Identifying Laffer Bounds: A Sufficient-Statistics Approach with an Application to Germany*

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June 2, 2015

Abstract

We derive a simple sufficient-statistics test for whether a nonlinear tax-transfer system is second-best Pareto efficient. If this is not the case, it is beyond the top of the Laffer curve and there exists a tax cut that is self-financing. The test depends on the income distribution, extensive and intensive labor supply elasticities and income effect parameters. A tax-transfer system is likely to be inefficient if marginal tax rates are quickly falling in income. We apply this test to the German tax-transfer system and find the structure of effective marginal tax rates likely to be inefficient in the region where transfers are phased-out.

JEL-**classification:** H 21, H 23.

Keywords: Income Taxation, Laffer Curve, Pareto Efficiency, Extensive Margin

*Contact: lorenzn@uni-trier.de, sachs@wiso.uni-koeln.de. We thank the editor Matti Liski, two anonymous referees, Friedrich Breyer, Sarah Brockhoff, Sebastian Findeisen, Laurence Jacquet, Mathias Kifmann, Fabian Krüger, Etienne Lehmann, Nick Netzer, Florian Scheuer, Dirk Schindler, Esther Schuch, Jean-François Tremblay and Ivàn Werning as well as seminar participants in Gerzensee and Hamburg and conference participants of the 4th RGS Doctoral Conference in Dortmund, the IIPF in Dresden, and the ESEM in Màlaga for helpful comments and suggestions. Dominik Sachs acknowledges the hospitality of Stanford University where part of this research was undertaken. Dominik Sachs' research was partly funded by a post-doc fellowship of the Fritz-Thyssen Foundation and the Cologne Graduate School in Management, Economics and Social Sciences.

1 Introduction

Although there is considerable disagreement on whether a more equitable tax system requires higher tax rates, there seems to be a consensus among economists that lowering income taxes from current levels does not lead to higher but to lower tax revenue.[1] This consensus regards the effect of lowering marginal tax rates for *all* income levels. But what about different reforms of the tax system? Since real world tax-transfer systems are nonlinear in most countries, there is no reason for economists to focus their analysis on linear tax reforms.

In this paper, we provide a simple sufficient-statistics test that can uncover whether a non-linear tax-transfer system (henceforth TTS) is beyond the Laffer bound and therefore inefficient. We do not only consider the case where marginal tax rates are too high, but also that they may have an inefficient structure. The test also provides guidance on how to reform the TTS in a Pareto improving manner.

The theoretical foundation of our work is the optimal nonlinear income tax literature à la Mirrlees (1971). This literature provides the methods to address the question of whether a TTS is designed efficiently. However, most of this literature has focused on characterizing properties of optimal TTS for a given social welfare function and some given economic primitives (preferences, skill distributions). Recently, this approach has also been inverted to determine for which social welfare function a given TTS is optimal (given the primitives). If the resulting welfare function implies social marginal welfare weights that are negative, the TTS is not second-best Pareto efficient.[2]

Werning (2007) elaborates this idea and provides conditions on the skill distribution and preferences under which a TTS is inefficient. His main conclusion is that from a theoretical point of view *anything goes*; thus, all kinds of TTS can be justified as Pareto optimal for some skill distribution and preferences. Given knowledge about the primitives, he then proposes a theoretical test on whether a given TTS is Pareto efficient.

This paper builds on the work of Werning (2007) and provides a sufficient-statistics test for the efficiency of a TTS. The innovation is that our analysis takes into account extensive labor supply responses and, more importantly, that our test function only depends on measurable statistics.[3] Information about the income distribution, extensive and intensive labor supply elasticities as well as income effect parameters are sufficient to detect an inefficiency of the TTS. Consequently, our empirical test can be easily conducted if information on these sufficient statistics is available.

In addition to the classical Laffer-argument – which states that marginal tax rates are inefficiently high when they are above the revenue-maximizing level – we argue that the *structure of marginal tax rates* can be a source of inefficiency. We show that a TTS is particularly likely to be inefficient if effective marginal tax rates are quickly falling in income.

Motivated by this theoretical observation, we apply our test to the German TTS because it is characterized by rapidly falling effective marginal tax rates at the income threshold where transfers are phased out. Our results indicate that the structure of marginal tax rates in this income region is inefficient and that a Pareto improvement can be achieved by decreasing marginal tax rates below

[1] In the IGM economics experts panel, none of the respondents was in favor of this idea for the US, see IGM (2012). Trabandt and Uhlig (2011) revisit the Laffer curve argument also for most European countries and conclude that decreasing taxes is not self-financing.

[2] It was Saez (2001, p. 221) who first suggested this approach.

[3] In this paper, we use the two terms 'extensive labor supply responses' and 'participation responses' interchangeably.

the threshold, and increasing them above. Since the absolute level of taxes does not increase for any income level, no individual is made worse off, but tax revenue increases due to the induced labor supply responses. Given that effective marginal tax rates are rapidly falling in many countries, where transfers are phased out up to an income threshold, we think that this result may not be confined to Germany, but may apply for other countries as well.

Related Literature Besides the papers already mentioned, our work is related to a recent literature that has inverted the optimal tax approach to income taxation. Bourguignon and Spadaro (2012), who study the French redistribution system, is probably closest to our paper. They find that for some specifications the top tax rate is above its Laffer value. Our analysis differs in that we study an environment with a continuous income distribution, intensive and extensive labor supply responses as well as income effects. Further, we theoretically elaborate conditions for the structure of marginal tax rates to be inefficient, which also motivates our application for the German TTS.

