in period two owing to the direct wealth effect.

(ii) In order to smooth consumption and leisure over time, she also lowers her savings.

(iii) The savings reaction leads to lower labor earnings in period 1, it dampens the labor earnings reaction of those who inherit in period 2, and implies an increase in labor earnings for those who did not inherit in period 2.

(iv) Finally, the household either increases (or decreases) gross bequests to her descendants, which has an additional positive (or negative) effect on labor supply.

2.2 Bequest Taxes and Cohorts' Life-Time Tax Payments

Knowing what happens to labor earnings when bequest levels change, we can now look at how a cohort's life-time tax payment changes upon the increase of bequest taxes. We therefore conduct the following thought experiment. We assume that our model is in a steady state. At some date s, the government changes the level of the bequest tax by a (marginal) amount $d\tau_b$. This change is not anticipated by households. Hence, the old generation at time s – the one born in $s-1$ – is surprised by this change. Since bequests are predetermined by the decisions of the generation born at date $s-2$, the change in bequest received by generation $s-1$ is

$$d\left[(1-\tau_b)\cdot b_s\right] = d(1-\tau_b)\cdot b_s = -d\tau_b \cdot b,$$

where b is the level of bequest in the steady state prior to the tax reform. Now as a result to this change in net bequests, the old households in period s adapt the amount of bequests they leave to their descendants, such that under the assumption of normal goods we should expect $b_{s+1} \leq b_s$. Having received a smaller amount of inheritance, the next generation then again changes its bequeathing behavior etc., which leads us to a series of new bequest levels

$$b = b_s \geq b_{s+1} \geq b_{s+2} \geq \ldots \quad \text{or in differences} \quad 0 \geq db_{s+1} \geq db_{s+2} \geq \ldots$$

until bequests finally converge to a new steady state value. Let us again define the elasticity of bequests that a household receives from her parent's generation at time t with respect to the net-of-tax-rate $1-\tau_b$ as[8]

$$\varepsilon_t = \frac{db_t}{d(1-\tau_b)} \cdot \frac{1-\tau_b}{b} \geq 0.$$

With this elasticity definition, we can obviously write

$$db_t = \varepsilon_t \cdot \frac{b}{1-\tau_b} \cdot d(1-\tau_b) \quad \text{where} \quad \varepsilon_s = 0.$$

Proposition 2 *The change in life-time tax payments of a cohort born at time $t \geq s$ to a change in bequest taxes $d\tau_b$ – which comes surprisingly at a date s – is given by*

$$dR_t = \pi \cdot \frac{d\left[\tau_b b_{t+1}\right]}{1+r} \cdot \left\{1 + \frac{\tau_l}{\pi\left[1 - \frac{\tau_b}{1-\tau_b} \cdot \varepsilon_{t+1}\right]} \cdot \left\{ (1+\varepsilon_{t+1}) \cdot \right.\right.$$
$$\left.\left. \left[\alpha\eta_1 + \pi\left[\eta_2^I - \alpha\eta_2^I\right] + (1-\pi)\left[-\alpha\eta_2^N\right]\right] - \left[\pi\eta_2^I\xi_\tau^I + (1-\pi)\eta_2^N\xi_\tau^N\right] \right\}\right\}. \quad (9)$$

For the cohort born at date $s-1$ we have

$$dR_{s-1} = \pi \cdot \frac{d\tau_b \cdot b_s}{1+r} \cdot \left\{1 + \frac{\tau_l}{\pi} \cdot \left[\pi\eta_2^I - \left[\pi\eta_2^I\xi_\tau^I + (1-\pi)\eta_2^N\xi_\tau^N\right]\right]\right\}.$$

Proof: see Appendix A. □

Before we interpret these equations, note that the total revenue effect of a change in bequest taxes has a direct component[9] as well as an additional component through changes in labor supply behavior and a corresponding impact on labor tax revenue. In order to isolate the latter and explore by how much life-time tax payments of a cohort rise in excess of the bequest taxes it pays, we normalize R_t by the expected bequest tax payment of the generation born in period t.

Corollary 2 *The change in life-time tax payments in excess of the bequest tax revenue effect is*

$$dE_t = \frac{\tau_l}{\pi\left[1 - \frac{\tau_b}{1-\tau_b} \cdot \varepsilon_{t+1}\right]} \cdot \left\{ (1+\varepsilon_{t+1}) \cdot \left[\alpha\eta_1 + \pi\left[\eta_2^I - \alpha\eta_2^I\right] + (1-\pi)\left[-\alpha\eta_2^N\right]\right]\right.$$
$$\left. - \left[\pi\eta_2^I\xi_\tau^I + (1-\pi)\eta_2^N\xi_\tau^N\right] \right\} \quad (10)$$

[8] In the same way as in Proposition 1.

[9] Reflected in the term 1 in parenthesis and simply indicating that higher bequest taxes will (at least on the upward sloping part of the Laffer curve) lead to higher bequest tax revenues.

for all generations born in period $t \geq s$ and

$$dE_{s-1} = \frac{\tau_l}{\pi} \cdot \left\{ \pi\eta_2^I - \left[\pi\eta_2^I \xi_\tau^I + (1-\pi)\eta_2^N \xi_\tau^N \right] \right\}. \tag{11}$$

Note that this corollary directly follows from

$$dE_t = \frac{dR_t}{\pi \cdot \frac{d\left[\tau_b b_{t+1}\right]}{1+r}} - 1.$$

Equations (10) and (11) tell us that for each dollar of bequest tax revenue the government receives (in present value terms) from a generation that is affected by an increase in proportional bequest taxes $d\tau_b$, we can expect dE_t additional cents of labor tax revenue. The effect dE_t thereby consists of multiple components. Starting with the old generation at the time of the bequest tax increase in equation (11), we can directly see two effects at work. All households of this generation are surprised by the change in taxes. Since they are already old, the only margin by which they can react to this change is by adjusting their current consumption and labor earnings as well as the amount of bequest they leave to their descendants. All households of type i who receive an inheritance therefore experience a negative *wealth effect* of $d\tau_b \cdot b$,[10] which directly translates into higher labor earnings. The size of this wealth effect is given by η_2^I, which measures the households willingness to earn out of unearned income, holding fix life cycle savings. Non-heirs, of course, experience no wealth effect.

Yet, an increase in bequest taxes also induces a *price effect*, which impacts on the households' willingness to leave bequests to their own descendants. This channel is summarized in the second term of equation (11). ξ_τ^K measures the extent to which households of type $K = I, N$ adjust their *gross bequests* to a change in the tax rate $d\tau_b$. Note that ξ_τ^K itself is a result of two effects. On the one hand, an increase in the tax rate τ_b makes bequeathing to the descendants less attractive, which is why – if all goods are normal – households want to reduce their level of *net bequests*. However, at the same time, the tax change $d\tau_b$ already mechanically reduces net bequest by an amount of $d\tau_b \cdot b_t^K$, where b_t^K is the level of *gross bequests*. If $d\tau_b \cdot b_t^K$ is smaller (larger) than the household's desired decline in net bequests, then the agent will also lower (increase) her gross bequest level b_t^K. As a result, she will require less (more) labor earnings which mitigates (reinforces) the wealth effects.

With these effects in mind, let us turn to the excess tax revenue of all generations born at time $t \geq s$ in equation (10). We can immediately see that the same effects are at work for this generation. However, the *wealth effect* is now a product of three subcomponents:

$$\underbrace{\alpha\eta_1}_{\text{Anticipation Effect}} + \underbrace{\pi\left[\eta_2^I - \alpha\eta_2^I\right]}_{\text{Effect on Heirs}} + \underbrace{(1-\pi)\left[-\alpha\eta_2^N\right]}_{\text{Effect on Non-Heirs}} \tag{12}$$

[10] Recall that bequest b_s are predetermined by the old generation in period $s-1$, which was totally unaffected by the tax increase

The term $\pi\eta_2^I$ again covers the direct wealth effect that we would observe if a generation was hit by the tax change unexpectedly in the middle of their life. Since all households born at a time $t \geq s$ however observe the increased bequest tax rate already in the first period of life, there is an *anticipation effect*. Specifically, all members of a cohort will try to smooth the impact of a smaller expected inheritance over their life cycle. As a result, if all goods are normal, they lower consumption in period one in order to increase savings into the next period. This leads labor earnings to already increase prior to a (potential) bequest tax receipt ($\alpha\eta_1$). The savings increase in turn induces an additional positive wealth effect on households when old. Hence, it mitigates the labor earnings reaction of heirs and induces non-heirs' labor earnings to even fall below their steady state earnings level.

Over and above the three labor supply effects discussed so far, there is a fourth effect in equation (10), which relates to the impact the tax increase $d\tau_b$ has on the equilibrium bequests received by generation t. By definition, bequests in the period of the reform are predetermined, i.e. $\varepsilon_s = 0$. The old generation at time s will, however, adjust their bequest level both owing to the wealth effect induced by a lower amount of inheritance as well as to the price effect. This induces aggregate bequest of the next generation to fall, which is why we should expect $\varepsilon_{s+1} \geq 0$. The old generation in period $s+1$ hence does not only experience a mechanical wealth effect due to the change in the tax rate, but an additional wealth effect owing to the decline in bequests. As a result, their bequest will fall even below the amount they received, leading to an elasticity of $\varepsilon_{s+2} \geq \varepsilon_{s+1}$. Following this logic period by period yields a sequence of elasticities

$$0 = \varepsilon_s \leq \varepsilon_{s+1} \leq \varepsilon_{s+2} \leq \ldots,$$

which converges to some steady state level. Summing up, the factor $1 + \varepsilon_{t+1}$ measures the *exposure* or *equilibrium effect* of each generation that results from intertemporal spillovers through the bequest channel. A greater ε_{t+1}, hence, leads to a stronger decline in the net bequests the generation born at time t receives and therefore induces stronger wealth effects on labor earnings. Note that the price effect does not depend on ε_{t+1}, as it is merely a consequence of the change in the price of bequests $d\tau_b$, where this price change is constant across all affected cohorts.

Summing up, we have shown that by increasing bequest taxes in our model, the government not only receives additional bequest tax revenue, it can also expect a rise in labor taxes paid by each generation. The extend by which labor earnings actually increase is the product of

1. a *direct wealth effect* on heirs through a fall in net inheritances,

2. an *anticipation effect* leading to a smoothing of labor earnings (also for individuals that are ex-post non-heirs) over the life cycle and therefore changes in savings,

3. a *price effect* associated with the behavioral reaction to a change in the price of net bequests, and

4. an *equilibrium effect* that results from intergenerational spill-overs and that leads to a different extent of the wealth and anticipation effect for generations born at different points in time.

In the following analysis, we concentrate on the first two effects, since they can be traced by suitably calibrating a quantitative model to quasi-experimental evidence on the wealth effects on labor earnings. The price and equilibrium effects, on the other hand, require a careful specification of bequest motives and the sensitivity of bequests with respect to tax rates. Since evidence on the effects of bequest taxes on intergenerational bequeathing behavior is scarce, we will leave these channels to future research. In terms of our model, one can interpret this exercise as setting $\xi_{\tau}^{K} = \varepsilon_t = 0$ for all $t = 0, 1, \infty$. In this case, the excess tax payments associated with a change in proportional bequest taxes can be summarized as

$$dE_t = \frac{\tau_l}{\pi} \cdot \left[\alpha\eta_1 + \pi \left[\eta_2^I - \alpha\eta_2^I \right] + (1-\pi) \left[-\alpha\eta_2^N \right] \right] \quad \text{and} \quad dE_{s-1} = \tau_l \cdot \eta_2^I.$$

3 Quantitative Life-Cycle Model

Our previous theoretical analysis has revealed that the anticipation of bequests plays a crucial role in determining the labor supply response to a change in bequest taxes. In the following sections we construct and calibrate a full life-cycle model that accounts for proper expectations and allows us to realistically quantify the effect of a change in bequest taxes on the labor supply of heirs.

Timing and endowments Time $t \in \{1, \ldots, T\}$ is discrete and period length is one year. The economy is populated by a continuum of mass one of heterogeneous households. Households enter the economy at age 20 (model age $t = 1$). At this point in time, they are endowed with an earnings ability level $e \in \{1, \ldots, E\}$ and a signal $s \in \{0, \ldots, n\}$ about the amount of inheritance they might receive. Agents work until they reach the (exogenous) retirement age t_r. They die with certainty at age T.

Bequest and expectations Throughout their life-cycle, households might receive a bequest. Bequests are stochastic both with respect to timing and size. We assume, for the sake of simplicity, that a household can only inherit once in her lifetime – at the age at which her ancestors pass away. Denote by $\{p_t^e\}_{t=1}^T$ the unconditional probability distribution of ancestors passing away when a household of ability e is of age t. We assume that the chance of parents surviving their children is zero, i.e. $\sum_{t=1}^T p_t^e = 1$.