A series of papers applies the inversion of the optimal tax approach more in a positive than in a normative way by asking for which underlying welfare weights a given TTS would be optimal. Using the inverse optimum approach to infer the implicit welfare weights of the government as done in most of these papers implies to assume that the government (i) chooses the tax schedule which maximizes a social welfare function and (ii) uses the correct estimates for the labor supply elasticities. However, under the assumption of (i) and (ii), it is, e.g., difficult to explain the development of the U.S. tax policy over the last 30 years, as (i) and (ii) imply that either labor supply elasticities or the welfare weights have changed dramatically over these years (Lockwood and Weinzierl 2014). Our approach differs from these studies in that our test applies to any tax schedule, no matter what labor supply elasticities the government thinks apply and no matter which objective the government has pursued when deciding on the tax schedule.

Blundell et al. (2009) infer the implicit welfare weights for lone mothers in Great Britain and Germany. They show that the current systems can only be optimal if the governments put much higher weights on the non-working poor than the working poor. Relatedly, Bargain et al. (2014) compare revealed social preferences for 17 EU countries and the US. For this purpose, they also estimate the elasticities for these countries. Whereas they find no significant differences in the elasticities, they do find differences in revealed social preferences. In particular they find preferences that are close to Rawlsian for the Nordic and some continental European countries.[4] These papers work with models with a discrete type space and look at a rather small number of income intervals. By contrast, we consider a continuous income distribution which enables us to examine the structure of marginal tax rates in greater detail.

Zoutman et al. (2014) invert the optimal tax approach for the Netherlands. They find that welfare weights increase until the modal income, decrease afterwards and become slightly negative for top earners, which implies a top tax rate above the Laffer value. Relatedly, Zoutman et al. (2015) study the implied redistributive preferences of different Dutch parties.

From a more theoretical point of view, our Pareto efficiency test is related to the test derived by Scheuer (2014), who considers differential taxation of entrepreneurs and workers. In his model the extensive margin captures the decision of being a worker or an entrepreneur. da Costa and Pereira

[4] In a follow up study, Bargain et al. (2013) investigate the implied social preferences and find some anomalies in the sense that welfare weights on the working poor are rather low. They show that this anomaly would disappear if extensive margin responses were ignored. This suggests that the implicit welfare weights of the governments may indeed not reflect this anomaly but that governments rather underestimated the extensive labor supply elasticities.

(2014) study the properties of tax schedules that satisfy a minimum equal sacrifice rule and use the Pareto efficiency test of Werning (2007) to test whether these schedules are Pareto efficient. Badel and Huggett (2015) study the Laffer bound of the top tax rate and derive a sufficient statistics formula that also captures aspects from dynamic models.

Finally, our paper is also connected to the quantitative macroeconomic literature on the Laffer bound. Trabandt and Uhlig (2011) study the Laffer value for linear tax rates for many European countries and the United States. Considering tax functions that are restricted within a class of two-parameter schedules, Holter, Krueger, and Stepanchuk (2014) study how the Laffer bound changes if the degree of progressivity of the tax function is altered.

To sum up, we connect the studies that invert the optimal income tax approach with the more theoretically motivated paper by Werning (2007) and the macroeconomic literature on Laffer bounds. It is shown how theory provides guidance on where to search for inefficiencies: rapidly falling marginal tax rates. This theoretical guidance motivates our application to the German TTS, for which we find an inefficiency to be likely.

The remainder of this paper is organized as follows. We derive our test in Section 2 and provide our application for Germany in Section 3. Section 4 concludes. Appendix A contains a detailed explanation of the construction of the German TTS.

2 Theoretical Considerations

In this section, we provide the theoretical foundation for our sufficient-statistics test. In Section 2.1, we explain the economic environment and provide some formal results from the literature as a starting point. In Section 2.2, we provide a simple characterization of the Laffer value of marginal tax rates within a nonlinear TTS. In Section 2.3, we show how to identify more subtle inefficiencies of a TTS that are due to an inefficient structure of marginal tax rates.

2.1 Formal Framework

We consider a static framework, where individuals choose their labor supply along the extensive and the intensive margin, giving rise to an income distribution $H(y)$, with corresponding density $h(y)$. The individuals' labor supply decisions depend on the tax-transfer system (TTS) and the underlying primitives. Note, that we do not make any concrete assumptions on these primitives. Thus, our main findings hold more general in the spirit of a sufficient-statistics approach (Chetty 2009): to detect an inefficiency, only the labor supply elasticities, but not the exact nature of the labor supply decisions has to be known.

An individual's gross income y determines the level of income taxes, contributions to social insurance and the transfers of all welfare programs, the sum of which we denote by $T(y)$. The derivative of the TTS, $T'(y)$, therefore denotes the effective marginal tax rate, comprising the welfare transfer phase out, the marginal tax rate of the statutory income tax and marginal contributions to social insurance. This effective marginal tax rate determines the distortion along the intensive labor supply margin.[5]

[5] In the following, we will simply speak of the marginal tax rate instead of the effective marginal tax rate. Also, we will sometimes use the term tax schedule instead of tax-transfer system.

For the extensive margin, what is important is not the marginal tax rate but the absolute increase in taxes when choosing to work over being unemployed. In the literature, this is typically called the participation tax. Formally, for income level y it is defined by $T_{part}(y) = T(y) - T(0)$, where $-T(0)$ denotes the welfare benefit of individuals not working at all.

In general, we could derive our results either in terms of $T(y)$ or in terms of $T'(y)$ and $T_{part}(y)$. As shown in the literature, it is more convenient and intuitive to derive the expressions in terms $T'(y)$ and $T_{part}(y)$, so we follow this approach.

Besides the income distribution, the following statistics are sufficient to determine whether a TTS is beyond the top of the Laffer curve: intensive and extensive labor supply elasticities and income effect parameters. Denote by $\varepsilon(y)$ the elasticity of taxable income with respect to (one minus) the marginal tax rate for individuals that earn income y, i.e., $\varepsilon(y) = \frac{dy}{d(1-T'(y))}\frac{1-T'(y)}{y}$. Note that the group of individuals earning income y may consist of several (or a large number of) different types regarding the primitives, so that $\varepsilon(y)$ captures the average labor supply response of all these types. Likewise, denote by $\xi(y)$ the respective extensive margin semi-elasticity, i.e., $\xi(y) = \frac{dh(y)}{d(-T_{part}(y))}\frac{1}{h(y)}$. Finally, denote by $\eta(y)$ the income effect parameter which captures by how much individuals with income y adjust their income to a marginal increase in their absolute tax payment, i.e. $\eta(y) = \frac{dy}{dT(y)}$.[6] As some simple standard arguments (explained in greater detail below) reveal, optimal marginal tax rates satisfy

$$\frac{T'(y)}{1-T'(y)} = \frac{\int_y^\infty [1 - g(\tilde{y}) + \eta(\tilde{y})T'(\tilde{y}) - \xi(\tilde{y})T_{part}(\tilde{y})]\, dH(\tilde{y})}{\varepsilon(y)h(y)y}, \tag{1}$$

where $g(y)$ is the social marginal welfare weight (measured in terms of public funds) of individuals that earn income y: Giving one dollar to an individual with income y increases welfare (measured in terms of public funds) by $g(y)$.

In what follows, we provide a standard heuristic derivation of this formula. Assume that the bold curve in Figure 1 represents the optimal tax schedule. Then a small perturbation where, in the interval $[y - dy, y]$, T' is increased by dT', must not have any first-order effect on welfare (Piketty 1997, Saez 2001). Implementing such a perturbation will have the following four effects on welfare (always measured in terms of public funds): (i) a substitution effect, (ii) a redistribution effect, (iii) a participation effect and (iv) an income effect. For the tax schedule to be optimal, these four effects have to add up to zero. Note that by the envelope theorem, any labor supply responses incurred by such a perturbation only change welfare by their impact on public funds.

Substitution effect: Individuals within the interval adjust their labor supply along the intensive margin. The mass of individuals within this interval is $h(y)dy$. The average increase in income is given by $\frac{\partial y}{\partial T'(y)}dT' = -\frac{y}{1-T'(y)}\varepsilon(y)dT'$, which, multiplied by $T'(y)$, yields the effect on public funds. The substitution effect in terms of welfare then is

$$dW^S(y) = -\frac{T'(y)}{1-T'(y)}y\varepsilon(y)h(y)dT'dy. \tag{2}$$

Redistribution effect: The increase of the marginal tax rate results in a higher overall tax of $dT'dy$ for all individuals earning more than y and thereby redistributes money from these individuals (valued by $g(y)$) to the government (valued by 1). This redistribution effect on welfare reads as

$$dW^R(y) = dT'dy \int_y^\infty (1 - g(\tilde{y}))dH(\tilde{y}). \tag{3}$$

[6]Note that we define $\varepsilon(y), \xi(y)$ and $\eta(y)$ in such a way that they are positive.

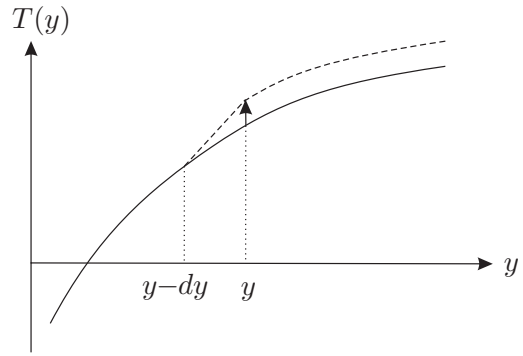


Figure 1: Tax Perturbation

Participation effect: Some of the individuals earning more than y will stop working due to the higher participation tax. For each income level $\tilde{y} \geq y$, their mass is captured by $\frac{\partial h(\tilde{y})}{\partial(-T_{part}(\tilde{y}))} dT' dy = \xi(\tilde{y}) h(\tilde{y}) dT' dy$. By choosing unemployment over employment, for each of these individuals tax revenue is decreased by the participation tax $T_{part}(\tilde{y})$. The participation effect therefore equals

$$dW^P(y) = -dT' dy \int_y^\infty \xi(\tilde{y}) T_{part}(\tilde{y}) dH(\tilde{y}). \tag{4}$$

Income effect: Individuals with earnings above y now face higher absolute taxes. Via an income effect they will increase their labor supply whenever leisure is a normal good. An individual with income y changes its income by $\frac{\partial y}{\partial T(y)} dT = \eta(y) dT' dy$, which, multiplied by T', yields the effect on public funds. Therefore

$$dW^I(y) = dT' dy \int_y^\infty \eta(\tilde{y}) T'(\tilde{y}) dH(\tilde{y}). \tag{5}$$

For the tax function to be optimal, we have to have $dW^S(y) + dW^R(y) + dW^P(y) + dW^I(y) = 0 \ \forall\, y$, or

$$\frac{T'(y)}{1 - T'(y)} \varepsilon(y) h(y) y - \int_y^\infty \left[1 + \eta(\tilde{y}) T'(\tilde{y}) - \xi(\tilde{y}) T_{part}(\tilde{y})\right] dH(\tilde{y}) = -\int_y^\infty g(\tilde{y}) dH(\tilde{y}), \tag{6}$$

yielding (1). Such a formula has been derived by Jacquet et al. (2013) in a framework where individuals respond along the extensive margin due to some fixed costs of work.[7] In the following we will show how this formula can be used to test whether a nonlinear TTS is beyond the top of the Laffer curve.