When a household's parents die at time t, their bequest can take one of $n+1$ different levels $\{b_{it}^e\}_{i=0}^n$, where $b_{0t}^e = 0$. We call $i \in \{0, \ldots, n\}$ a bequest class and assume that the conditional probability of the household's inheritance falling into such a class is time invariant. Agents form expectations about the class their inheritance will belong to according to the signal s they received at the beginning of their life cycle. A signal of perfect quality would imply that a household falls into inheritance class $i = s$ with certainty. We will also consider less precise signals and will be more specific about how we formalize the quality of the signal in the next section. For now, we just denote by π_{is}^{e} the time invariant probability that a household with signal s and earnings capacity e attaches to receiving an inheritance of class i. The probability that an individual of type (e, s) receives a bequest at age t that falls into class i is then given by $p_t^e \cdot \pi_{is}^e$.

While the probability distribution over bequest classes i is time invariant, bequest levels b_{it}^e in each class are allowed to vary over time t. This reflects, for example, that ancestors might run down their wealth throughout a prolonged retirement phase. The bequests levels b_{it}^e as well as depend on the individual earnings capacity e which can account for the empirical fact that higher earning children tend to have richer parents.

Preferences At any age t, households decide about how much to consume c_t, how much to work l_t and how much to save a_t. They have preferences over consumption and labor supply

$$U_0(e,s) = \mathbb{E}\left[\sum_{t=1}^{T} \beta^{t-1} \left(\frac{c_t^{1-\gamma}}{1-\gamma} - \lambda \frac{l_t^{1+\chi}}{1+\chi} \right) \middle| e, s \right]$$

and form expectations about inheritances according to the above probabilities. We assume utility of consumption and disutility of labor to be additively separable. The parameter λ denotes the relative weight of labor in the agent's utility, χ is the inverse of the Frisch elasticity of labor supply, β is the time discount factor, and γ is risk aversion.

Budget constraint The budget constraint is given by

$$c_t + a_{t+1} = w_t^e l_t - \mathcal{T}\left(w_t^e l_t\right) + \mathcal{P}_t^e + W_t.$$

Consumption and savings into the next period are financed out of gross labor income $w_t^e l_t$ minus taxes $\mathcal{T}\left(w_t^e l_t\right)$, pension income $\mathcal{P}_t^e$ and net wealth W_t. Gross labor income is the product of the wage rate w_t^e and labor effort l_t. The function $\mathcal{T}(.)$ maps gross labor income into a tax payment and is specified in more detail in the calibration section of this paper. Throughout retirement, the household receives pension income $\mathcal{P}_t^e$, which we assume to be constant and conditional on the household's earnings capacity. In particular, we set

$$\mathcal{P}_t^e = \begin{cases} 0 & \text{if } t < t_r \\ \mathcal{P}^e > 0 & \text{if } t \geq t_r. \end{cases}$$

Net wealth is a composite of both individual savings a_t and (potential) bequests b_{it}^e received

$$W_t = \left[1 + (1-\tau_k) r\right] a_t + (1-\tau_b) b_{it}^e,$$

where $(1-\tau_k)r$ is the net-of-tax interest rate on savings and τ_b is a proportional tax rate on bequests.

Finally, each household faces a borrowing constraint

$$a_{t+1} \geq a_{min},$$

with the minimal asset level being a number $a_{min} \in (-\infty, 0]$. Retirement at age t_r is mandatory. Hence labor supply needs to satisfy

$$l_t = 0 \quad \text{for all} \quad t \geq t_r.$$

Dynamic optimization problem The state space of the household optimization problem contains the individual's earnings capacity e, the signal about the size of bequests s as well as net wealth W_t. Since households only inherit once in their life time, the state space further contains an indicator $h_t \in \{0,1\}$ for whether the agent's parents already passed away prior to or at date t. The dynamic optimization problem of the household hence reads

$$V_t(e,s,h_t,W_t) = \max_{c_t,l_t,a_{t+1}} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} - \lambda \frac{l_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}\left[V_{t+1}\left(e,s,h_{t+1},W_{t+1}\right) \Big| e,s,h_t \right] \right\}.$$

If the household's parents are still alive, expectations are formed according to

$$\begin{aligned} \mathbb{E}\left[V_{t+1}\left(e,s,h_{t+1},W_{t+1}\right) \Big| e,s,h_t = 0 \right] = \tilde{p}_{t+1}^e \cdot \sum_{i=0}^{n} \pi_{is}^e \cdot V_{t+1}\left(e,s,1,W_{t+1,i}\right) \\ + \left[1 - \tilde{p}_{t+1}^e\right] V_{t+1}\left(e,s,0,W_{t+1}\right), \end{aligned}$$

where

$$\begin{aligned} W_{t+1,i} &= \left[1+(1-\tau_k)r\right] a_{t+1} + (1-\tau_b) b_{i,t+1}^e \quad \text{and} \\ W_{t+1} &= \left[1+(1-\tau_k)r\right] a_{t+1}. \end{aligned}$$

Furthermore,

$$\tilde{p}_{t+1}^e = \frac{p_{t+1}^e}{1 - \sum_{s=1}^{t} p_s^e}$$

is the conditional probability of receiving an inheritance at age $t+1$, given that one hasn't received an inheritance yet. In case the agent's ancestors already deceased, all uncertainty has been revealed and we can simply write

$$\mathbb{E}\left[V_{t+1}\left(e,s,h_{t+1},W_{t+1}\right) \Big| e,s,h_t = 1 \right] = V_{t+1}\left(e,s,1,W_{t+1}\right).$$

4 Parameterizing expectations about bequests

One crucial element of our life cycle model is the probability distribution π_{is}^e according to which a household forms expectations about the class i her inheritance can fall into, including the case where no inheritance is received $i = 0$. Measuring expectations about inheritances is complicated if one can only observe actual cases of inheritances. Whereas our data allows us to estimate the distribution of inheritances conditional on age and earnings of the heirs, this does not inform us about the expectations heirs in that age-earnings class actually had. We therefore suggest different parameterizations of the signal quality. We only require that they are all consistent with the conditional cross-sectional distribution of inheritances. On the one extreme, we will consider signals of perfect quality: conditional on the parents dying, heirs know for sure how much they inherit. On the other extreme, the signal contains no information at all: heirs just draw their inheritance from the estimated cross-sectional distribution. To elaborate how our results depend on expectations, we consider both extreme cases as well as intermediate ones.

More formally, the signal $s \in \{0, \ldots, n\}$ an agent receives is a discrete number that contains information about which class i her inheritance will fall into. The parameter $\sigma \in [0, 1]$ is an indicator for the quality of this signal. If $\sigma = 0$, the signal contains no information at all, while for $\sigma = 1$ the household knows with certainty that $i = s$. At the beginning of the life cycle, a fraction φ_s^e of households of ability e is equipped with the signal s. We now have to make a distinction between the individual specific probability distribution π_{is}^e, which depends on the individual signal s, as well as the population wide (cross-sectional) distribution ω_i^e of households of earnings class e over different bequest levels i. In order for the individual probability distributions to be consistent with the cross-sectional distribution, we require

$$\forall i, e: \sum_{s=0}^{n} \varphi_s^e \cdot \pi_{is}^e = \omega_i^e. \tag{13}$$

Note that when the signal is fully informative about the household's bequest class ($\sigma = 1$), the individual probability distribution is

$$\pi_{is}^e = \begin{cases} 1 & \text{if } i = s \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

On the other hand, if the signal contains no information ($\sigma = 0$), the best forecast a household can make about the class her inheritance will fall into is the cross-sectional distribution over all households of the same earnings level ω_i^e, meaning that $\pi_{is}^e = \omega_i^e$ for all $s = 0, \ldots, n$. For any intermediate signal quality, we let the individual probability distribution be a convex combination of the two. Hence, we have

$$\pi_{is}^e = (1 - \sigma)\omega_i^e + \sigma \cdot \mathbb{1}(i = s) \quad \text{for} \quad \sigma \in [0, 1],$$

where $\mathbb{1}(i = s)$ is an indicator function that takes a values of 1 if i is equal to s and 0 otherwise. For any $\sigma > 0$, equation (13) implies $\varphi_s^e = \omega_s^e$, meaning that the distribution of the population of an earnings level e over different signals s has to exactly equal the cross-sectional distribution of this population over inheritance levels i.

5 Calibration

We calibrate our model in three steps:

1. We first estimate labor earnings profiles $y_t^e = w_t^e l_t$, the probability of ancestral death p_t^e, the cross-sectional distribution ω_i^e as well as bequest levels b_{it}^e using data from the *German Socio-Economic Panel (GSOEP)*.
2. In a second step, we parameterize further model parameters, prices and government policies.
3. Finally, we jointly pin down both the labor supply elasticity parameter χ and risk aversion γ such that our model is consistent with recent empirical evidence on the effects of lottery wins on labor earnings provided in Cesarini et al. (2017).

5.1 Labor earnings and bequests

Our main data source is the GSOEP, an annual panel survey on German households.[11] We use data on age, education, labor income and inheritances on the household level in between the years 2000 and 2014, and pool together all data from these 15 different waves into one cross-section.[12] We assume that a household consists of either one or two persons, meaning that we abstract from the presence of children or any other relative or non-relative household members. For two person households we identify the household head as the primary earner and use the head's age and education level in all further calculations. We define household labor income as the sum of labor earnings, public transfers (such as social assistance) and pension payments. In addition to age, GSOEP provides data on whether the household has received an inheritance in a respective survey year and if yes, about its size. To account for different household sizes, we divide gross labor income and inheritances of two person households by 1.5, which equals the common scale parameter used by the OECD. Finally, we drop all observations for which information on either age, education level, labor income or inheritances are missing as well as all households aged 19 and below. This leaves us with a total of 163,369 observations.

[11] For detailed information about the GSOEP, see Wagner et al. (2007).

[12] Note that we can not use data on the individual level, as the household is the only unit on which inheritance data can be observed in the GSOEP.

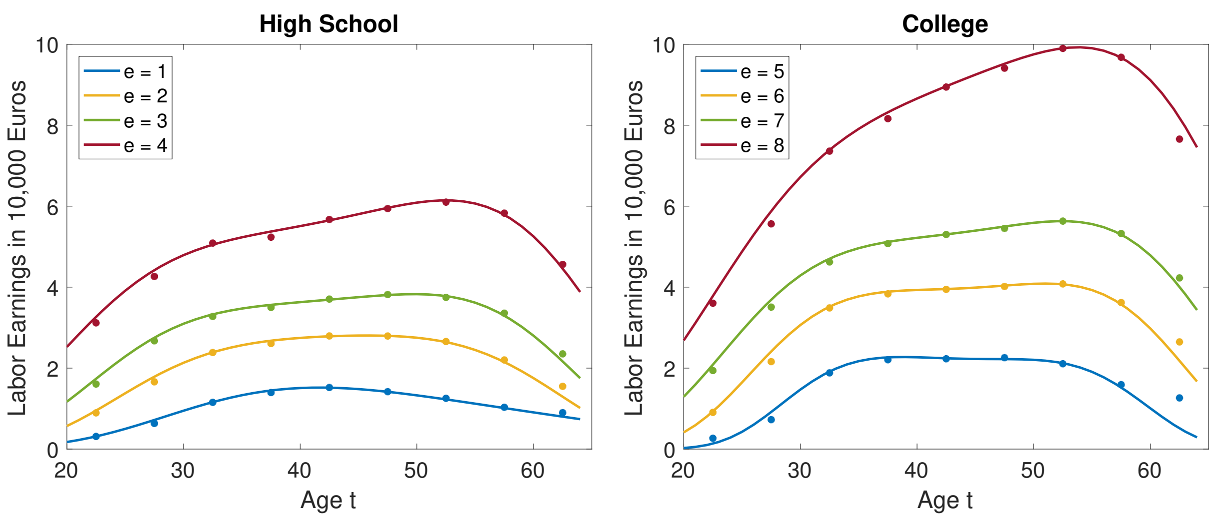
5.1.1 Labor earnings classes

We define a total of $E = 8$ different earnings classes, which results as a combination from two education levels and four income groups per education levels. We first stratify our sample according to the education level of the household. We say that a household has a low education, if the highest educational degree of the household head is a secondary or lower degree according to the ISCED97 education classification standard. All households with household head holding a tertiary education degree are considered highly educated. We assign households with low education into earnings classes $e = 1, 2, 3, 4$ and those with high education into $e = 5, 6, 7, 8$. We then group all households of an education level according to five year age bins, that is 20-24, 25-29, $\ldots$, 60-64, and pool all observations aged 65 and above into one bin. Within each education-age group, we separate households into four quartiles according to their labor income, leading to 4 earnings classes within each educational group. Table 6 in Appendix B summarizes mean earnings of the 8 earnings classes at different ages derived from the GSOEP. The last row of this table shows the shares of households in each earnings class in the total population. This shows that in our sample 28.4 percent of household heads hold a higher education degree. In order to feed our model with annual data, we fit polynomials of the form

$$y_t^e = \exp\left(\kappa_0^e + \kappa_1^e \cdot t + \kappa_2^e \cdot t^2 + \kappa_3^e \cdot t^3 + \kappa_4^e \cdot t^4\right) \tag{14}$$

for each earnings class e to our data. We derive the polynomial coefficients by minimizing a simple residual sum of squares between the data reported in Table 6 and the corresponding moments derived from the polynomial. Figure 1 shows the resulting age-earnings profiles.

Figure 1: Estimated age-earnings profiles for different earnings classes



Our model features endogenous labor supply decisions. Hence, labor earnings – the product of labor effort l_t and productivity w_t^e – are an endogenous object. In order to back out labor productivity profiles that lead to the labor earnings profiles shown in Figure 1, we follow the strategy proposed by Saez (2001). Note that, in our model, labor productivity is assumed to be deterministic over the life cycle and utility from consumption and disutility from labor are additively separable. In order to be able to apply the strategy of Saez (2001), we have to make on additional simplifying assumption, namely that instead of receiving bequests according to the risk process outlined above, households of each earnings class e receive a lump-sum transfer in each period of life that is equal to the average amount of bequest for this group, that is

$$\mathcal{Z}_t^e = p_t^e \cdot \sum_{i=0}^{n} \omega_i^e \cdot b_{it}^e.$$

In doing so, we eliminate all uncertainty from our model,[13] which allows us to write the household optimization problem as

$$\max_{c_t^e, y_t^e, a_{t+1}^e} \sum_{t=1}^{T} \beta^{t-1} \left(\frac{(c_t^e)^{1-\gamma}}{1-\gamma} - \lambda \frac{\left[\frac{y_t^e}{w_t^e}\right]^{1+\chi}}{1+\chi} \right)$$
$$\text{s.t.} \quad c_t^e + a_{t+1}^e = y_t^e - \mathcal{T}(y_t^e) + \mathcal{P}_t^e + \mathcal{Z}_t^e + (1+r)a_t^e \quad \text{and} \quad a_{t+1}^e \geq a_{min}.$$

The first order conditions of this problem read

$$(c_t^e)^{-\gamma} = \beta(1+r)\left(c_{t+1}^e\right)^{-\gamma} + \alpha_t \quad \text{with} \quad a_{t+1} \cdot \alpha_t = 0 \quad \text{and}$$
$$(w_t^e)^{1+\chi} = \frac{\lambda}{1-\mathcal{T}'(y_t^e)} \cdot \frac{(y_t^e)^{\chi}}{(c_t^e)^{-\gamma}},$$

where α_t is the Lagrangean multiplier on the minimum asset constraint in instantaneous utility values. Given a government policy $\mathcal{T}(\cdot)$ and $\mathcal{P}_t^e$, a set of lump sum transfers $\mathcal{Z}_t^e$ and a deterministic earnings path y_t^e, we can use the Euler equation together with the household budget constraint to calculate the deterministic consumption path c_t^e. We can then use the intra-period first order condition to back out the corresponding labor productivity profile w_t^e for households of earnings class e. Note that the resulting productivity profile is only approximately correct, owing to the assumption we made. However, comparing the model simulated average earnings path including bequest uncertainty for each earnings class to the earnings profiles estimated from the data showed only minor differences.

5.1.2 Probabilities of ancestral death and receiving and inheritance

Having grouped our observations into suitable earnings classes, we next have to estimate the age-dependent probability of ancestral death for members of each of these earnings groups. As inheritances arrive typically only onces or twice in a life-time,

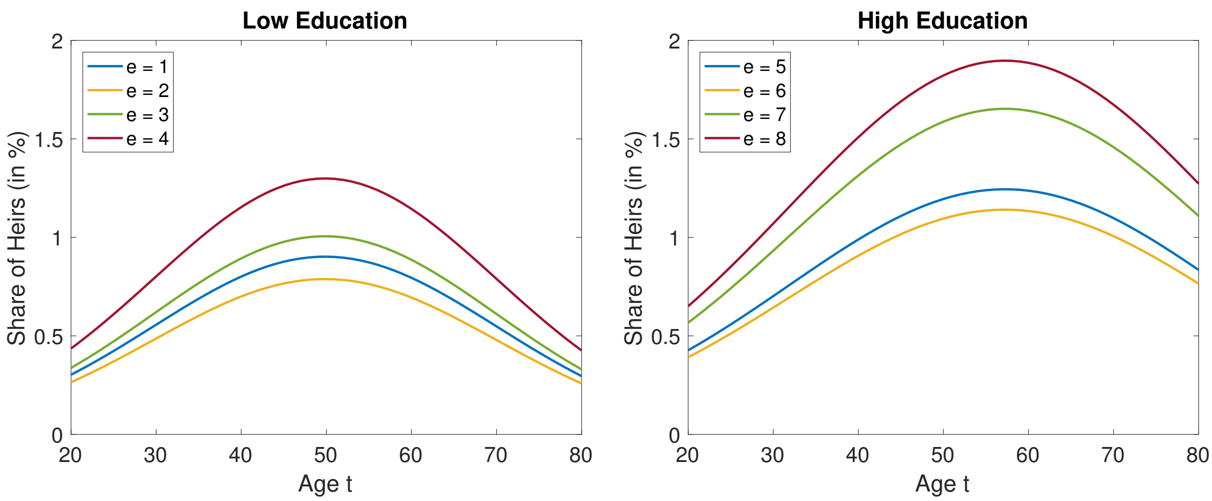
[13] Note that we only do this for the purpose of calibration, not in our main simulations.

receiving an inheritance is an infrequent event in our data. Hence, albeit the fact that we have 163,369 observations, only 2,394 observed households (1.47 percent of our sample) received an inheritance in the sample period. In order to guarantee somewhat reliable estimates, we therefore use a coarser definition of age groups, namely 20-34, 35-44, 45-54, 55-64 and 65+ in what follows. For each earnings class e and age group, we calculate the fraction of the observed population in the GSOEP that actually received an inheritance. The results are shown in Table 7 and 8 in Appendix B. We again fit this data using cubic log-polynomials

$$q_t^e = \exp\left(\kappa_0^e + \kappa_1^e \cdot t + \kappa_2^e \cdot t^2 + \kappa_3^e \cdot t^3\right). \tag{15}$$

We weigh each moment in the residual sum of squares with the inverse of its standard error in order to control for the varying precision of our estimates. In addition, to reduce the degrees of freedom, we assume that polynomials across households of different earnings classes, but within the same education level (low or high), are only allowed to vary in the intercept κ_0. All other polynomial coefficients need to be identical for households of the same education level. Finally, we have to control for the fact that a large number of households in our sample is composed of a head and a spouse, and such couples tend to receive an inheritance twice in their lifetime, once from the head's parents and once from the spouse's parents. In order to make the estimated polynomials consistent with our model, we therefore standardize them with a factor of $1 + \varsigma^e$, where ς^e is the fraction of two-person households in each earnings class e in the GSOEP data. Figure 2 shows the resulting polynomials. The share of heirs in a

Figure 2: Estimated age-inheritance relationship for different earnings classes



cohort is the highest around ages 50 to 60, which is consistent with a roughly 30 year age difference between parents and children as well as a life expectancy of around 80 years. Higher educated households are more likely to receive an inheritance and tend to get it later in life, mirroring a higher average life expectancy of their (potentially high skilled) parents.

Note that the estimated polynomials represent the share of a cohort that receives an inheritance. In terms of our model, this share is a combination of the probability of the parents deceasing and the likelihood that they pass a positive inheritance to their offspring. Consequently, the polynomials identify

$$q_t^e = p_t^e \cdot \sum_{i=1}^{n} \omega_i^e = p_t^e \cdot (1 - \omega_0^e).$$

Using our structural assumption that parents cannot outlive their children, we immediately get

$$\sum_{t=1}^{T} q_t^e = (1 - \omega_0^e) \sum_{t=1}^{T} p_t^e \quad \Leftrightarrow \quad \omega_0^e = 1 - \sum_{t=1}^{T} q_t^e.$$

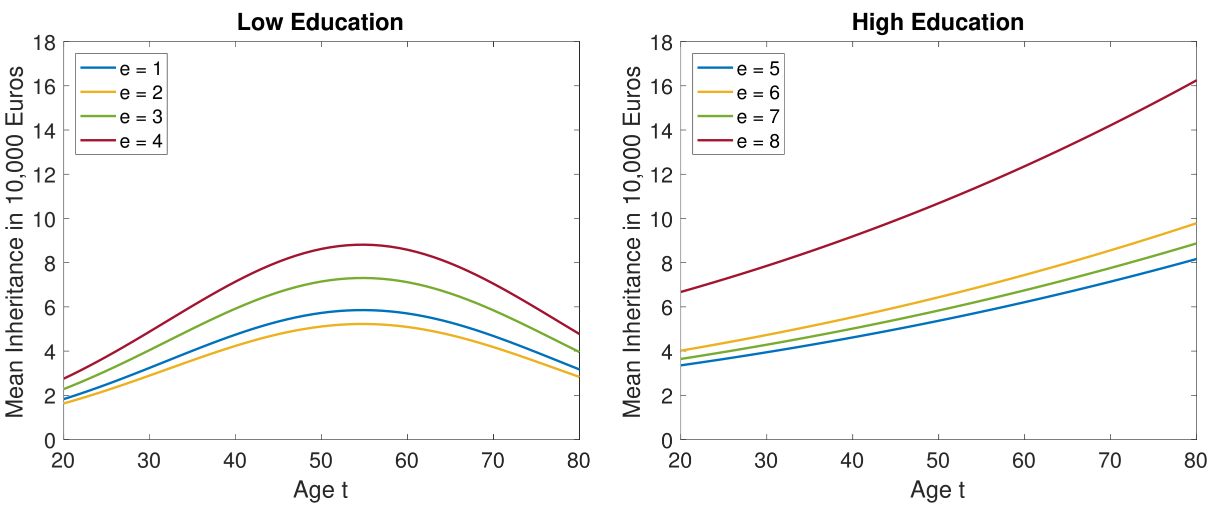
Furthermore, the probabilities of ancestral death are consequently given by

$$p_t^e = \frac{q_t^e}{\sum_{t=1}^{T} q_t^e}.$$

5.1.3 Bequest classes and bequest levels

In a last step, we have to determine the cross-sectional distribution over (positive) bequest classes ω_i^e, $i \in \{1, \ldots, n\}$ as well as the average bequest levels b_t^i. To this end, we first calculate mean bequests of households who received a positive inheritance for each age group and earnings class in the GSOEP, see Tables 7 and 8 in Appendix B. We again fit this data with cubic log-polynomials using the same methodology as described in the previous section. Figure 3 shows the resulting mean bequest profile by age and earnings level. Interestingly, the mean bequest profiles of the lower skilled

Figure 3: Estimated mean bequest profiles for different earnings classes



are hump-shaped over the life cycle, while those of the high skilled are strictly upward sloping. This could indicate that bequests of parents of lower skilled households tend to be accidental. Hence, if parents follow a regular life-cycle savings pattern and successively outlive their wealth at very high ages, bequests fall again. On the other hand, that bequests of parents of higher skilled households increase with the heirs' age indicates that parents consume less than their income speaking in favor of an active bequest motive. This is in line with the view of de Nardi et al. (2010), who model bequests as a luxury good.

In order to determine bequest levels in each bequest class i, we standardize the amount of inheritance of each household in the GSOEP who received a positive bequest by the age group and earnings class specific mean bequest level as reported in Tables 7 and 8. We then pool together all data for households of one education level, separate the data into quartiles and calculate the mean standardized bequest level for each of these quartiles. The resulting quartile means by education level are shown in Table 1. The

Table 1: Standardizes bequest quartile means by education

Education	Q1 ($i = 1$)	Q2 ($i = 2$)	Q3 ($i = 3$)	Q4 ($i = 4$)
Low	0.070	0.232	0.611	3.095
High	0.070	0.258	0.704	2.971

table reveals that the distribution of bequests within the group of heirs is very skewed. While the lowest quartile of heirs receives an average inheritance that amounts to 7 percent of the mean bequest level, the upper quartile's inheritance ranges around three times the mean. The distribution does not differ substantially across households of different education levels. We multiply the mean bequest profiles in Figure 3 with the factors in the above table in order to construct the bequest levels in each bequest class b^e_{it}. Since we divided bequests into quartiles, we set the cross-sectional distribution of households with positive inheritances over bequest classes to $\omega^e_i = 0.25 \cdot (1 - \omega^e_0)$.