2.2 Laffer Values of Marginal Tax Rates

We start by considering the natural extension of the Laffer bound to a nonlinear TTS, i.e., we ask under what conditions a decrease of the marginal tax rate at a certain income level is self-financing. Our formal result is summarized in the following proposition.[8]

[7] In an earlier version of this paper, Lorenz and Sachs (2012), we have shown that such a formula also applies for the case where the extensive margin is due to a minimum hours constraint; there, however, we abstracted from income effects. Under the assumption of no income effects, Saez (2002) has derived a discrete version of this formula; the corresponding working paper (Saez 2000) also captures the continuous case.

[8] Bourguignon and Spadaro (2012) have derived a related condition for the case without income effects (Proposition 1 in their paper) and the case with income effects (Proposition 4), but without extensive labour supply responses.

Proposition 1. *For given intensive elasticities $\varepsilon(y)$, extensive semi-elasticities $\xi(y)$, income effect parameters $\eta(y)$ and an income distribution $H(y)$, whenever a tax schedule $T(y)$ satisfies*

$$\frac{T'(y)}{1-T'(y)}\varepsilon(y)h(y)y + \int_y^\infty \left[\xi(\tilde{y})T_{part}(\tilde{y}) - \eta(\tilde{y})T'(\tilde{y})\right] dH(\tilde{y}) - [1-H(y)] > 0 \qquad (7)$$

for at least some y, then the tax schedule is second-best Pareto inefficient.

The first term and the integral in (7) capture the impact of the tax reduction – a perturbation of the tax schedule as shown in Figure 1, but with a decrease instead of an increase of T' – on public funds via (i) the substitution effect, (ii) the participation effect and (iii) the income effect. For this tax cut to be self-financing, the fiscal gains via these changes in behavior have to outweigh the mechanical loss via the reduction in taxes which is captured by $[1-H(y)]$ in (7).

Thus, if (7) is fulfilled, lowering the marginal tax rate at income level y will increase tax revenue; it will also reduce the tax for all income levels above y, which will make these individuals better off. A small reduction of the marginal tax rate at income level y therefore constitutes a Pareto improvement.

The test in Proposition 1 identifies a specific and simple form of inefficiency of nonlinear TTS. In the following section, we derive a test that can uncover more subtle forms of inefficiencies.

2.3 Inefficient Structure of Marginal Tax Rates

Relating (7) to the LHS of (6) shows that having a marginal tax rate at income level y above its Laffer value is equivalent to

$$\int_y^\infty g(\tilde{y})dH(\tilde{y}) < 0.$$

This implies that the government would choose a marginal tax rate above its Laffer value only if it put welfare weights on individuals with income above y which are negative on average. Therefore, (7) is a sufficient condition for Pareto inefficiency: if the average of the weights above some income level y is negative, at least one of the weights has to be negative. The weaker condition – that one of the weights is negative – can be obtained by taking the derivative of (6) with respect to y:

$$\frac{\partial}{\partial y}\left[\frac{T'(y)}{1-T'(y)}\varepsilon(y)h(y)y\right] + h(y) - \xi(y)T_{part}(y)h(y) + \eta(y)T'(y)h(y) = g(y)h(y). \qquad (8)$$

Whenever the left-hand side of (8) is negative, we must have $g(y) < 0$.

In order to be able to use the same test function for both tests, we express this condition using the left hand side of (7) instead of (8):[9]

Proposition 2. *For given intensive elasticities $\varepsilon(y)$, extensive semi-elasticities $\xi(y)$, income effect parameters $\eta(y)$ and an income distribution $H(y)$, a tax schedule $T(y)$ is second-best Pareto inefficient, if*

$$\frac{T'(y)}{1-T'(y)}\varepsilon(y)h(y)y - [1-H(y)] + \int_y^\infty \left[\xi(\tilde{y})T_{part}(\tilde{y}) - \eta(\tilde{y})T'(\tilde{y})\right] dH(\tilde{y}) \qquad (9)$$

[9] This condition is related to Proposition 1 in Werning (2007). The difference is that we take into account extensive labor supply responses and provide a formula in sufficient statistics. It is also related to Corollary 2 in Scheuer (2014) who theoretically characterizes optimal taxes for entrepreneurs and workers.

is decreasing in y for at least one y.

If the test indicates that a tax schedule is inefficient, then a reform as depicted in Figure 2, conducted at income level y, will yield a Pareto improvement.[10] Such a reform will then be self-financing or even increase tax revenue. Using equation (8) makes it easy to see when this will be the case: The mechanical loss in tax revenue is given by $h(y)$, the mass of individuals affected by the tax cut. The participation effect on public funds is captured by the third term on the left hand side of (8): The larger $T_{part}(y)$ and the larger the participation semi-elasticity $\xi(y)$, the larger this participation effect will be and the more likely it is that such a tax cut is self-financing. The income effect, captured by the fourth term on the left hand side of (8), works in the opposite direction: if leisure is a normal good, we have $\eta(y) > 0$ and individuals at this income level work less. Thus, the stronger the income effect, the less likely it is that such a tax cut is self-financing.

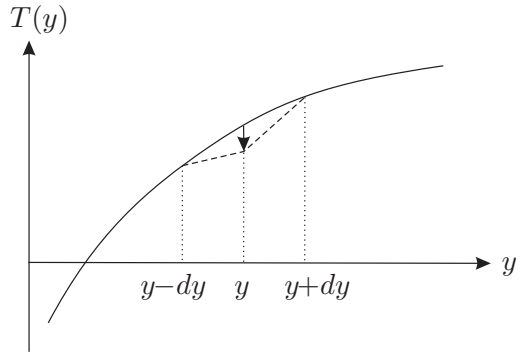


Figure 2: Tax Perturbation

The last effect is the substitution effect, which is more involved. A reform as depicted in Figure 2 decreases marginal tax rates for incomes slightly lower than y, and increases them for incomes slightly higher than y (see Figure 2). In the limit, (i.e., considering an infinitesimal tax cut at income y), the overall impact of these labor supply responses is captured by the derivative of the substitution effect, the first term on the left hand side of (8). The positive effect on public funds induced by the labor supply increase of those with slightly lower income than y is more likely to outweigh the negative effect of the labor supply decrease of those with slightly higher income than y if the marginal tax rate, the density or the elasticity is decreasing in income.

Thus, an inefficiency of the tax-transfer system is likely whenever the marginal tax rate, the density or the elasticity (strongly) decreases in income at a certain income level. This theoretical observation motivates the empirical application in the next section, where we analyze the German TTS which is characterized by a sharp decline in marginal tax rates as soon as welfare benefits are completely phased-out.

3 Application to Germany

In order to apply the Pareto efficiency test, the tax-transfer schedule has to be known. For Germany (and likely for other countries as well), it is not immediately apparent what this schedule looks like because it is the result of the interplay of three different systems. We discuss how to construct this

[10] For the case without extensive labor supply responses, such a reform has been proposed by Werning (2007).

schedule in the following Section 3.1; there we also show how we estimate the income distribution. The tests of Propositions 1 and 2 are applied in Section 3.2. Policy implications are discussed in Section 3.3.

3.1 Tax-Transfer System, Income Distribution, and Elasticities

As in most countries, the tax-transfer system conditions on marital status as well as on the number of children. As the taxation of families raises a number of additional issues, we focus on singles without children. In addition, eligibility for welfare benefits depends on assets. Therefore, we only consider individuals with sufficiently low assets so that eligibility for welfare benefits is ensured.

The tax-transfer system results from the interplay of three different systems: the income tax, the welfare benefit system ('Hartz IV') and social insurance. Several steps (presented in detail in Appendix A.1) are necessary to derive the effective tax schedule; here, we only sketch the main steps: Gross income determines payments to the social insurance system according to the Social Security Code. Gross income and social insurance contributions then determine the tax liability according to the Personal Income Tax Code. Transfers then depend on gross income, taxes and social insurance contributions. Integrating the three systems, we arrive at the schedule of effective marginal tax rates for the year 2011 as shown in Figure 3.[11] Marginal tax rates are very high for low incomes, but as soon as transfers are phased out, marginal tax rates decrease drastically.[12]

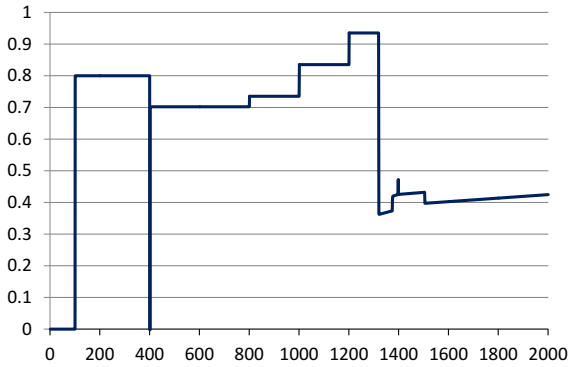


Figure 3: Marginal tax rates as a function of monthly income for the year 2011

In contrast to other studies (like Sinn et al. 2006), the highest phase-out rate is below 100%. This is because we consider contributions to the pension system not purely as a tax, as there is a Bismarckian pension system in place in Germany, see OECD (2011). Although the rate of return in the pension system is likely to be very low, it seems reasonable to assume that individuals will (on average) receive at least half of their marginal contributions as higher pensions; this reduces the effective marginal tax rate by about five percentage points.[13]

[11] In 2011 there was a reform of the welfare benefit system ('Hartz-IV-Reform 2011') that was explicitly targeted to the phase out region, where marginal tax rates were reduced. We present the results for 2011, i.e., after the reform.

[12] There is a small downward jump in the tax schedule at 400 euro, which is why T' tends to $-\infty$ at this income level. Also, there is a small spike at 1,398 euro, which is due to the way the tax formula is stated in the tax code. Both are of second-order importance, so we do not further comment on them. For income levels between 1,375 and 1,504 Euro, marginal tax rates are shifted upwards because of the solidarity surcharge which is phased in at a higher rate in this interval.

[13] The result of an inefficient structure of the marginal tax rates is robust with regard to how contributions to the pension system are taken into account.