5.2 Parameters, prices and government policy

Table 2 summarizes our choices for parameters, prices and government policy. Starting their life by the age of 20 $(t = 1)$ we let households live with certainty up to age 80 $(t = 61)$, which corresponds to the average life expectancy at birth of the German population. Retirement is mandatory at age 65.

We choose a time discount factor for the household of $\beta = 0.98$, such that the time preference rate is equal to the gross interest rate. We normalize $\lambda = 1$, which only has an implication for the endogenous labor productivity profiles w^e_t we estimate in the simplified model version in Section 5.1.1, but doesn't have an impact on our simulation results otherwise. We set the coefficient of risk aversion to $\gamma = 1$ and the labor supply elasticity parameter to $\chi = 4.37$, implying a Frisch elasticity of labor supply of 0.23.

Table 2: Parameters, prices and government policy

Parameter	Value	Note
T	61	Age of death = 80
t_r	46	Retirement age = 65
β	0.98	Time discount factor
λ	1	Coefficient for disutility of work
γ	1	Coefficient of risk aversion
χ	4.37	Frisch elasticity = 0.23
σ	0.75	Signal quality (benchmark)
r	2%	Interest rate
a_0	0	No initial wealth
a_{min}	$-\infty$	Only natural borrowing limit
$\mathcal{P}$	0.40	Pension = 40% of average gross income
τ_0	0.679	Average labor earnings tax rate
τ_1	0.128	Progressivity of labor tax
τ_k	0.25	Linear capital income tax
τ_b	0.00	Linear inheritance tax

Section 5.3 provides more details on how we jointly pin down the two. Finally, we set the signal quality to $\sigma = 0.75$ in our benchmark scenario. We, however, consider various other scenarios for σ in a sensitivity analysis.

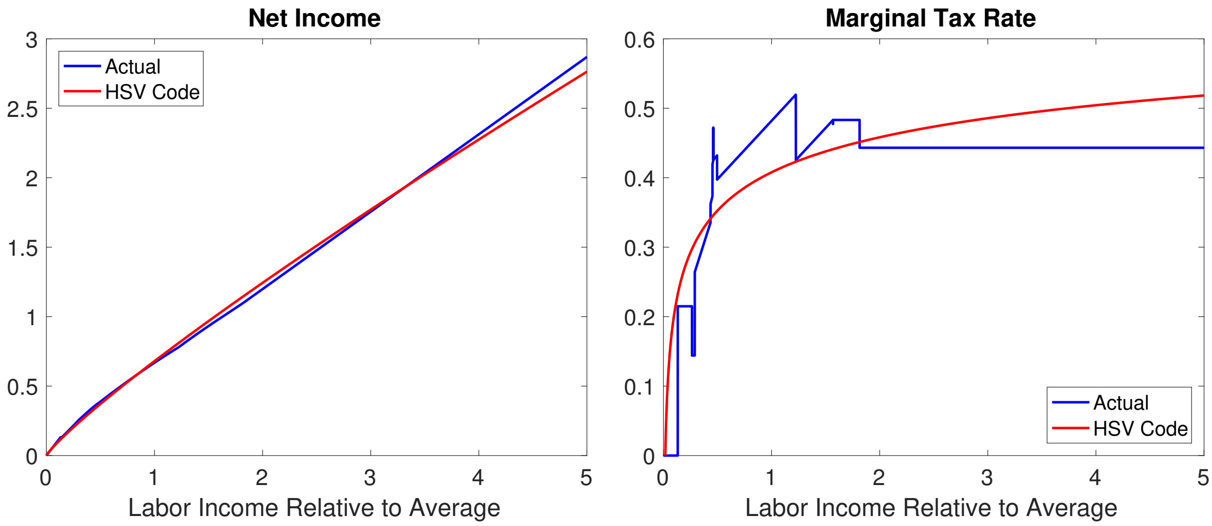
Taking a longer run perspective on savings, we take the annual interest rate to be 2%, which is equal to the current interest rate on outstanding household deposits with an agreed maturity of over two years. We furthermore assume that households start their life with zero own wealth. However, they might of course receive an inheritance early in life. Finally, we assume that the only borrowing limit the household faces is the natural borrowing limit, meaning that $a_{min} = -\infty$. We show in Appendix C that this choice provides the best fit for impulse responses to lottery gains, which we use to calibrate the extent of wealth effects on labor earnings in Section 5.3. This appendix also reveals that the policy implications do not depend starkly on this assumption.

Finally, we have to specify the tax and pension policy of the government. Starting with the latter, we set the replacement rate of pensions to 40% of average gross labor earnings over the life cycle, which matches the replacement rate reported by the OECD (2017). We calculate pension payments separately for households of different earnings classes, such that higher earners also receive a higher pension. With regard to labor income taxes, we use data on the mapping from gross into net income provided by Lorenz and Sachs (2016). We fit this data in a least squares sense using a functional form that was first proposed by Benabou (2002) and more recently applied by Heathcote et al. (2017). We therefore write net income as a function of gross income as

$$y_{net} = y - \mathcal{T}(y) = (1 - \tau_0) \cdot y^{1-\tau_1},$$

where τ_0 roughly captures the average tax rate of the system and τ_1 is an index for its progressivity. Figure 4 shows our original data as well as the fitted tax schedule. The parameter set that yields the best match is $\tau_0 = 0.321$ as well as $\tau_1 = 0.128$ with an R^2 value of 0.998. Last but not least, we set the flat capital income tax rate at $\tau_k = 0.25$,

Figure 4: Net Income and Marginal Tax Rates



which is equal to the statutory tax rate in Germany, and assume that in our benchmark simulation bequests are not taxed, which reflects very high exemption levels (400 000 Euro) for inheritances received from parents.

5.3 Pinning down wealth effects on labor earnings

In our model, the elasticity of labor earnings of a household with respect to an exogenous and unexpected change in wealth is given by

$$\eta_{y,t} = -\frac{W_t - a_{t+1} \cdot \eta_{a,t+1}}{\frac{\chi+\tau_1}{\gamma} \cdot c_t + (1-\tau_1) \cdot [y_t - \mathcal{T}(y_t)]},$$

where $\eta_{a,t+1}$ is the elasticity of savings into the next period with respect to current wealth, see Appendix D for a proof. With consumption in each period being a normal good, we can expect $a_{t+1}\eta_{a,t+1} < W_t$. Hence, labor earnings of a household unambiguously decline upon exogenous wealth changes. The extent of this decline depends both on the progressivity of the labor earnings tax schedule – measured by τ_1 – as well as on the preference parameters χ and γ. The greater is their ratio $\frac{\chi}{\gamma}$, the smaller we can expect the wealth effect on labor earnings to be.[14] Since we estimated τ_1 from the data, the only thing that remains to pin down the wealth effects on labor earnings are the preference parameters. Note that, if labor taxes were proportional ($\tau_1 = 0$), then the wealth effect on labor earnings would be solely identified by their ratio $\frac{\chi}{\gamma}$, which is not exactly true under a progressive tax system.

As outlined in the introduction, estimating the impact of inheritances on labor earnings is empirically difficult, as studies can be expected to produce only biased results. In particular, in the data – as in our model – inheritances are not a random and unexpected treatment. Instead, agents rater adjust their economic decisions (such as saving, consumption and labor supply) prior to their arrival, owing to an anticipation effect. A more reliable and convincing source of data comes from a recent study by Cesarini et al. (2017). They evaluate the effect of winning the lottery on individual labor earnings using a rich administrative data set of over 250,000 lottery winners in Sweden. Their empirical estimates indicate a marginal propensity to earn out of unearned income of -0.11 before labor taxes and social security contributions of employers. When including employer contributions this number declines to -0.14.[15]

In order to pin down the wealth effect on labor earnings in our model determined by the ratio between χ and γ, we directly use the evidence from Cesarini et al. (2017). More specifically, we randomly pay out lottery gains to our model households, using exactly the lottery size and age distribution provided in their Computational Online Appendix. We then calculate the reduction in labor earnings of all households in the first five years after they won the lottery, measured as a fraction of the amount gained. We target an average annual reduction in labor earnings of -1.07% of the lottery win. Our preferred choice of parameter that matches these targets is $\gamma = 1$ and $\chi = 4.37$. In our preference specification, this implies a value for the Frisch elasticity of labor supply of 0.23. This is within the range of estimates provided in MaCurdy (1981) and Altonji (1986) for prime age males. Blundell et al. (2016) find slightly higher values for the Frisch labor supply elasticity of males using a sample of married couples and values of around 1 for married females. Fiorito and Zanella (2012) reconcile the consistency between micro- and macro-level estimates.

A risk aversion of 1 and a Frisch labor supply elasticity of 0.23 both range at the lower end of the spectrum typically found in the life cycle and the macroeconomic literature. However, increasing both risk aversion and the Frisch labor supply elasticity to higher

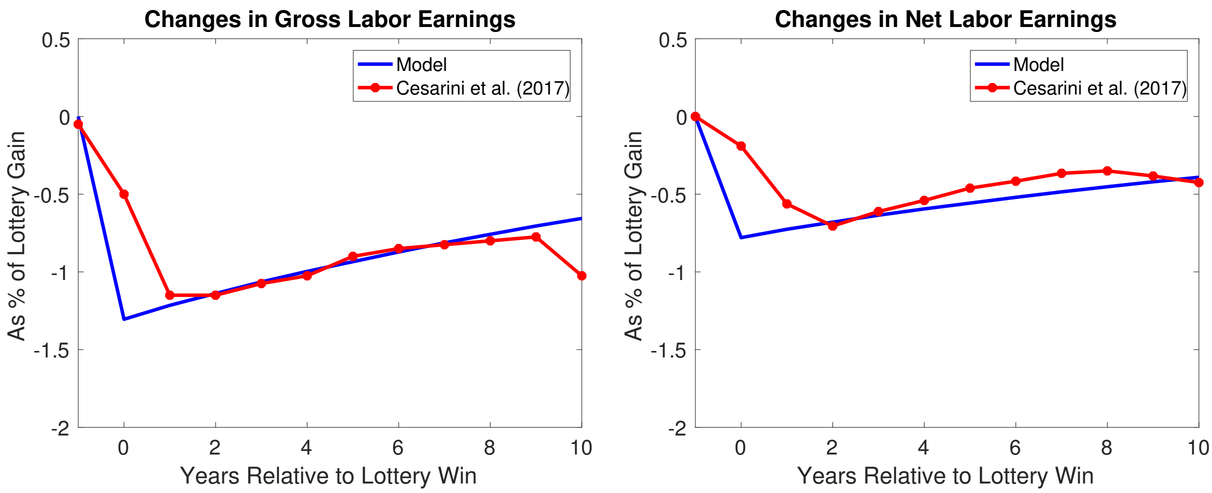
[14] Note that at this point it is easy to see that our choice of the level parameter λ does not influence the extend of labor earnings reactions to wealth changes, but only the level of labor effort l_t, which we do not necessarily have to interpret as labor hours.

[15] One concern of lottery studies typically is external validity, meaning that lottery players might be systematically different from the Swedish population at large. Cesarini et al. (2017) address this issue by pulling a random sample from the entire Swedish population, which can be done in Swedish register data. After reweighing this random sample to match the demographic characteristics of the sample of lottery winners, the authors find no significant difference in observable labor market characteristics between lottery players and the general population.

values would significantly increase the wealth effect on labor earnings, which would strongly enforce the labor tax revenue response to an increase in bequest taxes. However, this wealth effect would be inconsistent with empirical evidence. Yet, we provide some sensitivity checks with respect to our parameter choices in Section 6, where we set γ at a value smaller than 1, which directly implies a higher Frisch elasticity as well as a value of $\gamma = 4$, which implies a high risk aversion.

Figure 5 reports the average impulse response functions of gross and net labor earnings in our model for the first 10 years after a lottery win. Although we only targeted the average gross labor earnings response of households in the first five years after a lottery win to calibrate χ, both the gross as well as the (untargeted) net labor earnings response functions show a remarkably good fit with the impulse response data provided in Cesarini et al. (2017). This is of course only true starting from year one, the

Figure 5: Impulse Response Functions in Data and Model



year after the lottery gain, since lotteries are paid out at some date throughout year 0, which creates an upward bias in the labor supply response in the data. If at all, we slightly overestimate the net earnings response of individuals, indicating that either our average tax rate is too small or the employed tax code is not progressive enough. In either case, this only enforces our simulation results in the next section. Note further that, albeit the fact that we paired lottery evidence from Sweden with labor earnings data from Germany, we do get a good fit for both impulse responses in Figure 5, which makes us confident that we do provide valid estimates even with such a mixture of different data sources.[16]

[16] In future work, we plan to also estimate our model using register data on labor earnings and bequests from Sweden, which at the time this paper was written was not yet available to us.