To estimate the income distribution we use data of the German Socio-Economic Panel (SOEP), which is a representative sample of German households that are interviewed annually, see Wagner et al. (2007). Our sample for the year 2011 (of singles, aged 18 to 65, out of education, and with sufficiently low assets) consists of 627 observations. The minimum and maximum value of gross monthly income are 0 and 13,921 Euro. The mean income is 1,977 Euro (2,375 Euro if restricted to positive incomes).

We estimate the density of the income distribution nonparametrically (using the standard SOEP weights), employing an Epanechnikov kernel and Silverman's rule of thumb to determine the bandwidth, see Fan and Gijbels (2003). Because results for the Pareto efficiency test are basically identical for different values of the bandwidth, we refrain from applying any cross-validation procedure to determine the bandwidth. The distribution of monthly gross incomes is shown (up to 10,000 Euro) in Figure 4.

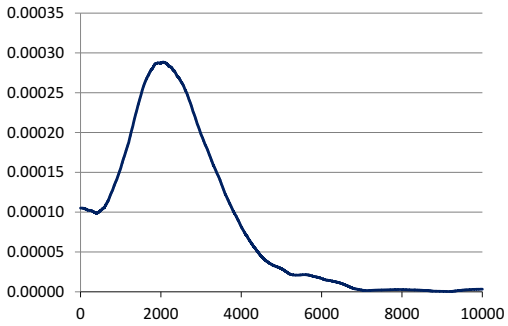


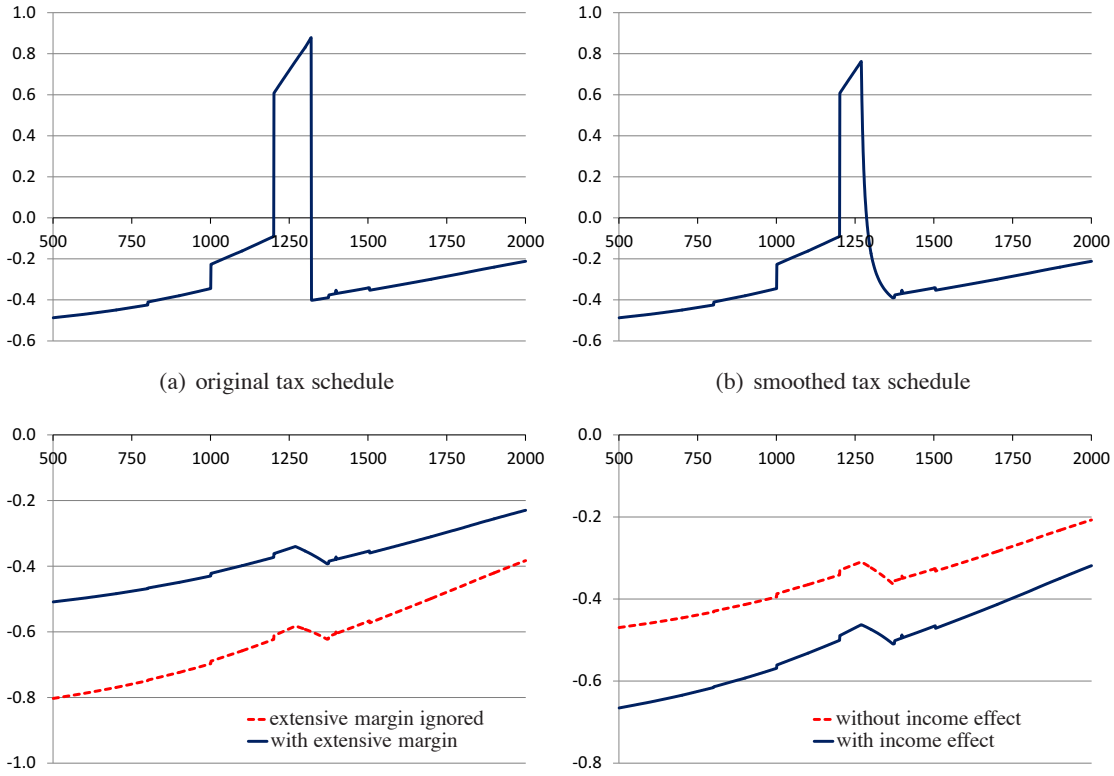
Figure 4: Density of the income distribution for the year 2011

For the elasticities we apply a range of values of the empirical literature. For the benchmark case we use 0.25 for the extensive elasticities (which we denote by ν), $\varepsilon = 0.33$ for the intensive elasticities (see Chetty et al. 2011) and 0.1 for the income effect parameter η (see Imbens et al. 2001). However, our main result holds for a large set of values (see below).

3.2 Results

Using the elasticities of the benchmark case and the income distribution as shown in Figure 4, the test function given in (7) and (9) applied to the German TTS is shown in Figure 5(a). Clearly, the test function is greater than zero in the interval 1201 to 1391 Euro and is decreasing at 1319 Euro, indicating inefficiencies with respect to both versions of the test. However, these results require the following qualification: Because there is a downward jump of the marginal tax rate at 1319 Euro, we would expect individuals not to choose income levels close to that jump.[14] One reason why we do not see a gap in the income distribution is that welfare benefits ("Hartz IV") are calculated on a monthly basis, which implies that the jump in marginal tax rates applies to monthly income. We derived the TTS in Section 3.1 under the assumption that monthly income is constant. However, if monthly income varies, it might well be that average monthly income is close to the jump in marginal tax rates even though in each single month individuals do not choose a point close to the jump.