6 Results

The policy experiment in our numerical simulation model is very similar to the one in the theoretical analysis. Specifically, we assume that the government unexpectedly increases the (proportional) tax rate on bequests by one percentage point. We start from a case without any inheritance taxes which reflects the large exemption levels for inheritance taxes in Germany. We, for now, focus on the effect such a tax increase has on the life cycle behavior of a generation that lives under the new bequest tax rate for all their life. In Section 6.4, we illustrate how to measure the effects on short-run generations, who get surprised by a bequest tax change at some date in the middle of their life cycle.

The column *Total* of Table 5 shows the effect of a one percentage point bequest tax increase on the labor earnings and labor tax payments of one cohort. In particular, we evaluate the change in the expected present value of labor earnings and labor tax payments of one generation and relate it to the change in this generation's expected present value of bequest tax payments. The resulting number can be interpreted as the excess tax revenue effect of a change in the bequest tax rate in the spirit of Corollary 2. We find that a one percentage point bequest tax increase leads to an increase in gross earnings of 18.5 cents for each Euro of additional bequest tax payments. This results in a labor tax revenue increase of more than 7.5 cents.

Table 3: Effect of a 1% increase in bequest taxes

		Decomposition		
	Total	*Anticipation*	*Heirs*	*Non-Heirs*
Gross Earnings	18.50	9.57	9.60	−0.67
Labor Taxes	7.64	3.93	3.98	−0.27

Effects are measured as fraction of change in bequest tax revenue.

Our theoretical analysis has shown that the present value of labor earnings and labor tax changes can be decomposed into three components, confer (12):

1. Labor supply of heirs increases owing to the direct negative wealth effect induced by a bequest tax increase.
2. The anticipation effect causes households to smooth their labor earnings reaction over the life cycle and leads to higher labor earnings and tax payments already prior to the arrival of an inheritance.
3. As the anticipation effect involves an increase in savings, the resulting negative wealth effect on older cohorts mitigates the earnings reaction fo heirs and leads to a decline in labor earnings for non-heirs.

The extent of these effects is shown in the last three columns of Table 5. Both in terms of labor earnings as well as in terms of tax payments the anticipation effect plays an equally important role as the wealth effect on heirs. From this follows that, if we were to treat changes in bequests and bequest taxes as totally exogenous and unanticipated and would therefore only look at the impact on heirs, we would suffer from a serious downward bias in the tax revenue effect, leading us to an estimate of around 4 cents instead of 7.5 for excess labor tax revenue. The impact on non-heirs, which is a result of increased savings, is only modest and reduces the overall excess labor tax payment by only around 0.3 cents.

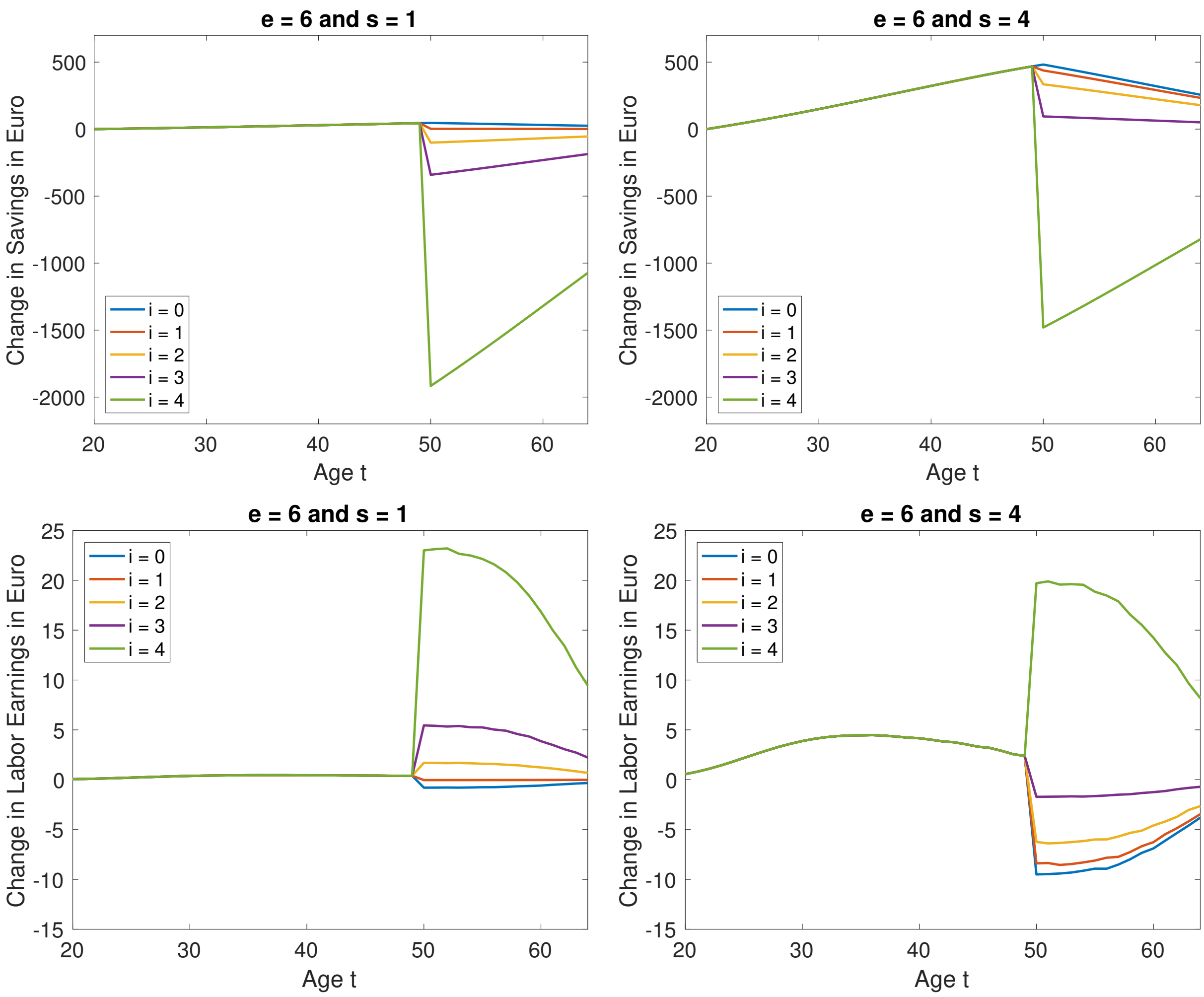
6.1 Illustrating the Mechanism

We now want to elaborate a bit more on the mechanism at work. To this end, Figure 6 shows the change in life cycle savings (upper panels) and earnings (lower panels) in Euro values that results from the one percentage point increase in bequest taxes. As an example, we picked households from a moderate earning class ($e = 6$), who's parents die at the age of 50. On the left hand side, we plot life-cycle graphs for agents who are endowed with a signal of $s = 1$ at the beginning of the life cycle, and therefore only expect a very small inheritance. The right hand side shows the same plots for households with a signal of $s = 4$, who consequently expect their inheritance to fall into class $i = 4$ with probability 0.78 (for a signal quality of $\sigma = 0.75$). The different lines denote the actual inheritance the household receives $i = 0, \ldots, 4$.

The figure shows that upon the increase in bequest taxes, both household types – those with a low and those with a high signal – increase their savings throughout the life cycle, up to the point where they receive an inheritance. Since households with a high signal expect a larger inheritance and therefore experience a greater wealth effect (at least in expectation), their savings reaction is much more pronounced than for the low signal households. Once the inheritance is received, on the other hand, savings typically drop below steady state levels, which is a direct result of the negative wealth effect induced by the bequest tax.

The lower panels of Figure 6 illustrate the importance of the anticipation effect, which first and foremost causes labor earnings to already increase prior to the date at which the household receives an inheritance. As with life-cycle savings, for individuals who expect a large inheritance ($s = 4$), this effect is much more pronounced than for agents with a low signal. Yet. the anticipation effect has a second component: It dampens the labor earning reaction in case the agent receives an inheritance that is greater than her expected inheritance level and causes labor earnings to fall below initial steady state levels in case the expected inheritance is small. Of course, the household endowed with signal $s = 4$ has a much higher expectation than the one with $s = 1$. Hence, labor earnings of the former fall for all inheritance levels but $i = 4$.

Figure 6: Change in life-cycle behavior of different households



6.2 Heterogeneity of Effects

Table 4 shows the effects of a one percentage point increase in the bequest tax for households of different earnings classes. In order to control for differences in expected bequests, we normalize the earnings and labor tax effects using the expected present value of bequest tax payments for each earnings level. We find a substantial amount of heterogeneity across labor productivity groups. Specifically, within each education group, higher earnings class households exhibit a greater reaction in labor supply. This relationship can be understood by realizing that the intratemporal first order condition in our model implies

$$y_t = \left[\frac{1 - \mathcal{T}'(y_t)}{\lambda}\right]^{\frac{1}{\chi}} \cdot w_t^{1+\frac{1}{\chi}} \cdot (c_t)^{-\frac{\gamma}{\chi}},$$

see (16) in Appendix D. From this follows that for any decline in consumption c_t (which would be the result of a bequest tax increase), a household with a higher labor produc-

Table 4: Effect of a 1% increase in bequest taxes by Earnings-Class

	Low Education				*High Education*			
$e =$	1	2	3	4	5	6	7	8
Earnings	12.99	17.90	18.66	21.11	13.31	16.60	19.69	20.36
Taxes	3.99	6.59	7.38	9.12	4.64	6.63	8.37	9.43

Effects are measured as fraction of change in bequest tax revenue by earnings class.

tivity will always increase her labor earnings to a greater extent than an agent with low labor productivity.

In economic terms, a higher labor productivity allows a household to counteract changes in exogenous income much easier than an agent with low labor productivity, since a one unit change in labor hours just leads to a much higher change in earnings for the former than for the latter. Or put it differently, a one hour reduction in leisure due to lower wealth translates into a larger increase in earnings and therefore consumption the larger the hourly wage is. Note that the heterogeneity in labor tax changes is larger than the heterogeneity in earnings effects across earnings classes. The reason is that, owing to the progressive labor tax schedule, households with higher labor productivity face much higher marginal tax rates.

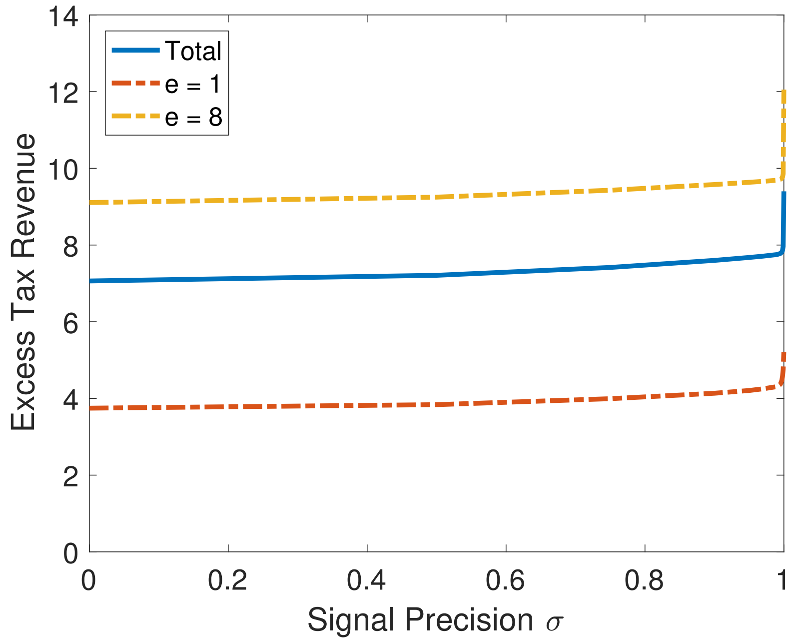
6.3 The Role of Signal Quality

In our benchmark simulation, we chose a signal quality of $\sigma = 0.75$. Figure 7 shows the sensitivity of our results with respect to this signal quality.[17] Recall that for $\sigma = 0$, the signal contains no information and all households use the cross-sectional distribution of bequests in their earnings class to forecast the size of their inheritance. For $\sigma = 1$, the signal is fully informative and households know exactly in which class their inheritance is going to fall. On the vertical axis of the figure, we again report the excess labor tax effect per unit of additional bequest tax revenue, when we increase the bequest tax rate by one percentage point. We find that, for any $\sigma \ll 1$, labor taxes increase by the same amount of roughly 7.5 cents per Euro of additional bequest tax revenue, regardless of the quality of the signal.

Only when the signal quality approaches 1, this suddenly changes and the excess labor tax revenue increases to almost 10 cents. The reason for this can be found in the natural borrowing constraint (Aiyagari, 1994) of a household. Whenever the signal is less than fully informative, a household can make some forecast about her future inheritance. Yet, there still is the possibility that the agent ends up inheriting nothing.

[17] Note that we only vary signal quality and do not recalibrate the labor supply elasticity parameter χ. We however checked for certain combinations that our results also hold under recalibration of χ.

Figure 7: Varying Signal Quality

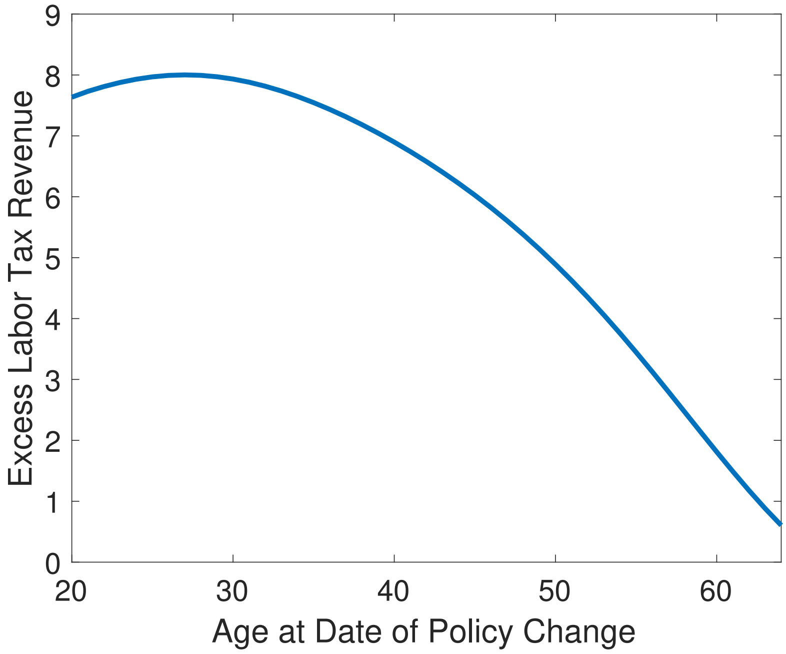


Households would obviously like to distribute the benefits of the expected bequest (that are typically received around the age of 50 to 60) evenly over the life cycle. Those with a higher expected inheritance might therefore even run into debt against future bequest transfers. The amount of debt they can hold is limited by the natural borrowing constraint. In case there is even a slight chance of inheriting nothing, the agent has to make sure that she can still service her debt in case she gets no bequest from her parents. Hence, her natural borrowing limit is relatively tight, even if on average she expects a large bequest. This only changes with a fully informative signal. In this case, the only remaining uncertainty is the uncertainty about timing. But eventually, every household with a positive signal will receive a positive bequest. Hence, life-cycle smoothing works much better in this scenario, as the natural borrowing constraint is relaxed. As a result, agents who have a high expectation about bequests will also react much stronger to changes in bequest taxes. In Figure 7 this fact can be seen when comparing the change in excess labor taxes for households from a low earnings class, who on average have low expectations about inheritances, with those from a high earnings class.

6.4 The Short vs. the Long Run

So far, we only focused on the effect of a change in the bequest tax rate on a cohort that has lived under the new bequest tax rate for their whole life. However, as already pointed out in the theoretical analysis, there is a difference between such cohorts and generations that are surprised by a change in bequest taxes at some date in the middle of their life cycle. In the following, we therefore conduct the same thought experiment as in our theoretical analysis. We assume that the economy is in a steady state with a bequest tax rate of 0%. Then, the government surprisingly increases the bequest tax rate by one percentage point. Figure 8 then shows the excess labor tax effect on cohorts with different ages at the time of the reform. Of course, for the cohort aged 1, we again get the very same number as in previous sections, as this cohort is the one that lives under the new tax system for their whole life span.

Figure 8: Short-run vs. Long-run Effects



The older a cohort is at the time the bequest tax rate changes, the less years of work remain to react to the tax change. Consequently, the excess labor tax effect declines in a cohort's age almost everywhere. Only for very young cohorts, we see a slight increase in excess tax revenue, which is due to a denominator effect. Since bequests are most likely to arrive at later ages, the labor earnings effect for cohorts between ages 20 and 30 at the time of the reform is almost identical. However, as some inheritances do arrive at these ages, the present value of bequest tax revenue (the denominator in the excess tax revenue effect) decreases in age, which causes the overall excess labor tax effect to increase slightly.

6.5 Sensitivity Analysis

As discussed in section 5.3, we have two parameters, the coefficient of relative risk aversion γ and the inverse of the Frisch elasticity of labor supply χ to match one target, the propensity to earn out of lottery gains in the five years following the lottery win. Our benchmark calibration of $\gamma = 1$ and $\chi = 4.37$ implies that both risk aversion and the Frisch elasticity of labor supply are in the range of empirical estimates, even though both are at the lower end of this range. In this section we provide robustness checks to this choice. Specifically, we consider the case of a relatively high Frisch elasticity of 0.5 ($\chi = 2.0$). In order for the model to match the lottery evidence on labor earnings, this yet implies that risk aversion needs to be extremely low ($\gamma = 0.475$). Similarly, we consider the other extreme case of a high risk aversion ($\gamma = 4.0$), even though this implies an extremely low Frisch labor supply elasticity of 0.06 ($\chi = 17.9$). For each of these calibrations we compute the effect of a marginal increase in bequest taxes on labor earnings and excess labor income taxes. Table 5 summarized the results.

Table 5: Effect of a 1% increase in bequest taxes

	$\gamma = 0.475$ *and* $\chi = 2.0$			
	Total	*Anticipation*	*Heirs*	*Non-Heirs*
Gross Earnings	19.36	10.75	9.37	–0.77
Labor Taxes	7.98	4.41	3.88	–0.31
	$\gamma = 4.0$ *and* $\chi = 17.9$			
[0.5ex]	*Total*	*Anticipation*	*Heirs*	*Non-Heirs*
Gross Earnings	15.79	5.60	10.57	–0.38
Labor Taxes	6.54	2.30	4.39	–0.16

Effects are measured as fraction of change in bequest tax revenue.

Despite the very different parameterizations, our number of interest is affected only modestly in both cases. In the case of a high labor supply elasticity and very low risk aversion, it increases by less than half a cent to 7.98, while in the case of high risk aversion and very low labor supply elasticity, it decreases by a bit more than one cent to 6.54. In general, the more elastic labor supply the higher the overall effect on labor earnings and hence tax revenue. We further observe that for the parameterization with high risk aversion, anticipation effects decline while post receipt effects increase. Under a high γ, a larger amount of savings is due to a precautionary motive. An increase in the bequest tax rate reduces not only the expected value of future bequests (increasing savings) but also the variance of potential bequests (reducing savings). With higher precautionary savings this second effect is more important. Lower precautionary savings prior to the receipt of inheritances in turn are financed with lower labor earnings. This, on the other hand, decreases wealth at the time of inheritance receipt further, triggering stronger responses in labor supply thereafter.

7 Conclusion

In this paper we elaborate one particular channel of how a change in inheritance taxes affects tax revenue: labor supply of the heirs. We quantify this effect through the lens of a state of the art life-cycle labor supply that is calibrated to match clean quasi-experimental evidence on wealth effects on labor supply. We show that this effect is positive and likely to be sizable and should therefore be taken into account in dynamic scoring exercises where revenues of such tax changes are simulated.

One margin that we were not accounting for and which could make the effect stronger is the education margin. It is likely that individuals do not only make their labor supply decisions conditional on their expectations about inheritances but also their education decisions. In that sense, an increase in inheritance taxes could also imply a positive effect on education of heirs which would imply another positive effect on labor income tax revenue

As we illustrated in our theoretical analysis, inheritance taxes are also likely to affect of labor supply of the bequeathers. This is an other channel that should be studied in future research.

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Appendix

A Proofs for 2 Period OLG model

A.1 Proof of Proposition 1

Let us assume that our model is in a steady state, meaning that all variables are constant over time. We will work ourselves backwards through the model, starting with period 2 of the household choice problem.

The household problem in period 2 Given a certain level of household savings a, a household of type $K = I, N$ maximizes her remaining life time utility given her instantaneous budget constraint. It is useful to write the optimization problem in terms of labor earnings $y_2^K = w_2 l_2^K$ as

$$\max_{c_2^K, y_2^K, b^K} \; v\left(c_2^K, \frac{y_2^K}{w_2}, (1-\tau_b)b^K\right)$$
$$\text{s.t.} \quad c_2^K + b^K \leq (1-\tau_l)y_2^K + (1+r)a + \mathbb{1}_{K=I}(1-\tau_b)b + T_2$$

Let's for expositional purposes write the net bequest level a household leaves to her descendants as $b_{net}^K = (1-\tau_b)b^K$. The first order conditions of the optimization problem then read

$$-\frac{v_l\left(c_2^K, \frac{y_2^K}{w_2}, b_{net}^K\right)}{w_2(1-\tau_l)} = v_c\left(c_2^K, \frac{y_2^K}{w_2}, b_{net}^K\right) = (1-\tau_b)v_b\left(c_2^K, \frac{y_2^K}{w_2}, b_{net}^K\right).$$

Using the implicit function theorem, we get

$$\begin{aligned}
&\left[v_{cc} + \frac{v_{lc}}{w_2(1-\tau_l)}\right] dc_2^K + \left[v_{cb} + \frac{v_{lb}}{w_2(1-\tau_l)}\right] db_2^K \\
&= -\left\{\left[\frac{v_{cl}}{w_2(1-\tau_l)} + \frac{v_{ll}}{[w_2(1-\tau_l)]^2}\right](1-\tau_l)dy_2^K + \left[v_{cb}\cdot\frac{b^K}{b} + \frac{v_{lb}\cdot\frac{b^K}{b}}{w_2(1-\tau_l)}\right] d(1-\tau_b)\cdot b\right\}
\end{aligned}$$

as well as

$$\begin{aligned}
&\left[(1-\tau_b)v_{bc} + \frac{v_{lc}}{w_2(1-\tau_l)}\right] dc_2^K + \left[(1-\tau_b)^2 v_{bb} + \frac{(1-\tau_b)v_{lb}}{w_2(1-\tau_l)}\right] db_2^K \\
&\qquad = -\left\{\left[\frac{(1-\tau_b)v_{bl}}{w_2(1-\tau_l)} + \frac{v_{ll}}{[w_2(1-\tau_l)]^2}\right](1-\tau_l)dy_2^K\right. \\
&\qquad\qquad \left. + \left[(1-\tau_b)v_{bb}\cdot\frac{b^K}{b} + \frac{v_{lb}\cdot\frac{b^K}{b}}{w_2(1-\tau_l)} + v_b\cdot\frac{b^K}{b}\right] d(1-\tau_b)\cdot b\right\}.
\end{aligned}$$

Note that we use v_{xy} as abbreviation for $v_{xy}\left(c_2^K, \frac{y_2^K}{w_2}, b_{net}^K\right)$.

These two equations constitute a linear equation system in dc_2^K and db_2^K, which (under some regularity assumptions) has a unique solution

$$\begin{bmatrix} dc_2^K \\ db^K \end{bmatrix} = - \begin{bmatrix} \xi_{2cy}^K & \xi_{2c\tau}^K \\ \xi_{2by}^K & \xi_{2b\tau}^K \end{bmatrix} \cdot \begin{bmatrix} (1-\tau_l)dy_2^K \\ d(1-\tau_b)\cdot b \end{bmatrix}$$

Assuming that no resources are put to waste, total differentiation of the budget constraint yields

$$dc_2^K + db^K = (1-\tau_l)dy_2^K + (1+r)da + \mathbb{1}_{i=k} \cdot d\left[(1-\tau_b)b\right] + dT_2$$

which under substitution of the above relationships brings us to

$$dy_2^K = \frac{-(1+r)da - \mathbb{1}_{i=k} \cdot d\left[(1-\tau_b)b\right] - dT_2 - \left(\xi_{2c\tau}^K + \xi_{2b\tau}^K\right) \cdot d(1-\tau_b) \cdot b}{(1-\tau_l)\left[1 + \xi_{2cy}^K + \xi_{2by}^K\right]}.$$

From this relationship, we directly see that the labor earnings reaction to a pure change in exogenous income dT_2, keeping savings da, bequests received $d\left[(1-\tau_b)b\right]$ and the net-of-tax rate $d(1-\tau_b)$ fixed, is

$$\left.\frac{dy_2^K}{dT_2}\right|_{da=0} = -\frac{1}{(1-\tau_l)\left[1 + \xi_{2cy}^K + \xi_{2by}^K\right]} =: -\eta_2^K.$$

At the same time, we immediately get with $dT_2 = 0$ that

$$dy_2^K = \eta_2^K \cdot \left\{ -\mathbb{1}_{i=k} \cdot d\left[(1-\tau_b)b\right] - (1+r)da - \left(\xi_{2c\tau}^K + \xi_{2b\tau}^K\right) \cdot d(1-\tau_b) \cdot b \right\}$$

from which follows that

$$\begin{aligned} \frac{dy_2^K}{d(1-\tau_b)\cdot b} &= \eta_2^K \cdot \frac{d\left[(1-\tau_b)b\right]}{d(1-\tau_b)\cdot b} \left\{ -\mathbb{1}_{i=k} - \frac{(1+r)da}{d\left[(1-\tau_b)b\right]} \right\} - \eta_2^K \cdot \left(\xi_{2c\tau}^K + \xi_{2b\tau}^K\right) \\ &= \eta_2^K \cdot (1+\varepsilon) \cdot \left[-\mathbb{1}_{i=k} + \alpha\right] - \eta_2^K \cdot \left(\xi_{2c\tau}^K + \xi_{2b\tau}^K\right), \end{aligned}$$

with ε being the elasticity of total bequests b received by the household with respect to the net of tax rate $1-\tau_b$. Let us further define $\xi_\tau^K = -\left(\xi_{2c\tau}^K + \xi_{2b\tau}^K\right)$, which measures the effect of a change in the net-of-tax-rate $1-\tau_b$ on the willingness of a household to bequeath to her own descendants. Then by substituting ξ_τ^K into the above equation, we obtain the second part of (7).