[14] With the usual convex indifference curves in leisure-consumption-space and a kink in the budget line there will be no tangency solution close to the kink.



(a) original tax schedule

(b) smoothed tax schedule

(c) $\varepsilon(T') = 0.5 - 0.5T'$

(d) with and without income effect of $\eta = 0.5$

Figure 5: Graph of the test function (7) and (9): (a) original tax schedule, $\varepsilon = 0.33$, $\nu = 0.25$, $\eta = 0.1$; (b) smoothed tax schedule, same elasticities as in (a); (c) smoothed tax schedule, $\varepsilon(T') = 0.5 - 0.5T'$, $\eta = 0.1$, with and without participation effect of $\nu = 0.25$; (d) smoothed tax schedule, $\varepsilon(T') = 0.5 - 0.5T'$, $\nu = 0.25$, with and without income effect of $\eta = 0.5$.

To not hinge our results on the discontinuity in marginal tax rates (which would imply an interval of zero density), we smooth the marginal tax rates within an interval around this downward jump. Importantly, our main result does not depend on the length of this smoothing interval. Figure 6 shows marginal tax rates with a smoothing interval of 100 Euro, 50 Euro below and above the jump, which we use in the following analysis. We consider larger smoothing intervals in Section 3.2.2 and show that our main result holds for intervals with length up to 300 Euro.

3.2.1 Laffer Values of Marginal Tax Rates

Figure 5(b) shows our test function for the TTS with smoothed marginal tax rates as depicted in Figure 6 and with the baseline (semi-)elasticities and income effects stated at the end of Section 3.1. For the interval where marginal tax rates are about 95%, they are above their Laffer value, because the test function is larger than zero.[15]

[15] As in most data sets, (very) high income levels are underrepresented in the SOEP data we use. Taking a higher number of very high income earners into account (by, e.g., appending a Pareto tail) would slightly weaken the case for the marginal tax rates being above their Laffer values unless the participation effect is very high (see equation (7)). However, our main result that the structure of marginal tax rates is inefficient, is independent of any underrepresentation of high incomes; this can be seen from equation (8), which does not depend on the share of high income earners, but only on how the local density evolves around the income level considered.

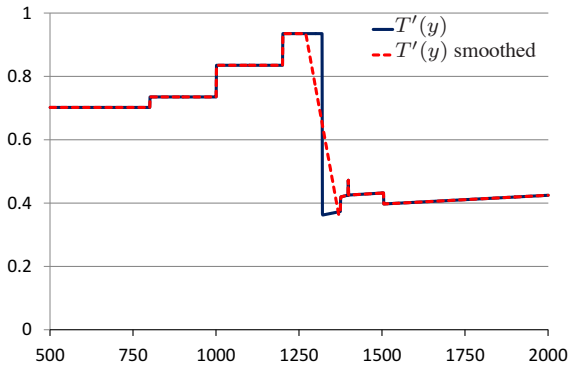


Figure 6: Marginal Tax Rates and smoothed Tax Rates as Function of Monthly Income for the year 2011

However, with a constant elasticity that does not depend on the level of the marginal tax rate the percentage increase in income due to a 1 percentage point increase in T' strongly increases in T'. For example, a decrease in T' from 95% to 94% induces a relative increase in income that is 10 times as high as for a decrease from 50% to 49%. This would imply a 10 times as large semi-elasticity. Such huge differences in the semi-elasticities might be considered too large. As a robustness check, we therefore also apply our test for the case of a constant semi-elasticity. To do so, we let the elasticity decrease linearly in T' from 0.5 to 0, i.e., we assume $\varepsilon(T') = 0.5 - 0.5T'$.[16] In this case, the inefficiency according to Proposition 1 vanishes, see the upper of the two curves in Figure 5(c), as our test function is now below zero.[17]

3.2.2 Inefficient Structure of Marginal Tax Rates

As the results from Figure 5 show, whether the German tax-transfer system passes the test of Proposition 1 is sensitive with respect to the elasticities. However, in both cases (Figure 5(b) and (c)) the test function is falling, so the test shows an inefficiency according to Proposition 2. This is a robust result, as it holds for large ranges of of elasticities: E.g., the lower of the two curves in Figure 5(c) depicts the test function with $\xi = 0$, i.e., when ignoring the extensive margin. In Figure 5(d), we depict the test function assuming either no income effects (the upper of the two curves) or very large income effects of $\eta = 0.5$ (the lower of the two curves). Clearly, the test function shows an inefficiency for very different elasticities. This is because the marginal tax rate is falling so quickly that the sign of equation (7) is dominated by the first term.

Finally, we determine whether the results also hold for different lengths of the smoothing interval. The larger this interval, the more slowly marginal tax rates decrease in income, so the more likely it is that the inefficiency according to Proposition 2 vanishes. However, even for considerably larger intervals the inefficiency remains; it only disappears for smoothing intervals larger than about 300 Euro.[18]

The decrease of marginal tax rates after transfers are phased out, as it is observed in many coun-

[16] For the intermediate value of $T' = 0.33$ we get the intensive elasticity $\varepsilon = 0.33$ that we assumed before.

[17] The inefficiency vanishes if ε decreases from 0.5 to some level x with $x \in [0, 0.1]$, i.e., for $\varepsilon(T') = 0.5 - (0.5 - x)T'$. Therefore the inefficiency exists as long as ε remains above 0.1 for all levels of T'.

[18] E.g., for the elasticities of the upper of the two curves in Figure 5(c), $\varepsilon(T') = 0.5 - 0.5T'$, $\nu = 0.25$, $\eta = 0.1$, the exact length of the smallest smoothing interval with no inefficiency is 306 Euro.

tries, has already been criticized by Kaplow (2007, p. 304). Referring to results from numerical simulations based on utilitarian welfare functions, he argues that marginal tax rates in the phase-out region are too high, and too low afterwards. We show that his results hold for Germany, too, but make the argument stronger since the tax-transfer system is (second-best) Pareto inefficient and can therefore not be justified by any welfare function.

3.3 Possible Reforms and Policy Implications

The German tax-transfer system has often been criticized for its disincentives to work for individuals with low incomes. One proposal has been to lower marginal tax rates in the phase-out region, financed by a decrease in the welfare benefit (Sinn et al. 2006). For individuals who cannot find a job this proposal also included a guaranteed job offer in the public sector; if accepted, transfers would then be as high as before the reform. Such a reform would increase employment, but its welfare consequences are ambiguous because at least some of the welfare recipients are worse off.

Our analysis suggests that the high marginal tax rates in the phase-out region are indeed hard to justify. If intensive elasticities in this region are above 0.1, they might even be above their Laffer value, which implies that slightly lowering them would increase tax revenue.

More importantly, the second test identifies an inefficient structure of marginal tax rates and this result holds for a large range of elasticities. We therefore conclude that there is room for a Pareto improving reform, where phase-out rates of the welfare benefits ('Hartz IV') are decreased which increases the threshold up to which individuals receive a transfer; as these new transfer recipients would then face transfer phase-out rates as well, their effective marginal tax rate would be increased. Since the absolute level of taxes does not increase for any income level, no individual is made worse off, but tax revenue increases due to the induced labor supply responses along the intensive and extensive margin.

Such a small reform typically does not imply that a Pareto optimum is achieved but only that the new allocation is closer to the Pareto frontier. To study large reforms of the TTS which can reach the Pareto frontier, one has to impose structural assumptions in order to predict labor supply responses due to substantial changes of the tax schedule.

4 Conclusion

We have derived a simple test for whether a given nonlinear tax-transfer system is beyond the top of the Laffer curve. The test only requires information on the income distribution, extensive and intensive margin elasticities as well as income effect parameters. A novel theoretical consideration is that a tax-transfer system is likely to be inefficient if marginal tax rates are quickly falling in income.

We have applied our test for the German tax-transfer system and took into account the interplay between the income tax, the welfare benefit system and the social insurance contributions. In the region where the welfare benefits are just phased out, effective marginal tax rates heavily decrease and our analysis indicates an inefficiency.

It is likely that this argument does not only apply to Germany but is more general. For example, Kotlikoff and Rapson (2006) find similar patterns of effective marginal tax rates for the United States when taking several welfare programs and their phase-out into account.

Lastly, our sufficient-statistics test provides guidance on how small tax reforms can lead to a Pareto improvement. Such small reforms will typically not reach the Pareto frontier, but studying larger reforms requires imposing structural assumptions in order to predict labor supply responses to major changes in the TTS. We leave this for future research.

A Appendix

A.1 Derivation of the effective tax schedule

In the following, the capital letters in parenthesis refer to columns in the Excel file containing the calculation of the effective tax schedule; the file can be downloaded at `http://www.dominiksachs.com/research.htm`.

An individual with gross income y (column A) may have to pay contributions to the pension fund (C to F), the sickness fund (G to J), the long term care fund (K to N) and to the unemployment insurance (O to R).[19]

The individual may also have to pay income taxes. The income tax schedule including the solidarity surcharge (AF to AP) is applied to the taxable income (AE) which is derived by subtracting several tax deductibles (S to AB, including part of the contributions to the social insurance system) from gross income (A).

An individual without income receives the welfare benefit of 675 Euro (AQ). If the individual earns a positive gross income (A), this welfare benefit is reduced by a fictitious 'net'-income (AZ): this 'net'-income equals gross income (A) minus a deductible (AW, which itself depends on gross income), the income tax (AR) and the contributions to the social insurance system (AS).[20]

The actual net income of the individual (including the welfare benefit) can be found in column BC. It is given by the income dependent welfare benefit (BA) – i.e. the constant benefit of 675 Euro minus the fictitious 'net'-income – plus gross income, minus contributions to the social security system, minus income taxes. Column BD then contains the marginal tax rate that results from this actual income.

Finally, one has to decide how to take contributions to the pension fund into account: Since there is a Bismarckian pension system in place in Germany, see OECD (2011), higher contributions imply higher pension benefits (with an approximately linear relationship between contributions and benefits). However, due to the demographic situation in Germany this Bismarckian pension system may not be sustainable, in which case higher contributions may not increase benefits. Column BG contains the effective marginal tax rate if contributions are considered purely as a tax, and column BO if contributions increase benefits by an equal amount; column BK contains the intermediate case where only half of the contributions are considered to be a tax. This is the effective marginal tax schedule that we use for our Pareto efficiency test.

[19] In each case, the first column contains the contribution if income is below 400 Euro, the second if it is above 400 but below 800, and the third if it is above 800 Euro.

[20] To be more precise, this 'net'-income is the maximum of zero and the gross income (A) minus the deductible (AW), the income tax (AR) and the contributions to the social insurance system (AS), i.e., if this 'net'-income is below zero, it is set to zero.

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