The household problem in period 1 Let us define

$$V(a) = \pi \cdot \max_{c_2^I, y_2^I, b^I} v\left(c_2^I, \frac{y_2^I}{w}, b^I\right) + (1-\pi) \max_{c_2^N, y_2^N, b^N} v\left(c_2^N, \frac{y_2^N}{w}, b^N\right)$$

subject to the second period budget constraints. Then, using Bellman's principle of optimality, we can write the first period optimization problem as

$$\max_{c_1, y_1, a} u\left(c_1, \frac{y_1}{w_1}\right) + \beta V(a) \quad \text{s.t.} \quad c_1 + a = (1-\tau_l)y_1 + T_1.$$

The first order conditions with respect to c_1 and y_1 read

$$-\frac{u_l\left(c_1, \frac{y_1}{w}\right)}{w(1-\tau_l)} = u_c\left(c_1, \frac{y_1}{w}\right).$$

Using the implicit function theorem yields

$$dc_1 = -\underbrace{\frac{u_{ll} + [w_1(1-\tau_l)] \cdot u_{cl}}{[w_1(1-\tau_l)]^2 \cdot u_{cc} + [w_1(1-\tau_l)] \cdot u_{lc}}}_{=:\xi_{c1}} \cdot (1-\tau_l)dy_1.$$

Assuming that no resources are put to waste, total differentiation of the budget constraint yields

$$dc_1 + da = (1-\tau_l)dy_1 + dT_1$$

which under substitution of the above relationships brings us to

$$dy_1 = -\frac{dT_1 - \frac{(1+r)da}{1+r}}{(1-\tau_l)\left[1+\xi_{c1}\right]}.$$

From this relationship, we directly see that the labor earnings reaction to a pure change in exogenous income is

$$\left.\frac{dy_1}{dT_1}\right|_{da=0} = -\frac{1}{(1-\tau_l)\left[1+\xi_{c1}\right]} =: -\eta_1.$$

At the same time, we immediately get with $dT = 0$ that

$$\begin{aligned} \frac{dy_1}{d(1-\tau_b)\cdot b} &= -\frac{\eta_1}{1+r} \cdot \frac{d\left[(1-\tau_b)b\right]}{d(1-\tau_b)\cdot b} \cdot \left[-\frac{(1+r)da}{d\left[(1-\tau_b)b\right]}\right] \\ &= -\frac{\eta_1}{1+r} \cdot (1+\varepsilon) \cdot \alpha. \end{aligned}$$

□

A.2 Proof of Proposition 2

The total differential of the life time tax revenue (5) of a generation born at date t is

$$dR_t = \tau_l \cdot \left[dy_{1t} + \frac{\pi dy_{2t+1}^I + (1-\pi)dy_{2t+1}^N}{1+r}\right] + \frac{\pi d\left[\tau_b b_{t+1}\right]}{1+r}.$$

Note that we made the assumption that neither the labor earnings tax rate nor lump-sum transfers are affected by the change in $d\tau_b$. We can write this equation as

$$dR_t = \frac{\pi d\left[\tau_b b_{t+1}\right]}{1+r} \cdot \Bigg\{ 1 + \tau_l \cdot \frac{d(1-\tau_b)\cdot b_{t+1}}{d\left[\tau_b b_{t+1}\right]} \cdot \frac{1+r}{\pi} \cdot \left[\frac{dy_{1t}}{d(1-\tau_b)\cdot b_{t+1}} + \frac{\pi \frac{dy^I_{2t+1}}{d(1-\tau_b)\cdot b_{t+1}} + (1-\pi)\frac{dy^N_{2t+1}}{d(1-\tau_b)\cdot b_{t+1}}}{1+r} \right] \Bigg\}.$$

We then obtain

$$\begin{aligned}
&\frac{1+r}{\pi} \cdot \left[\frac{dy_{1t}}{d(1-\tau_b)\cdot b_{t+1}} + \frac{\pi \frac{dy^I_{2t+1}}{d(1-\tau_b)\cdot b_{t+1}} + (1-\pi)\frac{dy^N_{2t+1}}{d(1-\tau_b)\cdot b_{t+1}}}{1+r} \right] \\
&= \frac{1+r}{\pi} \cdot \Bigg[-\frac{\eta_1(1+\varepsilon_{t+1})}{1+r} \cdot \alpha \\
&\quad + \frac{\pi\left[\eta^I_2(1+\varepsilon_{t+1})\left[-1+\alpha\right] + \eta^I_2 \cdot \xi^I_\tau\right] + (1-\pi)\left[\eta^N_2(1+\varepsilon_{t+1})\alpha + \eta^N_2 \xi^N_\tau\right]}{1+r} \Bigg] \\
&= -\frac{1}{\pi} \cdot \Bigg\{ (1+\varepsilon_{t+1})\left[\eta_1 \cdot \alpha + \pi\left[\eta^I_2 - \alpha\eta^I_2\right] + (1-\pi)\left[\alpha\eta^N_2\right]\right] \\
&\qquad - \left[\pi\eta^I_2\chi^I_\tau + (1-\pi)\eta^N_2\chi^N_\tau\right] \Bigg\}
\end{aligned}$$

Furthermore we get

$$\begin{aligned}
\frac{d(1-\tau_b)\cdot b_{t+1}}{d\left[\tau_b b_{t+1}\right]} &= \frac{d(1-\tau_b)\cdot b_{t+1}}{\tau_b db_{t+1} + d\tau_b b_{t+1}} = \frac{d(1-\tau_b)\cdot b_{t+1}}{\tau_b db_{t+1} - d(1-\tau_b)b_{t+1}} \\
&= \frac{1}{\frac{\tau_b}{1-\tau_b}\cdot\frac{(1-\tau_b)db_{t+1}}{d(1-\tau_b)b_{t+1}} - 1} = -\frac{1}{1-\frac{\tau_b}{1-\tau_b}\cdot\varepsilon_{t+1}}.
\end{aligned}$$

Putting all of this together yields (9).

The equation for the cohort born at time $s-1$, i.e. right before the bequest tax is increased, then simply follows from the fact that this cohort has – by definition – a savings reaction of $\alpha = 0$ and at the same time bequests are predetermined $\varepsilon_s = 0$. □

B Calibration data extracted from GSOEP

Table 6: Mean labor earnings in different earnings classes

	Low Education				High Education			
Age	$e = 1$	$e = 2$	$e = 3$	$e = 4$	$e = 5$	$e = 6$	$e = 7$	$e = 8$
20-24	3,126	8,947	16,061	31,182	2,676	9,070	19,407	36,026
25-29	6,342	16,614	26,748	42,639	7,274	21,607	35,064	55,638
30-34	11,544	23,854	32,762	50,884	18,828	34,868	46,228	73,596
35-39	13,965	26,082	34,988	52,340	22,071	38,341	50,761	81,618
40-44	15,216	27,946	37,049	56,708	22,313	39,453	53,004	89,428
45-49	14,184	27,929	38,173	59,408	22,582	40,171	54,511	94,091
50-54	12,547	26,578	37,469	60,999	21,083	40,803	56,316	98,965
55-59	10,328	22,015	33,568	58,279	15,927	36,203	53,249	96,778
60-64	9,002	15,500	23,521	45,613	12,640	26,474	42,283	76,568
65+	8,527	13,122	16,634	28,023	10,756	16,888	22,562	45,823
Share	0.179	0.179	0.179	0.179	0.071	0.071	0.071	0.071

Table 7: Fraction of heirs and mean bequest level by earnings class (low education)

Age	Frac. Heirs (in %)	Mean Bequest	Frac. Heirs (in %)	Mean Bequest
	$e = 1$		$e = 2$	
20-34	0.84	26,579	0.61	53,812
	(0.14)	(9,659)	(0.11)	(16,780)
35-44	0.81	39,176	1.19	31,761
	(0.13)	(10,543)	(0.15)	(6,165)
45-54	1.11	68,150	1.08	49,147
	(0.15)	(15,992)	(0.15)	(8,699)
55-64	1.25	52,864	1.17	51,501
	(0.18)	(10,495)	(0.16)	(8,282)
65+	0.60	46,869	0.52	46,197
	(0.09)	(9,562)	(0.08)	(11,311)
	$e = 3$		$e = 4$	
20-34	1.43	23,577	1.20	73,607
	(0.17)	(5,573)	(0.16)	(18,286)
35-44	0.92	73,587	1.37	52,417
	(0.14)	(20,388)	(0.16)	(15,080)
45-54	1.89	63,092	1.92	131,542
	(0.19)	(18,833)	(0.17)	(26,858)
55-64	1.54	93,182	2.51	70,160
	(0.18)	(16,922)	(0.21)	(10,216)
65+	0.58	47,055	1.04	62,391
	(0.09)	(9,451)	(0.11)	(17,901)

Standard errors are reported in parenthesis.

Table 8: Fraction of heirs and mean bequest by earnings class (high education)

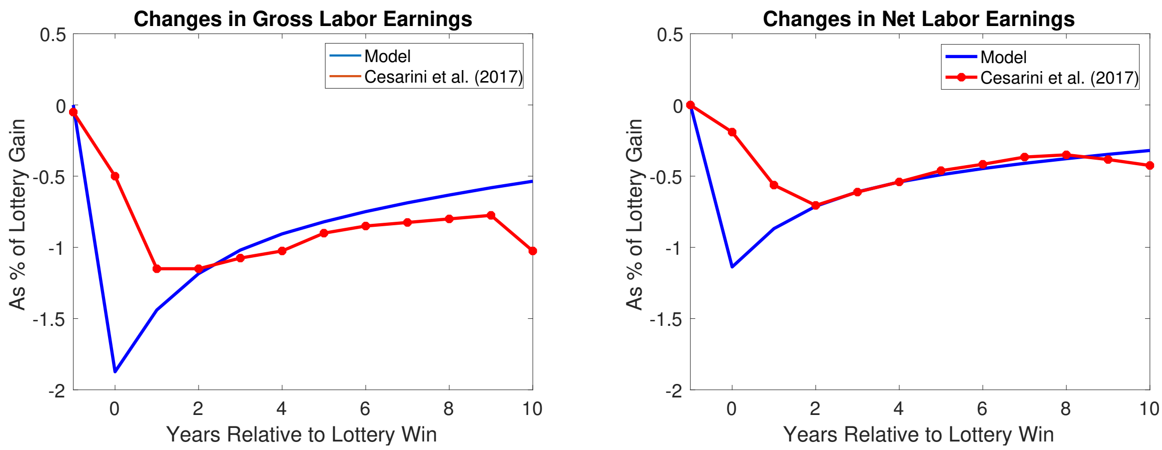
Age	Frac. Heirs (in %)	Mean Bequest	Frac. Heirs (in %)	Mean Bequest
	$e = 5$		$e = 6$	
20-34	1.73	72,007	1.14	33,552
	(0.34)	(28,507)	(0.26)	(10,246)
35-44	0.81	46,598	1.22	35,946
	(0.18)	(16,519)	(0.21)	(9,806)
45-54	2.38	54,616	1.67	68,809
	(0.31)	(10,300)	(0.24)	(18,128)
55-64	2.04	55,539	3.11	94,364
	(0.31)	(12,675)	(0.36)	(16,702)
65+	1.13	69,136	0.88	103,915
	(0.21)	(15,121)	(0.17)	(26,950)
	$e = 7$		$e = 8$	
20-34	2.03	281,532	2.05	81,609
	(0.36)	(107,188)	(0.38)	(22,610)
35-44	1.47	31,910	1.85	95,899
	(0.23)	(5,146)	(0.25)	(16,113)
45-54	2.68	55,250	2.50	112,098
	(0.28)	(11,426)	(0.25)	(24,719)
55-64	2.75	97,200	3.87	127,256
	(0.33)	(16,277)	(0.33)	(38,036)
65+	2.33	76,044	2.52	133,747
	(0.28)	(12,190)	(0.27)	(22,585)

Standard errors are reported in parenthesis.

C Borrowing limits

In our benchmark calibration we assumed that $a_{min} = -\infty$, i.e. as long as households can service their debt until they die, they can run into debt as much as they wish. In this section we perform the analysis for the other extreme case when no borrowing is allowed ($a_{min} = 0$). We again fix the parameter $\gamma = 1$ and re-calibrate χ such that the model replicates the evidence on earnings responses of lottery winners (Cesarini et al., 2017). Specifically, we need to increase the inverse of the Frisch elasticity to $\chi = 4.50$ such that in the five years following the lottery win, agents reduce their gross earnings on average by 1.07% of the lottery amount. Figure 9 again compares the average impulses of gross and net earnings in data and the model.

Figure 9: Impulse Response Functions in Data and Model with Strict Borrowing Limit



We observe that in the case of a strict no-borrowing limit the response is too strong in the first years following the lottery win, and too weak in later years (at least for gross earnings). The reason for the difference to the natural borrowing limit (compare Figure 5) is that in the no-borrowing case agents at the beginning of their economic life are (without a lottery win or inheritance) not able to borrow against their future income in order to smooth consumption. Instead they finance their whole consumption via labor earnings. Hence, labor earnings are higher than in the case without a constraint on borrowing. If an agent now receives an early lottery, it is not only an income effect that makes her work less but also the relaxation of the borrowing constraint. Further, in the no-borrowing case the limited ability to postpone labor to later periods (when wages are higher), not only makes them work more in early periods but also less in later periods compared to the case of unlimited borrowing. As a consequence with a strict no-borrowing limit a lottery win makes - on average - labor earnings drop a bit less in the years 4-10 after the lottery win than in the case of unlimited borrowing. Overall, this analysis suggests that borrowing constraints do not play a big role or, at least, that a strict no-borrowing limit is a too extreme assumption.

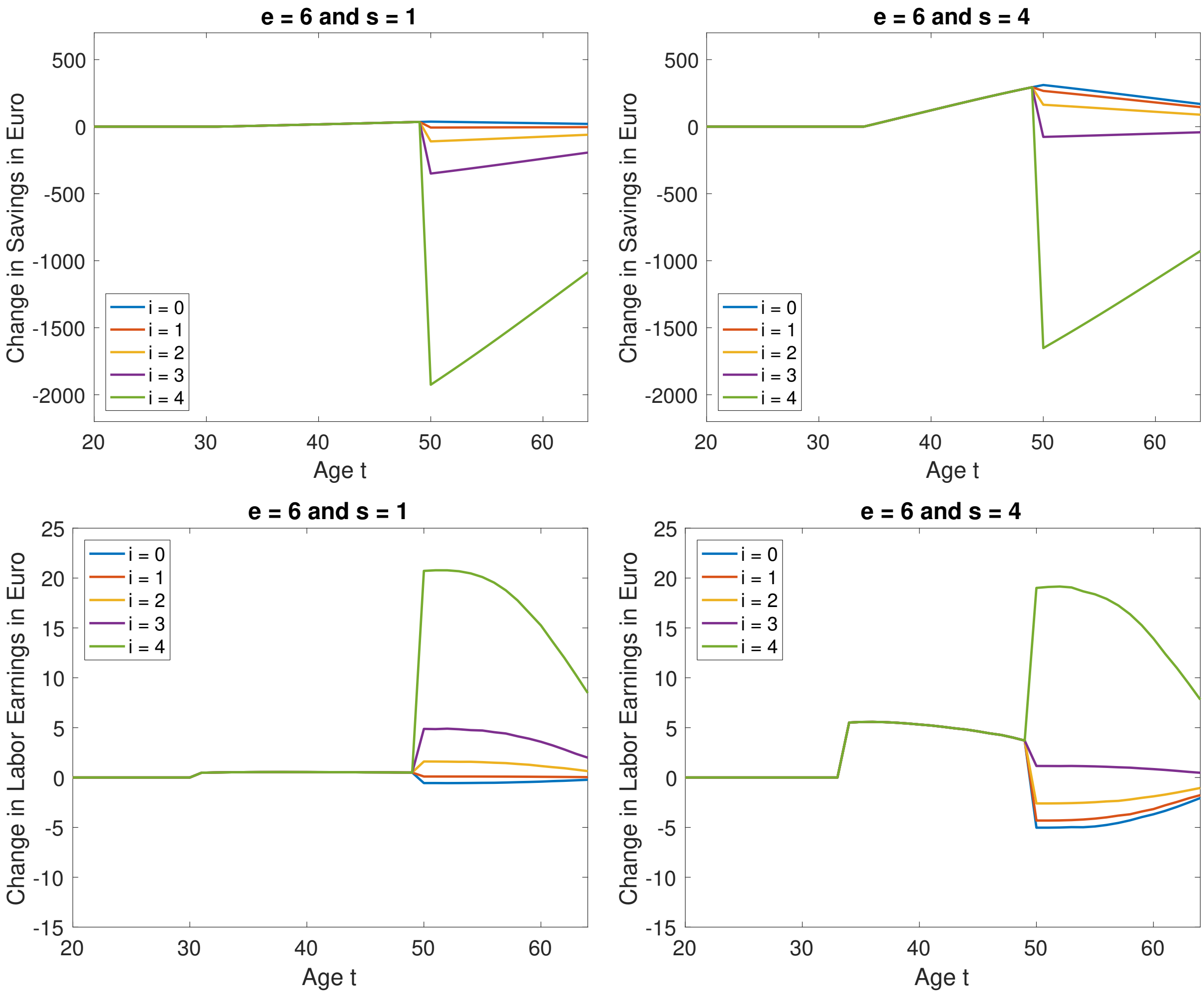
Since it seems to explain the data better, we chose the case of $a_{min} = -\infty$ in our standard parameterization.[18] Nevertheless, as another robustness check we present the results of our policy experiment (increasing the bequest tax from zero to 1%) for $a_{min} = 0$ in Table 9.

Table 9: Effect of a 1% increase in bequest taxes

		Decomposition		
	Total	*Anticipation*	*Heirs*	*Non-Heirs*
Gross Earnings	16.27	6.29	10.13	−0.31
Labor Taxes	6.83	2.66	4.29	−0.13

Effects are measured as fraction of change in bequest tax revenue.

Figure 10: Change in life-cycle behavior of different households



[18] Note that in practice individuals may not only borrow through bank loans but also from their parents.

We observe that our number of interest decreases slightly by about 0.8 cents to 6.83. Furthermore, anticipation effects are lower, while post-receipt effects are higher than with $a_{min} = -\infty$. To understand this result, it is again useful to look at the change in life-cycle profiles of savings and labor earnings depicted in Figure 10. The strict no-borrowing limit allows agents less to smooth consumption and labor supply over the life-cycle. While in the case of $a_{min} = -\infty$ the increase in the bequest tax resulted in an increase of earnings and savings already from the first year, now this is true only from the age onwards, at which the borrowing constraint seizes to bind (30-35 years). As agents save less early in their life at the time they receive an inheritance their wealth is lower than in the case with borrowing, triggering stronger responses thereafter.

D Wealth effect on labor earnings

The dynamic household optimization problem in our model reads

$$V_t(e,s,h_t,W_t) = \max_{c_t,l_t,a_{t+1}} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} - \lambda \frac{l_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}\left[V_{t+1}\left(e,s,h_{t+1},W_{t+1}\right) \middle| e,s,h_t \right] \right\}$$

subject to the budget constraint

$$c_t + a_{t+1} = w_t^e l_t - \mathcal{T}\left(w_t^e l_t\right) + \mathcal{P}_t^e + W_t,$$

where $\mathcal{P}_t^e = 0$ for all workers. We can write the Lagrangean for a working age household as

$$\mathcal{L} = \frac{c_t^{1-\gamma}}{1-\gamma} - \lambda \frac{l_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}\left[V_{t+1}\left(e,s,h_{t+1},W_{t+1}\right)\right] + \mu\left[w_t^e l_t - \mathcal{T}\left(w_t^e l_t\right) + W_t - c_t - a_{t+1}\right].$$

First order conditions with respect to consumption and labor effort are

$$(c_t)^{-\gamma} - \mu = 0 \quad \text{and} \quad \lambda(y_t)^{\chi} = \mu \cdot \left[1 - \mathcal{T}'(y_t)\right] \cdot (w_t^e)^{1+\chi}. \tag{16}$$

Together with the budget constraint, this leads to

$$F(y_t, W_t, a_{t+1}) := \lambda(y_t)^{\chi} - \left[y_t - \mathcal{T}\left(y_t\right) + W_t - a_{t+1}\right]^{-\gamma} \cdot \left[1 - \mathcal{T}'(y_t)\right] \cdot (w_t^e)^{1+\chi} = 0,$$

which implicitly defines labor earnings. The implicit function theorem then implies

$$\begin{aligned}
& \frac{\partial F}{\partial y_t} \cdot dy_t + \frac{\partial F}{\partial W_t} \cdot dW_t + \frac{\partial F}{\partial a_{t+1}} \cdot da_{t+1} = 0 \\
\Leftrightarrow \quad & \left[\chi\lambda(y_t)^{\chi-1} + \gamma(c_t)^{-\gamma-1} \cdot \left[1 - \mathcal{T}'(y_t)\right]^2 \cdot (w_t^e)^{1+\chi} - (c_t)^{-\gamma} \cdot \left(-\mathcal{T}''(y_t)\right) \cdot (w_t^e)^{1+\chi}\right] \cdot dy_t \\
& \quad + \left[\gamma(c_t)^{-\gamma-1} \cdot \left[1 - \mathcal{T}'(y_t)\right] \cdot (w_t^e)^{1+\chi}\right] \cdot dW_t \\
& \quad - \left[\gamma(c_t)^{-\gamma-1} \cdot \left[1 - \mathcal{T}'(y_t)\right] \cdot (w_t^e)^{1+\chi}\right] \cdot da_{t+1} = 0 \\
\Leftrightarrow \quad & \frac{\chi\lambda(y_t)^{\chi-1} + \gamma(c_t)^{-\gamma-1} \cdot \left[1 - \mathcal{T}'(y_t)\right]^2 \cdot (w_t^e)^{1+\chi} + (c_t)^{-\gamma} \cdot \mathcal{T}''(y_t) \cdot (w_t^e)^{1+\chi}}{\gamma(c_t)^{-\gamma-1} \cdot \left[1 - \mathcal{T}'(y_t)\right] \cdot (w_t^e)^{1+\chi}} \cdot dy_t \\
& \quad = -dW_t \cdot \left[1 - \frac{da_{t+1}}{dW_t}\right] \\
\Leftrightarrow \quad & \left[\frac{\chi}{\gamma} \cdot \frac{c_t}{y_t} \cdot \frac{\lambda(y_t)^{\chi}}{(c_t)^{-\gamma} \cdot \left[1 - \mathcal{T}'(y_t)\right] \cdot (w_t^e)^{1+\chi}} + 1 - \mathcal{T}'(y_t) + \frac{c_t}{\gamma} \cdot \frac{\mathcal{T}''(y_t)}{1 - \mathcal{T}'(y_t)}\right] \frac{dy_t}{dw_t} \\
& \quad = -\left[1 - \frac{da_{t+1}}{dW_t}\right]
\end{aligned}$$

From the first order conditions of the household problem, we directly get

$$\frac{\lambda(y_t)^{\chi}}{(c_t)^{-\gamma} \cdot \left[1 - \mathcal{T}'(y_t)\right] \cdot (w_t^e)^{1+\chi}} = 1.$$

Furthermore, using the functional form of our tax function yields

$$1 - \mathcal{T}'(y_t) = (1-\tau_1) \cdot \frac{y_t - \mathcal{T}(y_t)}{y_t} \quad \text{and} \quad \frac{\mathcal{T}''(y_t)}{1-\mathcal{T}'(y_t)} = -\frac{\tau_1}{y_t}.$$

Hence, we obtain

$$\frac{dy_t}{dW_t} = -\frac{1 - \frac{da_{t+1}}{dW_t}}{\frac{\chi+\tau_1}{\gamma} \cdot \frac{c_t}{y_t} + (1-\tau_1) \cdot \frac{y_t - \mathcal{T}(y_t)}{y_t}}.$$

Consequently, we can write the wealth effect on labor earnings in form of an elasticity as

$$\eta_{y,t} = \frac{dy_t}{dW_t} \cdot \frac{W_t}{y_t} = -\frac{W_t - a_{t+1} \cdot \eta_{a,t+1}}{\frac{\chi+\tau_1}{\gamma} \cdot c_t + (1-\tau_1) \cdot [y_t - \mathcal{T}(y_t)]},$$

with $\eta_{a,t+1}$ being the elasticity of savings into the next period with respect to current wealth.

□